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HAAR HUNGARIAN ALGORITHM TO SOLVE FUZZY ASSIGNMENT PROBLEM

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Abstract. Assignment problem is a well known topic and is used very often in solving problems of engineering and management sciences. If the parameters in the assignment problem are uncertain to determine, then the problem is said to be an assignment problem with fuzzy parameters or fuzzy assignment problem. In this article a new algorithm called Haar Hungarian algorithm is proposed. In this algorithm first the fuzzy parameters are converted in to Haar tuples using Haar wavelet technique and Hungarian algorithm is applied to get the solutiond.

1. **Introduction**

Assignment problem plays an important role in industry and other applications. In an assignment problem 'n' jobs are to be performed by 'n' persons depending upon their efficiency to do the job. The basic assumption in the assignment problem is that one person can be assigned exactly one job and also each person can do at most one job. The main objective of the assignment problem is to find the optimum assignment so that the total cost of performing all jobs is minimum or the total profit is maximum. If the parameters in the assignment problem are uncertain to determine, then the problem is said to be an assignment problem with fuzzy parameters or fuzzy assignment problem.

In this paper, assignment problem with fuzzy costs $\left(\tilde{C_{ij}}\right)$ is investigated. Assignment problems with fuzzy parameters have been studied by several authors, such as Balinski and Gomory [2], Chi-Jen Lin and Ue-pyng Wen [4], Dubois and Fortemps [9]. Chen [3] proposed a fuzzy assignment model that considers all individuals

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to have same skills and proved certain theorem related to fuzzy assignment problems. Wang [17] solved a fuzzy assignment problem in which the cost depends on the quality of the job. Mukerjee and Basu [12] proposed a new method for solving fuzzy assignment problems. Amit kumar and Anil gupta [1] proposed two methods for solving fuzzy assignment problems and fuzzy Travelling salesman problem. Dubois and Fortemps [9] proposed a flexible assignment problem, which combines with fuzzy theory, multiple criteria decision making and constrain-directed methodology. Long Sheng Huang and Guang-hui Xu [11] proposed a solution procedure for the assignment problems with restriction of qualification. S.Dhanasekar et al. [5] proposed a fuzzy Hungarian algorithm with element-wise subtraction of fuzzy numbers and also discussed about branch and bound algorithm to solve fuzzy assignment problem in [6]. Y.L.P.Thorani et al. [16] proposed algorithms in classical and linear programming for fuzzy assignment problem with fuzzy cost based on the ranking method.

P.Pandian and K.Kavitha [15] proposed parallel moving method to find an optimal solution to fuzzy assignment problem. Nagoor Gani et al[14] transformed the fuzzy assignment problem into crisp assignment problem in the LPP form and solved by using LINGO 9.0. S.Kar et al [10] solved the fuzzy assignment problem by modified Fuzzy Extremum Difference Method (FEDM) for initial Basic Feasible Solution (BFS) and to test the optimality Modified Distribution Method (MODI) method is used. S.Dhanasekar et al.[7] proposed fuzzified version of diagonal optimal algorithm to solve fuzzy assignment problem.

The choice of a ranking method is important in decision making. There are several ways to rank fuzzy numbers and there is no unique way to order the fuzzy numbers using existing ranking techniques. Furthermore some of the ranking techniques are giving different ranking orders in different α-cuts. Decision makers must consider the various characteristics of ranking methods in determining whether the chosen fuzzy ranking methods can support the features of decision making problems.

In this paper the ranking technique based on Haar wavelet is used [8], which satisfies the properties of compensation, linearity and additivity.. This ranking technique converts fuzzy number into an ordered pair of numbers so that we apply arithmetic operations of ordered pairs. The fuzzzification from the defuzzied value is very easy in this proposed ranking method. In this paper the Haar Hungarian method is used to solve assignment problem with fuzzy parameters.

In section-2 relevant definitions are given. The theorems related to the fuzzy Hungarian algorithm are also discussed in Section 2. Section 3 deals with the proposed new algorithm. Several examples are given in Section 4. Section-5 deals with the conclusion.

2. **Preliminaries:**

Definition 1. A *fuzzy set* can be mathematically constructed by assigning each possible individual in the universe of discourse to a value representing its grade of membership.

Definition 2. A *fuzzy number* $\tilde{A}$ is a fuzzy set whose membership function $\mu_{\bar{A}}(x)$ satisfies the following condition

(1) $\mu_{\tilde{A}}(x)$ piecewise continuous
(2) $\mu_{\tilde{A}}(x)$ is convex
(3) $\mu_{\tilde{A}}(x)$ is normal (i.e.,) $\mu_{\tilde{A}}(x_0) = 0$.

