The Magneto-hydrodynamic Lubrication of Curved Circular Plates With Couple Stress Fluid

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Abstract: In the present investigation, the effect of magneto-hydrodynamic squeeze film lubrication of curved circular plates with couple stress fluid has been analyzed. The lower plate curvature is described in hyperbolic form, where as upper plate curvature is in exponential form. A modified Reynolds equation is derived. The closed form solution is obtained for pressure, load carrying capacity and squeeze film time. It is found that the influence of applied magnetic field and couple stress parameter between the curved circular plates is to increase the pressure, load carrying capacity and squeeze film time.

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1 Introduction

Stokes [1] developed couplestress fluid model, which is one among polar fluid theory that describes the effects of couple stresses in fluids and all the major features which caused due to the mechanical interactions that occur inside a deforming continuum. It appears in the problems where thin film exists. Mokhiamer et. al [2] investigated the behavior of finite journal bearings lubricated with an electrically conducting fluid and seen that the effect of the couple stresses is more, when chain length of the additive molecule is larger. Hanumagowda et.al [3] studied Circular stepped plates lubricated with couple stress fluid by referring to the combined effect
of couple stress and pressure dependent viscosity. It was seen that the effect of couple stress and pressure dependency viscosity can improve dimensionless pressure, dimensionless load-carrying capacity, and the dimensionless response time when compared with iso-viscous lubricant case. Syeda Tasneem et al. [4, 5, 6] analyzed the behavior of squeeze film motion in presence of transverse magnetic field for distinct finite plates with an electrically conducting fluid and reported that there is steady growth in load-carrying capacity and delay in response time compared to the Newtonian fluid. Many researchers have used Stokes couple stress fluid model to study the various hydrodynamic lubrication problems [7, 8, 9]. Sukla [10] observed the squeeze film behavior of composite slider bearing in magnetic field and it may be noted that as the value of Hartmann number increases, load carrying capacity increases. Lin [11] showed the squeeze film motion between wide tapered-land slider bearings in presence of magnetic field and inferred that the effect of magnetic field could rise the load-carrying capacity and dynamic co-efficient decline the steady friction parameter. Naduvanamani et al. [12] presented the circular stepped plated lubricated with the couple stress fluid in the presence of magnetic field and they have determined that combined effect of MHD and couple stress will increase the load-carrying capacity and response time. Naduvanamani and Hanumagowda [13] also used couple stress model and MHD to carry out investigation on inclined stepped composite slider bearing. They reported that the load-carrying capacity, fluid film pressure, coefficient of friction and frictional force increases with increase in magnetic field.

In the present article, the influence of magnetic field on an electrically conducting couple stress between curved circular plates has been discussed. Results are obtained for film pressure, load-carrying capacity and squeeze film time.

### 2 Mathematical Formulation

Figure 1 display a schematic diagram of the squeeze-film lubrication between two curved circular plates with transverse magnetic field. \( h_0 \) central thickness of Fluid film between two curved circular plates, \( \beta \) and \( \gamma \) are curvature parameters of upper and lower plates, respectively. The uniform magnetic field \( B_0 \) is applied perpendicular to plates. The governing equations of motion for an electrically conducting fluid with applied transverse magnetic field are given by

\[
\begin{align*}
\mu \frac{\partial^2 u}{\partial z^2} - \eta \frac{\partial^4 u}{\partial z^4} - \sigma B_0^2 u &= \frac{\partial p}{\partial r}, \\
\frac{\partial p}{\partial z} &= 0
\end{align*}
\]
1. \frac{1}{r} \frac{\partial}{\partial r} (ru) + \frac{\partial w}{\partial z} = 0 \quad (3)

Where \( p \) is the pressure of fluid film region, \( \mu \) is the dynamic viscosity, \( u \) and \( w \) are the fluid velocity components in the radial \((r)\) and axial \((z)\) directions respectively and \( \eta \) is material constant responsible for the couple stress fluids.

