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An Approach of Solving Fuzzy Assignment Problem using Symmetric Triangular Fuzzy Number

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Abstract

Assignment problem is a special kind of Linear Programming Problem. In this paper, the cost values of the Fuzzy Assignment Problem are considered as Symmetric Triangular Fuzzy Numbers. First, the Symmetric Triangular Fuzzy Numbers are converted into crisp values using Yager's ranking method. Then the optimum assignment schedule of the Fuzzy Assignment Problem is obtained by usual Hungarian Method. The proposed approach is illustrated by a numerical example.

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1 INTRODUCTION

Assignment Problem is a special type of linear programming problem which deals with the allocation of the various resources to the various activities on one to one basis. It does it in such a way that the cost or time involved the process is minimum and profit or sale is maximum.

Suppose there are n facilities and n jobs it is clear that in this case, there will be n assignments. Each facility or say worker can perform each job, one at time. But there should be certain procedure by which assignment should be made so that the profit is maximized or the cost or time is minimized.

The concept of fuzzy set was introduced by Zadeh[12]in 1965 and it dealt with imprecision, vagueness in real life situations. In 1970, Bellman and Zadeh[1]proposed the concept of decision making problems involving uncertainty and imprecision.

P.K.De, Bharti Yadev[2]discussed a general approach for solving assignment problems involving with fuzzy cost coefficients. Jatinder Pal Singh, Neha Ishesh Thakur[3] proposed a novel method to solve assignment problem in fuzzy environment. Kadhirvel .K. Balamururgan .K[4] discussed method for solving Hungarian assignment problems using triangular and trapezoidal fuzzy number. A. Nagoor Gani, V.N. Mohamed[5] found the solution of a fuzzy assignment problem by using a new ranking method. S. Nareshkumar, S. Kumara Ghuru[6] discussed the solution of solving fuzzy transportation problem using symmetric triangular fuzzy number. G.Nirmala, R.Anju[7] discussed about cost minimization assignment problem using fuzzy quantifier. K.Ruth Isabels, G.Uthra[8] proposed an application of linguistic variables in assignment problem with fuzzy costs. K. Sangeetha, H. Haseena Begum, M. Pavithra[9] discussed about the ranking of triangular fuzzy number method to solve an unbalanced assignment problem. K.R. Shoba[10] proposed profit maximization of fuzzy assignment problem.

In this paper a Fuzzy Assignment problem is considered. The cost values of the Fuzzy Assignment Problem are taken as Symmetric Triangular Fuzzy Numbers. The Symmetric Triangular Fuzzy Numbers are converted into crisp values using Yager's Ranking Procedure. The problem is then solved by the usual Hungarian method.

The rest of this paper organized as follows. In section 2, some basic definitions and Yager's Ranking Procedure of Symmetric Triangular Fuzzy Number are given. Section 3, presents introduction of Fuzzy Assignment Problem. In section 4, procedure and numerical example for the proposed method are given followed by conclusion in section-5.

2 DEFINITIONS

Definition 2.1: Fuzzy number : Let A be a classical set, $\mu_A(x)$ be a function from A to [0,1]. A fuzzy set A with the membership function $\mu_A(x)$ is defined as $A = \{(x, \mu_A(x)); \text{x}\epsilon A\}$ and $\mu_A(x) \epsilon [0.1]$.

Definition 2.2: Triangular Fuzzy Number : A fuzzy number A=(a_1,a_2,a_3) is defined to be a triangular fuzzy number if its membership functions $\mu_A : R \rightarrow [0,1]$ is equal to

$$\mu_A(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} \; if \; x \in [a_1, a_2] \\ 1 \qquad\qquad x = a_2 \\ \frac{a_3-x}{a_3-a_2} \; if \; x \in [a_2, a_3] \\ 0 \qquad otherwise \end{cases}$$

It is denoted by A=(a_1,a_2,a_3) where a_1 is Core (A) , a_2 is left width and a_3 is right width. The geometric representation of Triangular Fuzzy Number is shown in Figure-1. Since, the shape of the Triangular Fuzzy Number A is usually in triangle it is called so.

