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MULTIPLE SLIP EFFETCS ON MHD AND HEAT TRANSFER IN A JEFFERY FLUID OVER AN INCLINED VERTICAL PLATE

C H Amanulla[1,2*]
N Nagendra[1] and M Suryanarayana Reddy [2]
[1]Department of Mathematics
Madanapalle Institute of Technology and Science
Madanapalle, India-517325.
[2]Department of Mathematics
JNTUA College of Engineering
Pulivendula, India-516390.
* Corresponding author: amanullamsc@gmail.com

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Abstract

In this paper, we study the multiple slip effects on magneto hydrodynamic flow of a viscous incompressible electrically conducting Jefferys fluid over an inclined vertical plate in the presence of transverse magnetic field. The governing equations are converted into ordinary once by using the non-dimensional quantities, which are solved logically by the finite difference method under appropriate boundary conditions. The profiles for the velocity and temperature distributions of the problem are displayed graphically for various values of pertinent parameters. Also, the skin friction coefficient Nusselt and Sherwood number are presented in tabular form.

AMS Subject Classification: 76A05, 76W05.

Key Words: Magneto hydrodynamics; inclined plate; non-Newtonian flow; thermal and momentum slip.

1 Introduction

The increasing demand of modern engineering applications involve the study of non-Newtonian fluids. The application of magnetic fields (static or alternating) has been shown to manipulate successfully the material characteristics of electro-conductive polymers which are finding new applications in aerospace,

offshore and naval industries[1], thin film fabrication processes [2] and design of shock dissipation systems with magnetic elastomers [3]. Relevant technologies in this regard are nuclear engineering [4], medical engineering exploiting stimuli-based polymers [5] and hydromagnetic energy generation [6]. Ellahi et al. [7] analyzed the problem of the MHD peristaltic flow of Jeffrey fluid in a non-uniform rectangular duct with effects of Hall and ion slip. Prasannakumara et al. [8] studied the Radiation and multiple slip effects on MHD Jeffery Nanofluid flow over a horizontal stretching surface. Kumari et al. [9] analyzed the slip effects on MHD flow of Jeffrey fluid in a channel. Jena et al. [10] have studied heat and mass transfer on MHD Jeffery Fluid Flow over a Stretching Sheet through Porous Media with presence of Chemical reaction and source/sink. Several authors [11-15] investigated heat transfer of the non-Newtonian Jeffrey fluid of various physical problems. Though various aspects of Jeffrey fluid flow over inclined vertical plate with no-slip boundary conditions have been extensively investigated over the past few decades, little attention has been paid to flow phenomena over inclined vertical plate with slip effect.

In the present investigation is MHD Jeffreys fluid flow past an inclined vertical plate with with momentum and thermal slip effects. The governing partial differential equations are converted into ordinary differential equations by using the non-dimensional quantities, which are solved numerically using the finite difference (Keller Box) method. The profiles for the velocity and temperature profiles of the problem are displayed graphically for various values of pertinent parameters.

2 Mathematical Model

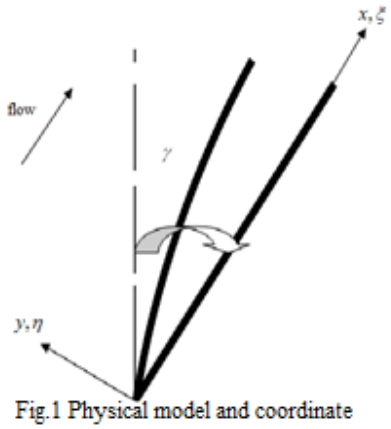


Fig.1 Physical model and coordinate

We examine steady buoyancy-driven convection heat transfer flow of Jeffreys non-Newtonian fluid over an inclined vertical plate. Figure 1 shows the flow model and associated coordinate system.