Required boundary conditions are:

**At upper plate surface** \( z = h \).

\[ u = 0, \quad \frac{\partial^2 u}{\partial z^2} = 0 \quad (4) \]
\[ w = V = -\frac{dh}{dt} \quad (5) \]

**At lower plate surface** \( z = 0 \).

\[ u = 0, \quad \frac{\partial^2 u}{\partial z^2} = 0 \quad (6) \]
\[ w = 0 \quad (7) \]

Upper plate surface is given by the relation:

\[ z_u = h_0 \exp(-\beta r^2); \quad 0 \leq r \leq a \quad (8) \]

Lower plate surface is given by the relation:

\[ z_i = h_0 \left( \frac{1}{1 + \gamma r} - 1 \right); \quad 0 \leq r \leq a \quad (9) \]
Where $\beta$ and $\gamma$ are the upper and lower plate curvature parameters respectively and $h_0$ is the central film thickness. Therefore, the film thickness $h(r)$ is considered as

$$h(r) = h_0 \left\{ \exp(-\beta r^2) - \frac{1}{1 + \gamma r} + 1 \right\}; \quad 0 \leq r \leq a$$

(10)

Solution of equation (1) using boundary conditions (4) and (6) is

$$u = \left[ (g_1 - g_2) - 1 \right] \frac{h_0^2}{\mu M_0^2} \frac{\partial p}{\partial r}$$

(11)

where $l = \left( \frac{a}{\mu} \right)^{\frac{1}{2}}$ is the couple stress parameter, $M_0 = B_0 h_0 \left( \frac{a}{\mu} \right)^{\frac{1}{2}}$ is the Hartmann number.

$$g_1 = g_{11}, g_2 = g_{12}, \quad \text{for } 4M_0^2l^2/h_0^2 < 1$$

(12)

$$g_1 = g_{21}, g_2 = g_{22}, \quad \text{for } 4M_0^2l^2/h_0^2 = 1$$

(13)

$$g_1 = g_{31}, g_2 = g_{32}, \quad \text{for } 4M_0^2l^2/h_0^2 > 1$$

(14)

The associated relations in equations (12), (13) and (14) are given in Appendix A.

Integrating the continuity equation (3), using the conditions given by (5) and (7) we get Reynolds equation for film pressure as

$$\frac{1}{r} \frac{\partial}{\partial r} \left\{ rf(h, l, M_0) \frac{\partial p}{\partial r} \right\} = \mu V$$

(15)

where

$$f(h,l,M_0) = \begin{cases} \frac{h_0^2}{h_0^2} \left( \frac{2l}{A^2 - B^2} \left( \frac{B^2}{A} \tanh \frac{Ah}{2l} - \frac{A^2}{B} \tanh \frac{Bh}{2l} \right) + h \right) & \text{for } M_0^2l^2/h_0^2 < 1, \\ \frac{h_0^2}{h_0^2} \left( \frac{h}{2 \sqrt{2}} \right) - 3 \sqrt{2} \tanh \left( \frac{h}{2 \sqrt{2}} \right) + h & \text{for } M_0^2l^2/h_0^2 = 1, \\ \frac{h_0^2}{h_0^2} \left( \frac{2lh_0}{M_0} \left( (A_2 \cot \theta - B_2) \sin B_2h - (B_2 \cot \theta + A_2) \sinh A_2h \right) \cos B_2h + \cosh A_2h \right) + h & \text{for } M_0^2l^2/h_0^2 > 1. \end{cases}$$

Introducing the following dimensionless variables

$$r^* = \frac{r}{a}, \quad h^* = \frac{h}{h_0}, \quad l^* = \frac{2l}{h_0}, \quad K = \beta a^2, \quad C = \gamma a, \quad P^* = \frac{-h_0^3p}{\mu a^2V}$$