Definition 3. A fuzzy number $\tilde{A} = (a, b, c)$ with membership function of the form

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a}{b-a} & a \le x \le b \\ 1 & x = b \\ \dfrac{c-x}{c-b} & b \le x \le c \\ 0 & otherwise \end{cases}$$

is called a *triangular fuzzy number* and a fuzzy number $\tilde{A} = (a, b, c, d)$ with membership function of the form

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a}{b-a} & a \le x \le b \\ 1 & b \le x \le c \\ \dfrac{d-x}{d-c} & c \le x \le d \\ 0 & otherwise \end{cases}$$

is called a *trapezoidal fuzzy number.*

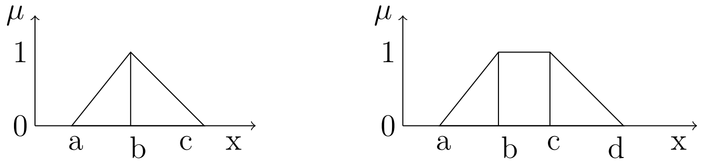


Fig 1. a) Triangular Fuzzy Number b) Trapezoidal Fuzzy Number

Definition 4. The *Fuzzy Operations* of fuzzy numbers are defined as
Fuzzy Addition:
$(a_1, b_1, c_1, d_1) + (a_2, b_2, c_2, d_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2)$
$(a_1, b_1, c_1) + (a_2, b_2, c_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2)$
Fuzzy Subtraction:
$(a_1, b_1, c_1, d_1) - (a_2, b_2, c_2, d_2) = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2)$
$(a_1, b_1, c_1) - (a_2, b_2, c_2) = (a_1 - c_2, b_1 - b_2, c_1 - a_2)$

Definition 5. Haar ranking: [8]
For a given fuzzy number $\tilde{A} = (a, b, c, d)$ the average and detailed coefficients namely the scaling and wavelet coefficients can be calculated using $\alpha = ((a+b+c+d))/4, \beta = ((a+b)-(c+d))/4, \gamma = (a-b)/2, \delta = (c-d)/4$ and call this new 4-tuple as $R(\tilde{A}) = (\alpha, \beta, \gamma, \delta)$.

- $\tilde{A} \prec \tilde{B}$, if the first element of the ordered tuple of $R(\tilde{A})$ is less than the first element of the ordered tuple of $R(\tilde{B})$.
- $\tilde{A} \succ \tilde{B}$, if the first element of the ordered tuple of $R(\tilde{A})$ is greater than the first element of the ordered tuple of $R(\tilde{B})$.
- $\tilde{A} \approx \tilde{B}$ if and only if all the elements of $R(\tilde{A})$ and $R(\tilde{B})$ are term wise equal. (i.e) $\alpha_1 = \alpha_2, \beta_1 = \beta_2, \gamma_1 = \gamma_2, \delta_1 = \delta_2$.

Definition 6. Elementwise Addition:
$(a_1, b_1, c_1, d_1) + (a_2, b_2, c_2, d_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2)$
Elementwise Subtraction:
$(a_1, b_1, c_1, d_1) - (a_2, b_2, c_2, d_2) = (a_1 - a_2, b_1 - b_2, c_1 - c_2, d_1 - d_2)$
$(a_1, b_1, c_1) - (a_2, b_2, c_2) = (a_1 - a_2, b_1 - b_2, c_1 - c_2)$

Definition 7. The fuzzy assignment problem can be defined in the form of an $n \times n$ cost matrix as follows:

	job1	job2		jobn
person1	$\tilde{C_{11}}$	$\tilde{C_{12}}$	$\cdots$	$\tilde{C_{1n}}$
person2	$\tilde{C_{21}}$	$\tilde{C_{22}}$	$\cdots$	$\tilde{C_{2n}}$
	$\cdots$	$\cdots$	$\cdots$	$\cdots$
	$\cdots$	$\cdots$	$\cdots$	$\cdots$
person n	$\tilde{C_{n1}}$	$\tilde{C_{n2}}$	$\cdots$	$\tilde{C_{nn}}$

Mathematically it can be given as

$$\min \tilde{Z} \approx \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{C_{ij}} x_{ij}$$

subject to

$$\sum_{i=1}^{n} x_{ij} = 1, \sum_{j=1}^{n} x_{ij} = 1$$

,

$$x_{ij} = \begin{cases} 1, \text{if } i^{th}\text{person is assigned to the } j^{th}\text{job} \\ 0, \text{otherwise} \end{cases}$$

3. **Haar Hungarian Algorithm**

Consider the matrix representation of the fuzzy assignment problem. Suppose the matrix is not a square matrix, then make it as a square matrix by adding fuzzy zero element rows or fuzzy zero element columns accordingly and call it as effectiveness matrix with number of rows and columns as n.
Step: 1
Convert all the fuzzy parameters in to Haar tuples.
Step: 2
Apply the Hungarian algorithm to get the fuzzy optimal solution.