The governing conservation equations can be written as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\frac{v}{1+\lambda}\left[\frac{\partial^2 u}{\partial y^2}+\lambda_1\left(u\frac{\partial^3 u}{\partial x\partial y^2}-\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}+\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}+v\frac{\partial^3 u}{\partial y^3}\right)\right]$$

$$+g\beta\left(T-T_\infty\right)cos\gamma-u\frac{\sigma B_0^2}{\rho} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

The boundary conditions are prescribed at the surface and the edge of the boundary layer regime, respectively as follows:

$$u=u_\infty,\ T=T_\infty,\ \text{at x=0},\ u=N_0\tfrac{\partial u}{\partial y}, v=0, T=T_w+K_0\tfrac{\partial T}{\partial y}\ \text{at y=0}$$

$$u\to u_\infty, T=T_\infty, as\quad y\to\infty \tag{4}$$

The stream function ψ is defined by $u=\frac{\partial\psi}{\partial y}$ and $v=-\frac{\partial\psi}{\partial x}$, and therefore, the continuity equation is automatically satisfied. In order to write the governing equations and the boundary conditions in dimensionless form, the following non-dimensional quantities are introduced

$\eta=\left(\frac{u_\infty}{vx}\right)^{1/2}, \xi=\xi(x), f(\xi,\eta)=\frac{\psi(x,y))}{(vu_\infty x)^{1/2}}, \theta(\xi,\eta)=\frac{(T-T_\infty)}{(T_w-T_\infty)}$

$$Gr_x=\frac{g\beta(T_w-T_\infty)x^3}{v^2}, De=\frac{\lambda_1 U_\infty}{x}, Pr=\frac{v}{\alpha}, Re_x=\frac{U_\infty x}{v} \tag{5}$$

$$\frac{1}{1+\lambda}f''+\frac{1}{2}ff'-\frac{De}{1+\lambda}\left(ff''+\frac{1}{2}f''^2-\frac{1}{2}ff^{iv}\right)+\xi\theta\cos\gamma-Mf'=$$

$$\xi\left(f'\frac{\partial f'}{\partial\xi}-f''\frac{\partial f}{\partial\xi}-\frac{De}{1+\lambda}\left(f'\frac{\partial f'''}{\partial\xi}-f''\frac{\partial f'}{\partial\xi}+f'\frac{\partial f''}{\partial\xi}-f^{iv}\frac{\partial f}{\partial\xi}\right)\right) \tag{6}$$

$$\frac{\theta''}{Pr}+\frac{1}{2}f\theta'=\xi\left(f'\frac{\partial\theta}{\partial\xi}-\theta'\frac{\partial f}{\partial\xi}\right) \tag{7}$$

The dimensionless form of the boundary conditions are:

$$\text{At } \eta=0,\quad f=0,\quad f'=S_f f''(0),\quad \theta=1+S_T\theta'(0)$$

$$As\quad \eta\to\infty,\qquad f'=1,\qquad \theta=0 \tag{8}$$

The engineering design quantities of physical interest include the skin-friction coefficient and Nusselt number, which are given by:

$$\frac{1}{2}C_f Re_x^{-1/2}=f''(\xi,0) \tag{9}$$

$$\frac{Nu}{\sqrt{Re_x}}=-\theta'(\xi,0) \tag{10}$$

3 Numerical Solution

The transformed, nonlinear, multi-physical boundary value problem defined by Eqns. (6)-(7) can be solved via a number of numerical schemes. Keller box method was originally settled for low speed aerodynamic problems and this system is established by Keller [16]. This method remains among the most powerful, versatile and accurate computational finite difference schemes employed in modern viscous fluid dynamics simulations. This method has been used extensively and effectively for over three decades in a large spectrum of nonlinear fluid mechanics problems. Kellers method provides unconditional stability and rapid convergence for strongly non-linear flows.It has been used recently in polymeric flow dynamics by Rao et al. [22].

4 Results and discussions

Extensive computations have been conducted using the Keller box code to study the influence of the key thermo-physical parameters on velocity and temperature. These are visualized in figs. 2 to 9. **Table 1** present numerical values for the influence of the various parameters on skin friction and Nusselt number functions.

Table 1: Values of $f''(\xi,0)$ and $-\theta'(\xi,0)$ for different De, λ, M and ξ.

De	λ	M	$\xi=1.0$		$\xi=2.0$		$\xi=3.0$	
			$f''(\xi,0)$	$-\theta'(\xi,0)$	$f''(\xi,0)$	$-\theta'(\xi,0)$	$f''(\xi,0)$	$-\theta'(\xi,0)$
0.5	0.2	0.001	0.9082	0.3359	2.7080	0.3659	5.2872	0.3899
1.0			0.9141	0.3347	2.7411	0.3647	5.3802	0.3875
2.0			0.9207	0.3342	2.7721	0.3629	5.4735	0.3836
0.1	0.0		0.8081	0.3264	2.4160	0.3576	4.7070	0.3809
	1.0		1.0399	0.3543	3.7167	0.3885	7.2949	0.4142
	2.0		1.5449	0.3695	4.7588	0.4052	9.3744	0.4321
	0.2	0.0	0.9029	0.3341	2.7116	0.3661	5.2924	0.3900
		0.01	0.8851	0.3321	2.6767	0.3645	5.2407	0.3887
		0.05	0.8192	0.3247	2.5467	0.3587	5.0456	0.3837
		0.1	0.7480	0.3167	2.4024	0.3522	4.8267	0.3781