(16)

in equation (15)

$$\frac{1}{r^*} \frac{\partial}{\partial r^*} \left\{ r^* f^*(h^*, l^*, M_0) \frac{\partial P^*}{\partial r^*} \right\} = -1$$

(17)
where
\[
f^*(h^*, l^*, M_0) = \begin{cases} 
\frac{1}{M_0^2} \left\{ \frac{l^*}{(A^* - B^*)} \left( \frac{B^*}{A^*} \tanh \frac{A^*}{l^*} - \frac{A^*}{B^*} \tanh \frac{B^*}{l^*} \right) + h^* \right\} & \text{for } M_0^2 l^* < 1, \\
\frac{1}{M_0^2} \left\{ \frac{h^*}{2} \sec h^2 \left( \frac{h^*}{\sqrt{2}} \right) - \frac{3l^*}{2} \tanh \left( \frac{h^*}{\sqrt{2}} \right) + h^* \right\} & \text{for } M_0^2 l^* = 1, \\
\frac{1}{M_0^2} \left\{ \frac{l^*}{M_0} (A^* \cot \theta^* - B^*) \sinh B^* h^* - l^* (B^* \cot \theta^* + A^*) \sin A^* h^* + h^* \right\} & \text{for } M_0^2 l^* > 1, 
\end{cases}
\] (18)

The expression for dimensionless pressure is
\[
P^* = -\frac{1}{2} \int_1^{r^*} \frac{r^*}{f^*(h^*, l^*, M_0)} dr^* \quad (19)
\]

The bearing load-carrying capacity in dimensionless form can be obtained by integrating the pressure over the film region:
\[
W^* = \frac{w h_0^3}{\pi \mu a^4 V} = -\frac{1}{2} \int_0^{h_2^*} \left\{ \int_1^{r^*} \frac{r^*}{f^*(h^*, l^*, M_0)} dr^* \right\} r^* dr^* \quad (20)
\]

The non dimensional squeeze film time is given by
\[
T^* = \frac{w h_0^2}{\pi \mu a^3} = -\frac{1}{2} \int_{h_1^*}^{h_2^*} \left[ \int_0^{r^*} \left\{ \int_1^{r^*} \frac{r^*}{f^*(h^*, l^*, M_0)} dr^* \right\} r^* dr^* \right] dh^* \quad (21)
\]

where
\[
h^* = \left\{ \exp(-K r^*^2) - \frac{1}{1 + C r^*} + 1 \right\}, \quad dh^* = \left\{ -2 Kr^* \exp(-K r^*^2) + \frac{C}{(1 + C r^*)^2} \right\} dr^*
\]
\[
h_1^* = \exp(-K r^*), \quad h_2^* = \frac{1}{1 + C r^*} - 1
\]

3 Results and Discussion.

The effect of magnetic field and couple stresses on characteristics of squeeze film is studied through parameters $M_0, l^*, \beta$ and $\gamma$. For the discussion of squeeze film characteristics the parametric values of $l^* = 0, 0.3$, $M_0 = 0, 2, 4, 6, \beta=0.4, \gamma=0.4$ and $r^*=0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1$ are chosen.
3.1 Non-dimensional pressure distribution.

Figure-2, depicts the dimensionless film pressure $P^*$ is plotted versus $r^*$ for distinct values of $\beta$ and $\gamma$ with $l^*=0.3$, $M_0=3$. It is found that the dimensionless film pressure increase for increasing values of $\beta$ and it decreases for increasing values of $\gamma$. In Fig-3, depicts the dimensionless film pressure $P^*$ is plotted versus $r^*$ for distinct values of $l^*$ and $M_0$ with $\beta=0.4$, $\gamma=0.4$, it is seen that for larger numerical values of the Hartmann number $M_0$ and couple stress parameter $l^*$, the pressure $P^*$ increases.