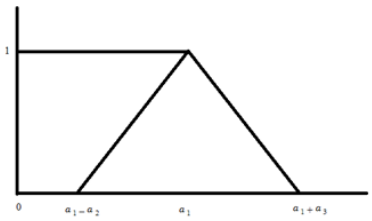


Figure 1 : Membership Function of Triangular Fuzzy Number

The Parametric form of a Triangular Fuzzy Number is represented by A=$[a_1- \ a_2 \ (1-r), \ a_1+ \ a_3 \ (1-r)]$

Definition 2.3: Symmetric Triangular Fuzzy Number : If $a_2 = a_3$ then the Triangular Fuzzy Number A=(a_1, a_2, a_3) is called Symmetric Triangular Fuzzy Number. It is denoted by A=(a_1, a_2) where a_1 is Core(A), a_2 is left width and right width of Core(A).

The Parametric form of a Symmetric Triangular Fuzzy Number is represented by A=$[a_1- \ a_2 \ (1-r), \ a_1+ \ a_2 \ (1-r)]$

Definition 2.4: Ranking of Triangular Fuzzy Number : Yager's ranking technique [11] which satisfy compensation, linearity, additively properties and provides results which consists of human intuition. IfA=$(a_1, a_2, a_3)$is a Fuzzy Number then the Yager's ranking is defined by

$\mathrm{R}(A) = \int_0^1 0.5(a_\alpha^L, a_\alpha^U)d\alpha$

Where $(a_\alpha^L, a_\alpha^U) = \{(a_2 - a_1)\alpha + a_1, a_3 - (a_3 - a_2)\alpha\}$ Since $a_3 = a_2$

Definition 2.5: Arithmetic Operations of Symmetric Triangular Fuzzy Number : The Fuzzy Number is fully and uniquely represented by its r-cut, since the r-cut of each fuzzy number are closed intervals of real numbers for all $r \epsilon [0, 1]$. This enables us to define arithmetic operations on Fuzzy number in terms of arithmetic operations on their r-cut.

Let A^* and B^* by arbitrary fuzzy numbers with the r-cut A^*=[A(r), A^*(r)] and B^* =[B(r),B^*(r)]. Then the arithmetic operations be-

tween A^* and B^* are denoted by
(i) $A^* + B^* = [\mathrm{A(r)+B(r)},\ A^*(r) + B^*(r)]$

(ii) $A^* - B^* = [\mathrm{A(r)}\text{-}B^*(r),\ A^*(r) - B(r)]$

(iii)$A^*B^* = \mathrm{H} = [\mathrm{H(r)}, H^*(r)]$
Where $\mathrm{H(r)} = \min\{\mathrm{A(r)B(r)},\ A^*(r)B^*(r), A^*(r)B(r), A(r)B^*(r)\}$
$H^*(r) = \max\{\mathrm{A(r)B(r)},\ A^*(r)B^*(r), A^*(r)B(r), A(r)B^*(r)\}$

(iv) $A^*/B^* = \mathrm{H} = [\mathrm{H(r)}, H^*(r)]$
Where $\mathrm{H(r)} = \min\{\mathrm{A(r)/B(r)},\ A^*(r)/B^*(r), A^*(r)/B(r), A(r)/B^*(r)\}$
$H^*(r) = \max\{\mathrm{A(r)/B(r)},\ A^*(r)/B^*(r), A^*(r)/B(r), A(r)/B^*(r)\}$
(v)

$$KA = \begin{cases} [KA(r), KA^*(r)], if K \geq 0 \\ [KA(r), KA^*(r)], if K < 0 \end{cases}$$

3 Fuzzy Assignment Problem

Consider the situation of assigning n machines to n jobs and each machine is capable of doing each job at different costs. Let C^*_{ij} be an fuzzy cost of assigning the i^{th} machine to the j^{th} job. Let x_{ij} be the decision variable denoting the assignment of the i^{th} machine to the j^{th} job. The objective is to minimize the total cost.