Table 1 presents the influence of increasing magnetic body force parameter (M), De on skin friction, heat transfer rate, along with a variation in the parameter (λ),ratio of relaxation and retardation times and transverse coordinate (ξ). With an increase in λ, all two (i.e., the skin friction and heat transfer rate) are increased. This implies that as the relaxation time is reduced (and the retardation time increased), the polymer flows faster and also transfers heat more efficiently from the plate surface. This appears consistent with other studies [21, 22], whereas an increasing De reduces heat transfer rate, whereas skin friction is enhanced. With increasing magnetic body force parameter, M, the skin friction, heat transfer rate reduced markedly. **Figs. 2-3** illustrate the impact of the momentum (hydrodynamic) slip parameter (S_f) on the velocity and temperature distributions. With increasing values of S_f the polymer slips i.e. shears more easily against the plate surface. This boosts momentum in the boundary layer and accelerates the flow (fig. 2). The velocity slip effect is

strongest at the plate surface ($\eta = 0$). A similar observation has been made by Yarin and Graham [17] and also by Rao et al. [18]. Temperature is conversely reduced consistently throughout the boundary layer with greater momentum slip.

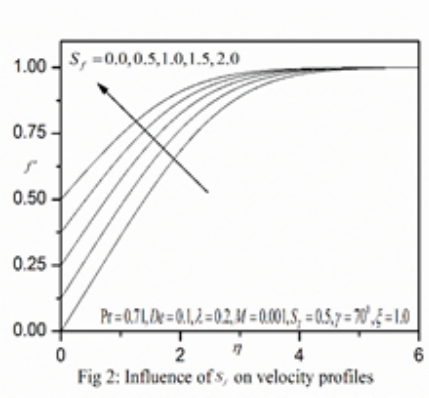


Fig 2: Influence of s_f on velocity profiles

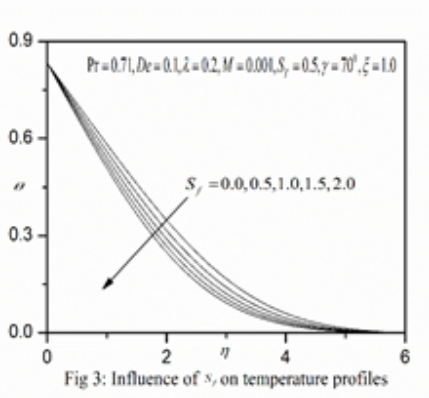


Fig 3: Influence of s_f on temperature profiles

Figs. 4-5 presents the variation of fluid temperature with thermal slip parameter (S_T). It is evident that the fluid temperature decreases for increasing values of thermal slip parameter. As the thermal slip parameter(S_T) increases, less heat is transferred from the plate to the fluid and hence the temperature decreases. We also observe that the thermal boundary layer thickness becomes thicker for increasing values of(S_T).**Figs. 6-7** illustrates that the velocity is reduced with an increase in . This indicates that the transverse magnetic field opposes the transport phenomena since an increase in leads to an increase in the Lorentz force, which opposes the transport process. This stronger Lorentz force produces more resistance to the transport. The higher the value of , the more prominent is the reduction in hydrodynamic boundary layer thickness. But from Fig. 7, the opposite phenomenon is observed with an increase in Magnetic field parameter M on temperature field.