3.2 Non-dimensional load carrying capacity

Non-dimensional load capacity $W^*$ is plotted versus $\beta$ for various values of $l^*$ and $M_0$ with $\gamma=0.4$ as shown in Fig.4, it is seen that the load capacity $W^*$ increases subsequently as increasing the values of $\beta$, $l^*$ and $M_0$. Further, the effect of $M_0$ and $l^*$ on $W^*$ is elaborated in Fig.5 with $\beta=0.4$, it is interesting to note that for increasing the values of $\gamma$ load decreases with increasing values of $M_0$ and $l^*$. This shows that the lower plate curvature parameter should be kept at minimum for obtaining a better performance.
3.3 Non-dimensional squeeze film time

In Fig.6, Non-dimensional squeeze film time $T^*$ is plotted versus $\beta$ for distinct values of $M_0$ and $l^*$ with $\gamma=0.4$ and observed that the effect of the magnetic field $M_0$ and couple stress parameter $l^*$ increases subsequently squeeze film time $T^*$. Further, variation of $T^*$ is plotted versus $\gamma$ for distinct values of $M_0$ and $l^*$ with $\beta=0.4$ is shown in Fig.7. It is reported that for larger numerical values of $M_0$ and $l^*$ along with $\beta$, the squeeze film time increases as compared to $M_0=0$ (Non Magnetic case) and $l^*=0$ (Newtonian case). From above results and discussions, it should be noted that the film pressure and load capacity increase with increasing couple stress and magnetic parameter for greater values of $\beta$ and lower values of $\gamma$.

4 Conclusions.

Based on couple-stress fluid theory by Stokes [1], in present article we analyzed the combined effects of MHD and couple stress between curved circular plates. From the obtained results the following interpretation may be drawn.

1. The film pressure, load capacity and squeeze film time increases with effect of MHD and couple stress parameter.
2. The film pressure, load capacity and squeeze film time increases with increasing the values of upper plate curvature parameter.
3. The film pressure and load capacity are decreased with increasing values of lower plate curvature parameter but squeeze film time increases with increasing values of lower plate curvature parameter.

To obtain the better results the lower plate curvature parameter is kept at minimum.
5 Appendix: A

\[ g_{11} = \frac{A^2}{(A^2 - B^2)} \cosh \left\{ \frac{B(2z-h)}{2l} \right\}, \quad g_{12} = \frac{B^2}{(A^2 - B^2)} \cosh \left\{ \frac{A(2z-h)}{2l} \right\}, \quad A = \left\{ \frac{1 + (1 - 4M_0^2/h_0^2)^{1/2}}{2} \right\}^{1/2}, \]

\[ B = \left\{ \frac{1 - (1 - 4M_0^2/h_0^2)^{1/2}}{2} \right\}^{1/2}, \quad g_{21} = \frac{2 \cosh \left\{ \frac{(z-h)/\sqrt{2l}}{l} \right\} + 2 \cosh \left\{ z/\sqrt{2l} \right\}}{2 \left\{ \cosh \left( h/\sqrt{2l} \right) + 1 \right\}}, \]

\[ g_{22} = \frac{z/\sqrt{2l} \sinh \left\{ \frac{(z-h)/\sqrt{2l}}{l} \right\} + \left\{ (z-h)/\sqrt{2l} \right\} \sinh \left\{ z/\sqrt{2l} \right\}}{2 \left\{ \cosh \left( h/\sqrt{2l} \right) + 1 \right\}}, \quad g_{31} = \frac{\cos B_2 \cosh A_2(z - h) + \cosh A_2 \cos B_2(z - h)}{\cosh A_2h + \cos B_2h}, \]

\[ A_2 = \sqrt{M_0/h_0} \cos (\theta/2), \quad B_2 = \sqrt{M_0/h_0} \sin (\theta/2), \quad \theta = \tan^{-1} \left( \frac{\sqrt{4I^2 M_0^2/h_0^4}}{4l} - 1 \right) \]

References


