The Intuitionistic Fuzzy Set Approach for Gray Level Image Thresholding using Normalized Graph cuts

S. Narayanamoorthy\textsuperscript{1} and P. Karthick\textsuperscript{2}
\textsuperscript{1,2}Department of Mathematics, Bharathiar University, Coimbatore - 641 046, India.
\textsuperscript{1}snm\_phd@yahoo.co.in and \textsuperscript{2}karthickphd91@gmail.com

February 3, 2017

Abstract

A novel thresholding algorithm is introduced in this research paper to determine the optimal threshold value for gray level image using Atanassov’s intuitionistic fuzzy sets and Normalized graph cut measure. Atanassov’s intuitionistic fuzzy index values have been used to representing the degree of unknowledge/ ignorance of an expert on finding whether a pixel of the image belongs to the background or the object of the image. Then we are selecting from a set of threshold of an image the best one. This process is based on the concept of normalized graph cut measure. Numerical results also represent the our algorithm is a better thresholding principle compared to the existing methods.

AMS Subject Classification: 03F55, 05C85, 68R10, 03E72, 68U10.

Key Words and Phrases: Intuitionistic fuzzy set, graph cut, image thresholding.
1 Introduction

Thresholding segmentation for image based on fuzzy set theory, to authorize effectively membership degree for pixels is mainstream [6, 4]. In order to determine this issue, Bustine et al. [3] fixed these numerical values of the associated pixels. A pixel has a value of zero, it means that the pixel is either belonging to the background or to the object. But, if the expert must represent its membership value to both with value 0.5 he does not know that the pixel belongs to the background or to the object. Under these reason expert use the Atanassov’s Intuitionistic Fuzzy Sets (A-IFSs) [1, 5, 9]. Graph partitioning methods can effectually be used for image thresholding [2].


In this paper, we use A-IFSs and normalized graph cut measure as the thresholding principle to separate objects from the background. Related to the existing techniques, the suggested method formulates the fuzzy sets of background and object then find the Atanassov fuzzy index \( \pi \). Using this \( \pi \) value we calculate A-IFSs and then measure the weighted graph by treating each pixel as a vertex and joining each pair of pixels by an edge. The purpose of this paper is to advance a simple and effective thresholding approach using IFSs. Such thresholding value is obtained by representing a graph using a symmetrical matrix based on gray levels and their corresponding pixel’s membership values.

2 General Framework for Intuitionistic Fuzzy Sets and Normalized Graph Cuts based Image Threshold

In this section, we prefer a novel approach which is represent by the pursuing steps:

a) An image \( X \) of \( I \times J \) dimensions and \( L \) denote the gray levels of the image \( X \), i.e. \( L = \{0, 1, ..., 255\} \). Let \( V = \{(i, j) : i \in I = 0, 1, ..., L - 1; j \in J = 0, 1, ..., L - 1\} \) and \( x(i, j) \) be the gray level.
value of the image at pixel \((i, j)\). \(V\) and \(x(i, j)\) satisfy

\[
x(i, j) \in L \quad \forall (i, j) \in V.
\]

\((1)\)

\[
V_k = \{(i, j) : x(i, j) = k, (i, j) \in V\}, \quad k \in L
\]

\((2)\)

\[
\bigcup_{k=0}^{255} V_k = V, \quad V_j \cap V_k = \emptyset, \quad k \neq j, \quad k, j \in L.
\]

\((3)\)

b) For any \(t (0 \leq t \leq 255)\), we can attain a unique bipartition \(V=(A, B)\) of the corresponding graph \(G=(V, E)\), where sets \(A\) and \(B\) can be generated as

\[
A = \bigcup_{k=0}^{t} V_k, \quad B = \bigcup_{k=t+1}^{255} V_k, \quad k \in L.
\]

\((4)\)

c) Assign fuzzy sets \(\mu_{QBt} (A)\) associated with the background and fuzzy sets \(\mu_{QOt} (B)\) associated with the object. The gray level \(t\) of the grayscale \(L\) used to obtain these fuzzy sets. The restricted dissimilarity functions are defined the membership functions of these sets and the expressions are:

\[
\mu_{QBt}(q) = F \left( d \left( \frac{q}{L-1}, \frac{m_B(t)}{L-1} \right) \right)
\]

\((5)\)

\[
\mu_{QOt}(q) = F \left( d \left( \frac{q}{L-1}, \frac{m_O(t)}{L-1} \right) \right)
\]

\((6)\)

where

\[
m_B(t) = \frac{\sum_{q=0}^{t}qh(q)}{\sum_{q=0}^{L}h(q)}
\]

\((7)\)

\[
m_O(t) = \frac{\sum_{q=t+1}^{L-1}qh(q)}{\sum_{q=t+1}^{L-1}h(q)}
\]

\((8)\)

and

\[
F(x) = 1 - 0.5x.
\]

being \(h(q)\) the number of pixels of the image with the gray level \(q\).

d) The following expression is used to calculate \(\pi\):

\[
\pi(q) = (1 - \mu_{QBt}(q))(1 - \mu_{QOt}(q))
\]

\((9)\)
e) Construct an A-IFSs, using $\pi$, with each one of the fuzzy sets $Q_{Bt}$ and $Q_{Ot}$.