The mathematical model of the Fuzzy Assignment Problem is given by

Minimize $z^* = \sum_{i=1}^{n} \sum_{j=1}^{n} C^*_{ij}\ x_{ij}$

Subject to

$$\sum_{i=1}^{n} x_{ij} = 1, \quad \textit{for j=1,2..n}$$

$$\sum_{j=1}^{n} x_{ij} = 1, \quad \textit{for i=1,2..n}$$

$$x_{ij} \in \textit{[0,1]}$$

$$\textit{where } x_{ij} = \begin{cases} 1, \textit{ if the } i^{th}\textit{machine is assigned to } j^{th} \textit{ job} \\ 0, \textit{ if the } i^{th}\textit{machine is not assigned to } j^{th} \textit{ job} \end{cases}$$

$$C^*_{ij} = (C^1_{ij}, C^2_{ij}, C^2_{ij})$$

4 PROCEDURE

Step 1: First convert the cost values of the fuzzy assignment problem which are all in symmetric triangular fuzzy numbers into crisp values by using Yager's Ranking.

Step 2: Check the condition that the fuzzy assignment problem is balanced.

(i) If balanced go to step 4. (Number of rows = Number of Columns)

(ii) If not balanced go to step 3. (Number of rows $\neq$ Number of Columns)

Step 3: If the given Fuzzy Assignment problem is not balanced then add dummy row (or) dummy Column with cost value as zero to make the fuzzy assignment problem balanced.

Step 4: Obtain the optimum assignment schedule by Hungarian method.

Example : A Fuzzy Assignment Problem with rows representing 4 machines M_1, M_2, M_3, M_4 and columns representing the 4 Jobs J_1, J_2, J_3, J_4 is considered. The cost matrix C^*_{ij} whose elements are Symmetric Triangular Fuzzy Numbers is given below. The problem is to find the minimum cost.

$$\begin{pmatrix} (0,2,2) & (4,8,8) & (28,30,30) & (44,48,48) \\ (4,8,8) & (1,2,2) & (33,36,36) & (37,40,40) \\ (23,25,25) & (44,48,48) & (1,2,2) & (0,2,2) \\ (37,40,40) & (4,8,8) & (15,18,18) & (15,18,18) \end{pmatrix}$$

Using yager's ranking, the crisp values of the above problem is

$$\begin{pmatrix} 1.5 & 7 & 29.5 & 47 \\ 7 & 1.75 & 35.25 & 39.25 \\ 24.5 & 47 & 1.75 & 1.5 \\ 39.25 & 7 & 17.25 & 17.25 \end{pmatrix}$$

Number of rows = Number of columns.
Therefore the fuzzy assignment problem is balanced.
Now, the fuzzy assignment problem is solved by Hungarian method.
The assignment schedule of the fuzzy assignment problem as follows.

$$\begin{pmatrix} (0) & 10.75 & 23 & 40.75 \\ \emptyset & (0) & 18.5 & 22.75 \\ 27.75 & 55.5 & \emptyset & (0) \\ 27 & \emptyset & (0) & 0.25 \end{pmatrix}$$

The assignment schedule of this problem is

$$M_1 \to J_1, M_2 \to J_2, M_3 \to J_4, M_4 \to J_3.$$

The assignment cost = 1.5 + 1.75 + 1.5 + 17.25 = 22.

5 CONCLUSION:

In this paper, Fuzzy Assignment problem with cost values as Symmetric Triangular Fuzzy Numbers is considered. The Symmetric Triangular Fuzzy Numbers are converted into crisp values using Yager's ranking. The optimum assignment schedule of the Fuzzy Assignment Problem is then obtained by Hungarian Method. We hope that this approach will be effective in assignment problems involving imprecise data.

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