International Journal of Pure and Applied Mathematics
Volume 113 No. 7 2017, 8 – 15
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
Special Issue

ijpam.eu

RADIO ODD MEAN NUMBER OF COMPLETE GRAPHS

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Abstract

A radio odd mean labeling of a connected graph G is a one to one map from the vertex set $V(G)$ of G to Z^+ such that for two distinct vertices u and v of G, $d(u,v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ is odd $\geq 1 + diam(G)$. The radio odd mean number of f, *romn (f)* is the maximum number assigned to any vertex of G. The radio odd mean number of G, $romn(G)$ is the minimum value of $romn(f)$ taken over all radio odd mean labeling of G. In this paper we have introduced the concept of radio odd mean labeling and determined radio odd mean number of wheel, complete graph, and complete tripartite graph.

AMS Mathematical Classification: 05C78
Key Words: Radio Odd Mean Labeling, Radio Odd Mean Graph

1 INTRODUCTION

All graphs in this paper are finite, simple and undirected. Terms not defined here are used in the sense of Harary [6]. The symbols $V(G)$ and $E(G)$ will denote the vertex set and edge set of a graph G. Radio labeling [1] is motivated by the channel assignment problem introduced by Hale et al [5] in 1980.To avoid interference, transmitters that are geographically close must be assigned channel with large frequency difference, transmitters that are further apart may receive channels with relatively close frequencies. The general solution is modeled by identifying transmitters with the vertices of a graph subject to a restriction conenering the distance between the vertices. The goal is to minimize the largest integer used. The radio labeling of a graph is most useful in FM radio channel restrictions to overcome from the effect of noise [2]. This problem turns out to find the minimum of maximum frequencies of all the radio stations considered under the network. Ponraj et al. [11] discussed the radio mean labeling. Motivated by the notion of radio mean labeling we have introduced the concept of radio odd mean labeling and determine radio odd mean number of wheel, complete graph and complete tripartite graph.

2 Definitions

2.1 Definition

Let G be a connected graph, the distance $d(u,v)$ between any pair of vertices u, v is the length of the shortest path between them.

2.2 Definition

The diameter of a graph is denoted by $diam(G)$ and defined as the maximum distance between any two vertices, that is, $diam(G) = max\{d(u,v); u,v \in G\}$

2.3 Definition

A radio labeling is one to one mapping $f : V(G) \to Z^+$ satisfying the condition $d(u,v) + |f(u) - f(v)| \geq 1 + diam(G)$, for every vertices u, v in G.

2.4 Definition

A radio odd mean labeling of a connected graph G is a one to one map from the vertex set $V(G)$ of G to Z^+ such that for two distinct vertices u and v of G, $d(u,v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ is odd $\geq 1 + diam(G)$. A graph that admits a radio odd mean labeling is

2.5 Definition

The span of a labeling f is the maximum integer that f maps to a vertex of graph G.

2.6 Definition

Radio odd mean number of graph G denoted by $romn(G)$ is defined as the lowest span taken over all radio labeling of graph G.

3 MAIN RESULTS

3.1 Observation

The labels 0 and 1 can be the labels of two adjacent vertices of radio odd mean graph iff $diam(G) = 1$

3.2 Observation

Let $f(u)$ and $f(v)$ be the labels of two adjacent vertices of a radio odd mean graph then $\left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ is not congruent to 0 (mod 4).

3.3 Result

Let $f(u)$ and $f(v)$ be the adjacent vertex labels of radio odd mean graph, then

(i) If $f(u) \equiv 0(mod4)$, then $f(v) \equiv 1(mod4)$ or $\equiv 2(mod4)$.

(ii) If $f(u) \equiv 1(mod4)$, then $f(v) \equiv 0(mod4)$ or $\equiv 1(mod4)$.

(iii) If $f(u) \equiv 2(mod4)$, then $f(v) \equiv 0(mod4)$ or $\equiv 3(mod4)$.

(iv) If $f(u) \equiv 3(mod4)$, then $f(v) \equiv 2(mod4)$ or $\equiv 3(mod4)$.

3.4 Result

Let $f(u)$ and $f(v)$ be the labels of two adjacent vertices of a radio odd mean graph. If $f(u) \equiv 0(mod4)$ and $f(v) \equiv 2(mod4)$, then the vertex which is adjacent to both u and v cannot be labeled with any label.

3.5 Theorem

The radio odd mean number of W_3 is 9.

Proof:
Let us denote the central vertex of W_3 by v and the rim vertices by v_i, $1 \leq i \leq 3$.
The diameter of W_3 is 1
So, as in observation 3.1, the labels 0 and 1 can be assigned to any adjacent vertices.
The rim vertices are labeled with $4i-3, 1 \leq i \leq 3$
case(i)
Suppose, we assign the label 0 to the central vertex. We have to show that

$$d(u,v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \text{ is odd} \geq 1 + diam(G) \tag{3.1}$$

for every pair of vertices $(u,v), u \neq v$.