$$\tilde{Q}_{Bt} = \{(q, \mu_{\tilde{Q}_{Bt}}(q), \nu_{\tilde{Q}_{Bt}}(q)) \mid q = 0, 1, ..., L - 1\}, \text{ given by}$$

$$\mu_{\tilde{Q}_{Bt}}(q) = \mu_{Q_{Bt}}(q)$$

$$\nu_{\tilde{Q}_{Bt}}(q) = 1 - \mu_{\tilde{Q}_{Bt}}(q) - \pi(q) = (1 - \mu_{Q_{Bt}}(q)) \cdot \mu_{Q_{Ot}}(q)$$

and

$$\tilde{Q}_{Ot} = \{(q, \mu_{\tilde{Q}_{Ot}}(q), \nu_{\tilde{Q}_{Ot}}(q)) \mid q = 0, 1, ..., L - 1\}, \text{ given by}$$

$$\mu_{\tilde{Q}_{Ot}}(q) = \mu_{Q_{Ot}}(q)$$

$$\nu_{\tilde{Q}_{Ot}}(q) = 1 - \mu_{\tilde{Q}_{Ot}}(q) - \pi(q) = (1 - \mu_{Q_{Ot}}(q)) \cdot \mu_{Q_{Bt}}(q)$$

f) Using the brightness of the pixels, spatial location of the pixels, we can define the weight of the graph edge joining two vertices $u$ and $v$ as (12), where $F(u)$ and $X(u)$ are the gray scale and spatial location value of vertex $u$, respectively, and $\| . \|_2$ denotes the vector norm. $d_I$ and $d_X$ are positive scaling factors that determine the sensitivity of $w(u, v)$ to the intensity difference and spatial location between two vertices, respectively, and $r$ is a positive integer that indicate the number of adjacent vertices involved the weight computations.

$$w(u, v) = \begin{cases} e^{-\left[ \frac{\|F(u)-F(v)\|_2}{d_I} + \frac{\|X(u)-X(v)\|_2}{d_X} \right]}, & \text{if } \|X(u)-X(v)\|_2 < r \\ 0, & \text{otherwise} \end{cases}$$

(12)

g) Construct the weighted graph $G=(V, E)$ by taking each pixel as a vertex and joining each pair of pixels by an edge. The weight on an edge should resonate the prospect that the two pixels belong the same object.

$$\text{cut}(A, B) = \sum_{i=0}^{t} \sum_{t+1}^{255} \text{cut}(V_i, V_j)$$

(13)

where

$$\text{cut}(V_i, V_j) = \sum_{u \in V_i, v \in V_j} w(u, v) \ast (1 - \mu_{Q_{Bt}}(u)) \cdot \mu_{Q_{Ot}}(v)$$

(14)
is the total collection between all vertices in $V_i$ (whose gray level is $i$) and all vertices in $V_j$ (whose gray level is $j$). Similarly

$$asso(A, A) = \sum_{u \in A, v \in A} w(u, v) \times \mu_{QB}^B(u) \times \mu_{QB}^B(v)$$

$$= \sum_{i=0}^t \sum_{j=i}^t \left[ \sum_{u \in V_i, v \in V_j} w(u, v) \times \mu_{QB}^B(u) \times \mu_{QB}^B(v) \right]$$

$$= \sum_{i=0}^t \sum_{j=i}^t \text{cut}(V_i, V_j)$$  \hspace{1cm} (15)$$

$$asso(B, B) = \sum_{i=t+1}^{255} \sum_{j=i}^{255} \text{cut}(V_i, V_j)$$  \hspace{1cm} (16)$$

Therefore,

$$Ncut(A, B) = \frac{\text{cut}(A, B)}{asso(A, A) + \text{cut}(A, B) + \text{cut}(A, B)}$$

$$+ \frac{\text{cut}(A, B)}{asso(B, B) + \text{cut}(A, B) + \text{cut}(A, B)}$$  \hspace{1cm} (17)$$

3 Selection from Kittler method, a Graph cut based segmentation method and the One Obtained with A-IFSs

In this section, we will select the best threshold of the image $X$ from a set of three thresholds: The one obtained with the classical Kittler method (see [8]), the one achieved with the graph cut based segmentation described in [11] and the one obtained with above method.

In Figs. 1 and 2, we present a sample composed of 2 images and their corresponding binary images using the methods Kittler, Graph and A-IFS. The Table.1 shows the numerical values corresponding, in each case, to the similarity between the Ncut constructed from the corresponding threshold (Kittler, Graph cut and
Figure 1: Performance comparison for Lena image

Figure 2: Performance comparison for Cameraman image
A-IFS). We have indicated in bold type the lowest Ncut value. We would point out that the best threshold selection carried out by above Algorithm that visually segments background from the object.

<table>
<thead>
<tr>
<th>Image</th>
<th>Kittler</th>
<th>Graph cut</th>
<th>A-IFS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>T</td>
<td>Ncut</td>
<td>T</td>
</tr>
<tr>
<td>Lena</td>
<td>36</td>
<td>0.2681</td>
<td>128</td>
</tr>
<tr>
<td>Cameraman</td>
<td>34</td>
<td>0.2363</td>
<td>70</td>
</tr>
</tbody>
</table>

Table 1. Ncut measure for Kittler, Graph cut and A-IFS

4 Conclusion

In this research paper we have presented a method that deals the difficulty encountered by the intuitionistic indices of the A-IFS and Normalized graph cut measure. Considering the experimental results, we can conclude that, the algorithm proposed in this paper gives us with better results than the existing methods via Ncut values. This happens for the two reasons, 1. The use of Atanassov fuzzy index values in the construction of membership values of the image. 2. The representation of the ignorance/unknowledge of the image through the intuitionistic fuzzy matrix N.

References