4. **Numerical Examples**

4.1. **Example.** Let us consider a fuzzy assignment problem with rows representing 5 persons A,B,C,D,E and columns representing 5 jobs Job1,Job2,Job3,Job4 and Job5. The cost matrix is $\tilde{C_{ij}}$ with trapezoidal fuzzy numbers is given in the following table

	Job1	Job2	Job3	Job4	Job5
A	(4,6,7,9)	(3,5,7,9)	(5,7,10,12)	(3,4,6,9)	(4,5,7,10)
B	(2,3,5,9)	(5,7,9,13)	(4,6,9,12)	(5,6,7,10)	(2,3,5,7)
C	(7,9,10,12)	(6,7,9,10)	(7,9,10,13)	(6,7,10,13)	(7,10,13,14)
D	(4,5,7,9)	(5,7,12,15)	(7,9,13,15)	(2,9,10,13)	(5,7,10,14)
E	(4,10,13,15)	(3,7,9,13)	(2,3,10,14)	(3,7,10,13)	(4,7,10,14)

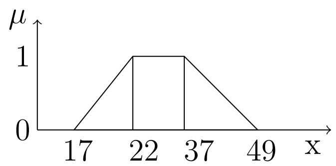
By using definition 5, the fuzzy numbers can be converted to Haar tubles. The following table gives the cost matrix with Haar tuples

	Job1	Job2	Job3	Job4	Job5
A	(6.5,-1.5,-1,-1)	(6,-2,-1,-1)	(8.5,-2.5,-1,-1)	(5.5,-2,-.5,-1.5)	(6.5,-2,-.5,-1.5)
B	(4.75,-2.5,-.5,-2)	(8.5,-2.25,-1,-2)	(7.75,-2.75,-1,-1.5)	(7,-1.5,-.5,-1.5)	(4.25,-1.75,-.5,-1)
C	(9.5,-1.5,-1,-1)	(8,-1.5,-.5,-.5)	(9.75,-1.75,-1,-1.5)	(9,-2.5,-.5,-1.5)	(11,-2.5,-1.5,-.5)
D	(6.25,-1.75,-.5,-1)	(9.75,-3.75,-1,-1.5)	(11,-3,-1,-1)	(8.5,-3,-3.5,-1.5)	(9,-3,-1,-2)
E	(10.5,-3.5,-3,-1)	(7.75,-2.75,-2,-2)	(7.25,-4.75,-.5,-2)	(8.25,-3.25,-2,-1.5)	(8.75,-2.25,-1.5,-2)

After applying the Hungarian algorithm the updated cost matrix is

	Job1	Job2	Job3	Job4	Job5
A	(1,.5,-.5,.5)	(.5,.5,-.5,.5)	(3,-.5,-.5,.5)	**(0,0,0,0)**	(1,0,0,0)
B	(.5,-.75,0,-1)	(4.25,-.5,-.5,-1)	(3.5,-1,-.5,-.5)	(2.75,.25,0,0.5)	**(0,0,0,0)**
C	(1.5,0,-.5,-.5)	**(0,0,0,0)**	(1.75,-0.25,-.5,-1)	(1,-1.5,0,1)	(3,-1,-1,0)
D	**(0,0,0,0)**	(3.5,-2,-.5,.5)	(4.75,-1.25,-.5,0)	(2.25,-1.25,-3,-.5)	(2.75,-1.25,-.5,-1)
E	(2.75,-1.25,-2.5,1)	(.5,2,-1.5,0)	**(0,0,0,0)**	(1,1.5,-1.5,.5)	(1.5,2,-1,0)

$\Rightarrow A \rightarrow 4, B \rightarrow 5, C \rightarrow 2, D \rightarrow 1, E \rightarrow 3$ The assignment cost is $= (5.5, -2, -.5, -1.5) + (4.25, -1.75, -.5, -1) + (8, -1.5, -.5, -.5) + (6.25, -1.75, -.5, -1) + (7.25, -4.75, -.5, -2) = (31.25, -11.755, -2.5, -6)$.
The corresponding fuzzy number is (17,22,37,49) and is matching with the result obtained in [13].
The membership function for the obtained result is



- According to the decision maker the minimum assignment cost will lie between 17 units and 49 units.

- The overall level of satisfaction of the decision maker about the statement that the minimum assignment cost will lie between 22 and 37 units is 100 percent.

- The overall level of satisfaction of the decision maker for the remaining values of minimum assignment cost can be obtained as follows: Let x_0 represents the minimum assignment cost then the overall level of satisfaction of the decision maker for x_0 is $\mu_{\tilde{A}}(x_0) \times 100$
where

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-17}{5} & 17 \leq x \leq 22 \\ 1 & 22 \leq x \leq 37 \\ \dfrac{49-x}{12} & 37 \leq x \leq 49 \\ 0 & otherwise \end{cases}$$

5. Conclusion

A Haar Hungarian algorithm is proposed for solving fuzzy assignment problem using Haar ranking technique. This algorithm is effective and easy to understand because of its similarity to Hungarian method. The arithmetic is easy since it is an ordered pair arithmetic techniques. When compared with the existing method this method is effective since we get the results in ordered pairs and further converted in to fuzzy number. This method is effective even for solving unbalanced assignment problem.

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