In order to satisfy the definition of radio odd mean labeling

$$\begin{aligned} d(u,v)+ & \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \text{ is odd} \geq 1 + diam(G) \\ \Rightarrow & \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \text{ is odd} \geq 1 + diam(G) \end{aligned}$$

By Result 3.3, the vertex labels adjacent to 0 are

$$f(v_i) \equiv 1(mod4) \text{ or } f(v_i) \equiv 2(mod4).$$

But $f(v_i) \neq 2(mod4)$ by Result 3.4
Therefore $f(v_i) \equiv 1(mod4)$
Thus $f(v_i) \equiv 4i-3, 1 \leq i \leq 3$
Hence,

$$romn(W_3) = 9$$

.
case(ii)
Suppose, we assign the label 0 to any one of the rim vertices $v_i, 1 \leq i \leq 3$
Now by Result 3.3, the vertex labels adjacent to 0 are $1(mod4)$ or $2(mod4)$.
But the remaining rim vertices and the central vertex cannot be label with $2(mod4)$ as in Result 3.4
Therefore, the remaining rim vertices and the central vertex get the label $1(mod4)$,

$$f(v_i) = 4i-3, 1 \leq i \leq 3$$

.
Hence,

$$romn(W_3) = 9$$

.

3.6 Theorem

The radio odd mean number of W_n is $4n-3$ for $n \geq 4$.

Proof
Let us denote the central vertex by v and the rim vertices by $v_i, 1 \leq i \leq n$
The diameter of Wn for $n \geq 4$ is 2
Assign the label 4 to the central vertex v and the rim vertices by $4i-3, 1 \leq i \leq n$ in any order. We have to prove that

$$d(u,v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \text{ is odd} \geq 1 + diam(W_n) \tag{3.2}$$

for every pair of vertices $(u,v), u \neq v$.

Case (i) Verify the pair$(v, v_i), 1 \leq i \leq n$

$$\text{If} f(v_i) \geq 1, \text{then} \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \text{ is odd} \geq 1+3 \geq 4$$

Therefore, 3.2 holds.
Case (ii) Examine the pair of $(v_i, v_j), 1 \leq i, j \leq n$
Subcase (i) $d(v_i, v_j) = 1$, for$i \neq j$.
Let $f(vi) \geq 1$ and $f(vj) \geq 5$.

$$d(v_i,v_j) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \text{ is odd} \geq 1+3 \geq 4$$

Therefore, 3.2 holds.
Subcase (ii) $d(v_i, v_j) = 2$, for $i \neq j$. Let $f(v_i) \geq 1$ and $f(v_j) \geq 5$.

$$d(v_i,v_j) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \text{ is odd} \geq 2+3 \geq 5$$

Therefore, 3.2 holds.
Hence,

$$romn(W_n) = 4n-3, \quad \text{for} \quad n \geq 4$$

.

3.7 Theorem

The radio odd mean number of complete graph K_n is $4n-7$ for $n \geq 2$.

Proof
Let us denote the vertex set and edge set of complete graph K_n by

$$V(K_n) = u_i, \text{for} 1 \leq i \leq n$$

$$E(K_n) = e_i = (u_i u_{i+1}), \text{for} 1 \leq i \leq n-1 \cup e_n = (u_n u_1)$$

The diameter of complete graph K_n is 1
Since the diameter is 1, as in observation 3.1 we can assign 0 and 1 to any adjacent vertices.
Assign the label 0 to the vertex u_1.

Assign the remaining vertices by $4i-7, 2 \leq i \leq n$ in any order.
We have to show that

$$d(u,v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \text{ is odd} \geq 2 \tag{3.3}$$

holds for all pairs of vertices $(u,v), u \neq v$.
Case (i) Check the pair $(u_1,u_i), 2 \leq i \leq n$
Let $f(u_1)=0$, $f(u_i)=4i-7$

$$d(u,u_i) + \left\lceil \frac{f(u)+f(u_i)}{2} \right\rceil \text{ is odd} \geq 2 + \left\lceil \frac{0+1}{2} \right\rceil \text{ is odd} \geq 2$$

Therefore, 3.3 holds.

Case (iii) Consider the pair $(u_i,u_j), i \neq j$

$$d(u_i,u_j) + \left\lceil \frac{f(u_i)+f(u_j)}{2} \right\rceil \text{ is odd} \geq 2 + \left\lceil \frac{1+5}{2} \right\rceil \text{ is odd} \geq 5$$

Therefore, 3.3 holds.
Hence,

$$romn(K_n) = 4n-7 \quad \text{for} \quad n \geq 2$$

.

3.8 Theorem

The radio odd mean of complete tripartite graph $K_{m,n,t}$ is $4(m+n+t)-7$ for $1 \leq i \leq m, 1 \leq j \leq n$, and$1 \leq k \leq t$.