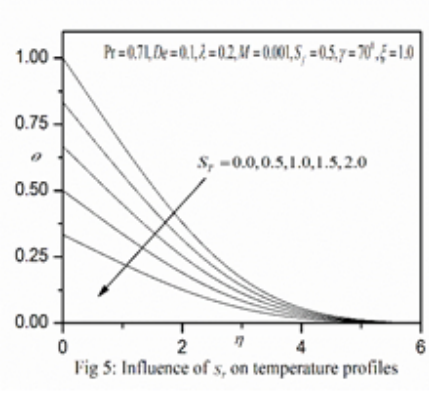


Fig 5: Influence of s_f on temperature profiles

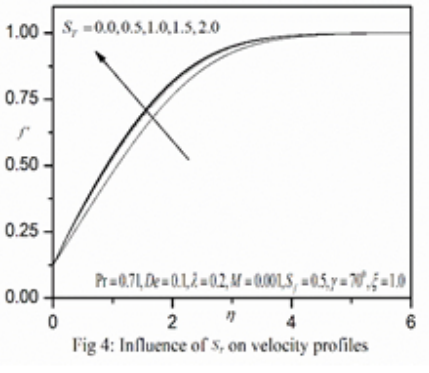


Fig 4: Influence of s_f on velocity profiles

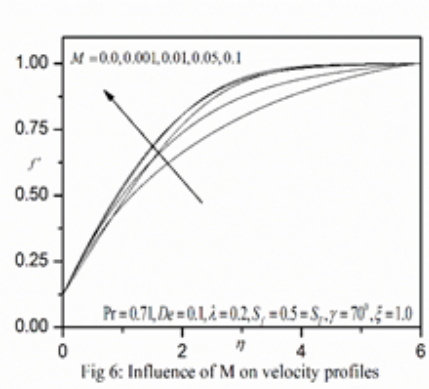


Fig 6: Influence of M on velocity profiles

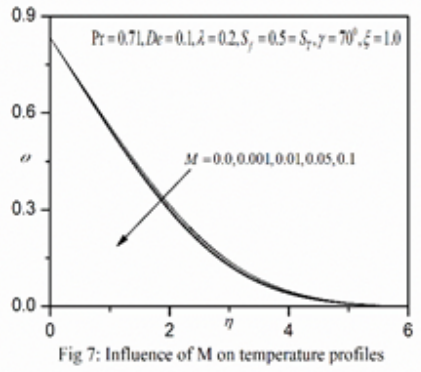


Fig 7: Influence of M on temperature profiles

Figs 8-9 presents the influence of the plate inclination on the dimensionless velocity and temperature. When $\gamma < 0$ i.e. negative plate inclination, in fig 8, the velocity is enhanced at first i.e. flow is initially accelerated nearer the plate surface; however further away it is decelerated. For the case of the vertical plate ($\gamma = 0$ degrees) and for positive inclination, velocities are always monotonic distributions. With $\gamma > 0$ i.e. 30 degrees and 80 degrees, velocity is enhanced i.e. flow is accelerated, largely owing to gravitational effects. Conversely in fig 9, with negative plate inclination ($\gamma < 0$) the temperature increases slightly; for a vertical plate temperatures are greater than for the negatively inclined plate; temperatures are further decreased marginally with positive inclination of the plate.

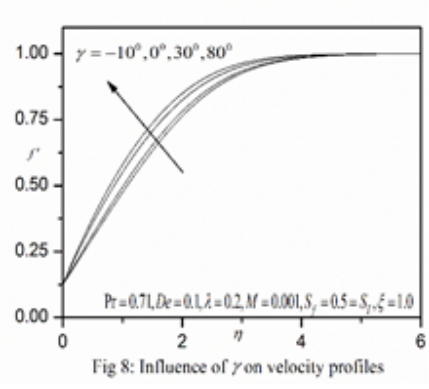


Fig 8: Influence of γ on velocity profiles

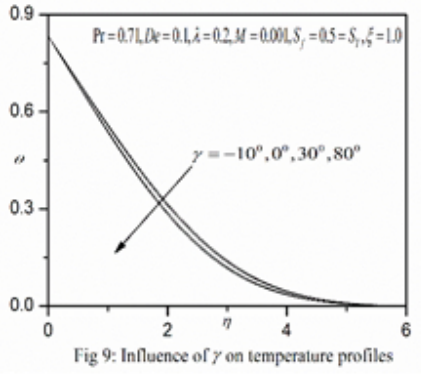


Fig 9: Influence of γ on temperature profiles

5 CONCLUSIONS

In this paper, we presented a mathematical model for steady, MHD, two-dimensional incompressible Jefferys fluid past an inclined vertical plate under transverse magnetic field. To simulate slippery polymer interfacial effects, both thermal and momentum slip have been incorporated into the model. The

present computations have shown that increasing Deborah number accelerates the near-wall flow and decreases temperatures (i.e reduces Nusselt number). Stronger magnetic parameter serves to accelerate the flow and to promote temperatures i.e. decreases Nusselt numbers. With greater momentum and thermal slip, the flow is accelerated near the plate surface whereas temperatures are depressed.

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