Proof
Let

$$V(K_{m,n,t}) = u_i, v_j, w_k, 1 \leq i \leq m, 1 \leq j \leq n, 1 \leq k \leq t$$

and

$$E(K_{m,n,t}) = u_iv_j, u_iw_k, v_jw_k, 1 \leq i \leq m, 1 \leq j \leq n, 1 \leq k \leq t$$

The diameter of complete tripartite graph $K_{m,n,t}$ is 2
Assign the vertex labels as follows:

$$f(u_1) = 0,$$

$$f(u_i) = 4i-7, 2 \leq i \leq m$$

$$f(v_j) = \begin{cases} 4(m+j)-3, \text{if} m=1 \\ 4(m+j)-7, \text{if} m>1 \end{cases}$$

$$f(w_k) = \begin{cases} 4(m+n+k)-3, \text{if} m=1 \\ 4(m+n+k)-7, \text{if} m>1 \end{cases}$$

We have to show that

$$d(u,v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \text{ is odd} \geq 3 \tag{3.4}$$

for every pair of vertices.

Case (i)

Consider the pair $(u_i, u_j), 1 \leq i, j \leq m$

$$d(u_i, u_j) + \left\lceil \frac{f(u_i) + f(u_j)}{2} \right\rceil \text{ is odd} \geq 2 + \left\lceil \frac{0+1}{2} \right\rceil \text{ is odd} \geq 2 + 1 \geq 3$$

Therefore, 3.4 holds.

Case (ii)

Verify the pair $(v_i, v_j), i \neq j, 1 \leq i, j \leq n$

$$d(v_i, v_j) + \left\lceil \frac{f(v_i) + f(v_j)}{2} \right\rceil \text{ is odd} \geq 2 + \left\lceil \frac{(4m-3) + (4m+1)}{2} \right\rceil \geq 5$$

Therefore, 3.4 holds.

Case (iii)

Examine the pair $(w_i, w_j), i \neq j, 1 \leq i, j \leq n$

$$d(w_i, w_j) + \left\lceil \frac{f(w_i) + f(w_j)}{2} \right\rceil \text{ is odd} \geq 2 + \left\lceil \frac{(4m+n) - 3 + 4(m+n) + 1}{2} \right\rceil \geq 9$$

Therefore, 3.4 holds.

Case (iv)

Consider the pair $(u_i, v_j), 1 \leq i \leq m, 1 \leq j \leq n$

Subcase (i) $m = 1$

$$d(u_i, v_j) + \left\lceil \frac{f(u_i) + f(v_j)}{2} \right\rceil \text{ is odd} \geq 1 + \left\lceil \frac{0 + (4m+1)}{2} \right\rceil \geq 4$$

Therefore, 3.4 holds.

Subcase (ii) $m > 1$

$$d(u_i, v_j) + \left\lceil \frac{f(u_i) + f(v_j)}{2} \right\rceil]\text{is odd} \geq 1 + \left\lceil \frac{0 + (4m-3)}{2} \right\rceil \geq 4$$

Therefore, 3.4 holds.

Case (v)

Verify the pair $(u_i, w_k), 1 \leq i \leq m, and 1 \leq k \leq t$

Subcase (i) $m = 1$

$$d(u_i, w_k) + \left\lceil \frac{f(u_i) + f(w_k)}{2} \right\rceil \text{ is odd} \geq 1 + \left\lceil \frac{0 + 4(m+n) + 1}{2} \right\rceil \geq 6$$

Therefore, 3.4 holds.

Subcase (ii) $m > 1$

$$d(u_i, w_k) + \left\lceil \frac{f(u_i) + f(w_k)}{2} \right\rceil \text{ is odd} \geq 1 + \left\lceil \frac{0 + 4(m+n) - 3}{2} \right\rceil \geq 4$$

Therefore, 3.4 holds.

Case (vi)

Examine the pair $(v_j, w_k), 1 \leq j \leq n, and 1 \leq k \leq t$
Subcase (i) $m = 1$

$$d(v_j, w_k) + \left\lceil \frac{f(v_j) + f(w_k)}{2} \right\rceil \text{is odd} \geq 1 + \left\lceil \frac{(4m+1) + 4(m+n) + 1}{2} \right\rceil \geq 8$$

Therefore, 3.4 holds.
Subcase (ii) $m > 1$

$$d(v_j, w_k) + \left\lceil \frac{f(v_j) + f(w_k)}{2} \right\rceil \text{is odd} \geq 1 + \left\lceil \frac{(4m-3) + 4(m+n) - 3}{2} \right\rceil \geq 4$$

Therefore, 3.4 holds.
Hence,

$$romn(K_{m,n,t}) = 4(m+n+t) - 7 \quad \text{for} \quad 1 \leq i \leq m, 1 \leq j \leq n \text{and} 1 \leq k \leq t$$

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