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SIGNED PRODUCT CORDIAL LABELING ON SOME CONNECTED GRAPHS

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Abstract

A vertex labeling of a graph G is a function $f : V(G) \to \{-1, 1\}$ with an induced edge labeling $f^* : E(G) \to \{-1, 1\}$ defined by $f^*(uv) = f(u)f(v)$ is called a signed product cordial labeling if $|v_f(-1) - v_f(1)| \leq 1$ and $|e_{f^*}(-1) - e_{f^*}(1)| \leq 1$, where $v_f(-1)$ and $v_f(1)$ are the number of vertices labeled with -1 and $+1$ respectively and $e_{f^*}(-1)$ and $e_{f^*}(1)$ are the number of edges labeled with -1 and $+1$ respectively. This paper focuses on the existence of signed product cordial labeling for removal of few edges from a complete graph, the maximal planar graph, $Go(P_n)$, $n > 4$, $G\hat{o}e$ and $G - e$.

AMS Subject Classification: 05C78.

Key Words and Phrases: Graphs, cordial, bijective function.

1 Introduction

A vertex labeling of a graph G is a mapping f from the set of vertices of G to a set of elements, often integers. Each edge xy has a label that depends on the vertices x and y and their labels $f(x)$ and $f(y)$. Graph labeling methods began with Rosa [?] in 1967. In 2011, Babujee and Shobana [?] introduced the concept of signed product cordial labeling. A vertex labeling of a graph G $f : V(G) \to \{-1, 1\}$ with an induced edge labeling $f^* : E(G) \to \{-1, 1\}$ defined by $f^*(uv) = f(u)f(v)$ is called a signed product cordial labeling if $|v_f(-1) - v_f(1)| \leq 1$ and $|e_{f^*}(-1) - e_{f^*}(1)| \leq 1$, where $v_f(-1)$ is the number of vertices labeled with -1, $v_f(1)$ is the number of vertices labeled with 1, $e_{f^*}(-1)$ is the number of edges labeled with -1 and $e_{f^*}(1)$ is the number of edges labeled with 1. In this paper we proved the existence of signed product cordial labeling for removal of few edges from a complete graph, the maximal planar graph, $Go(P_n)$, $n > 4$, $G\hat{o}e$ and $G - e$.

Definition 1. [?] A graph $Go(P_n)$ is obtained from a path P_n by introducing new edges between any two vertices if they are at odd distance. $Go(P_n)$ has n vertices and $(n^2 - 1)/4$ edges if $n$is odd and $n^2/4$ edges if n is even.

Definition 2. [?] If $G_1(p_1, q_1)$ and $G_2(p_2, q_2)$ are two connected graphs, $G_1\hat{o}G_2$ is obtained by superimposing any selected vertex of G_2 on any selected vertex of G_1. The resultant graph $G = G_1\hat{o}G_2$ consists of $p_1 + p_2 - 1$ vertices and $q_1 + q_2$ edges.

Definition 3. [?] Let $K_n = (V, E)$ be the complete graph with n vertices $V(K_n) = \{1, 2, \ldots, n\}$ and $n(n - 1)/2$ edges $E(K_n) = \{(i, j) : 1 \leq i, j \leq n \text{ and } i < j\}$.

Definition 4. [?] The class of planar graphs with maximal edges over n vertices derived from complete graph K_n is defined as $Pl_n = (V, E)$, where $V = \{1, 2, \ldots, n\}$ and $E = E(K_n)\backslash\{(k, l) : k = 3 \text{ to } n - 2, l = k + 2 \text{ to } n\}$. The number of edges in $Pl_n : n \geq 5$ is $3(n - 2)$.

2 Main Results

Theorem 5. *The removal of few edges from a complete graph K_n, $n \geq 4$ admits signed product cordial labeling.*

Proof. Let K_n be a complete graph with n vertices and nC_2 edges. Let the vertex set be $V = \{v_1, v_2, v_3 \ldots v_n\}$ and the edge set be $E = \{v_iv_j/1 \leq i, j \leq n, j > i\}$. The following two cases for the edge set are to be considered after the removal of few edges from the complete graph K_n.

Case (i) $n \equiv 1 \pmod 2$

Remove $(n-3)/2$ edges from K_n and define the edge set as $E = \{v_iv_j/1 \leq i,j \leq n, j > i\} - \{v_1v_{2j}/1 \leq j \leq (n-3)/2\}$. Then the number of edges of K_n reduces to $(n^2 - 2n + 3)/2$.

Case (ii) $n \equiv 0 \pmod 2$

Remove $(n-2)/2$ edges from K_n and define the edge set as $E = \{v_iv_j/1 \leq i,j \leq n, j > i\} - \{v_1v_{2j}/1 \leq j \leq (n-2)/2\}$. Then the number of edges of K_n reduces to $(n^2 - 2n + 2)/2$.

Define a vertex labeling $f : V \to \{1, -1\}$ as

$$f(v_i) = \begin{cases} 1 & i \equiv 1 \pmod 2 \\ -1 & i \equiv 0 \pmod 2; 1 \leq i \leq n \end{cases}$$

Then the induced edge labeling $f^* : E(G) \to \{1, -1\}$ is as follows:

$$f^*(v_iv_j) = \begin{cases} 1 & \text{if } f(v_i) \text{ and } f(v_j) \text{ have same sign} \\ -1 & \text{if } f(v_i) \text{ and } f(v_j) \text{ have oppositesigns;} \\ & 1 \leq i,j \leq n; j > i \end{cases}$$

In view of the above labeling pattern, the vertex and the edge labeling conditions are as follows

Table 1: Vertex and Edge Conditions

n	$v_f(1)$	$v_f(-1)$	$\lvert v_f(1) - v_f(-1)\rvert$
$n \equiv 0 \pmod 2$	$n/2$	$n/2$	0
$n \equiv 1 \pmod 2$	$(n+1)/2$	$(n-1)/2$	1
n	$e_{f^*}(1)$	$e_{f^*}(-1)$	$\lvert e_{f^*}(1) - e_{f^*}(-1)\rvert$
$n \equiv 0 \pmod 2$	$(n^2 - 2n)/4$	$(n^2 - 2n + 4)/4$	1
$n \equiv 1 \pmod 2$	$(n^2 - 2n + 1)/4$	$(n^2 - 2n + 5)/4$	1

From the above table, we infer that the complete graph with removal of few edges admits signed product cordial labeling. □

Theorem 6. *The maximal planar graph Pl_n, $n > 4$ is a signed product cordial labeling except for $n \equiv 0 \pmod 4$.*

Proof. Let pl_n be a maximal planar graph with n vertices and $(3n-6)$ edges. The vertex set and edge set of a planar graph are defined as $V = \{v_1, v_2, \ldots, v_n\}$ and $E = E_1 \cup E_2 \cup E_3 \cup E_4$ where

$$E_1 = \{v_i v_{i+1} : 3 \leq i \leq n-1\} \quad E_2 = \{v_1 v_{i+2} : 1 \leq i \leq n-2\}$$
$$E_3 = \{v_2 v_{i+2} : 2 \leq i \leq n-2\} \quad E_4 = \{v_1 v_2, v_2 v_3\}$$

Define $f : V \to \{1, -1\}$ as a bijective function such that

$$f(v_i) = \begin{cases} 1 & i \equiv 0, 3 \pmod 4 \\ -1 & i \equiv 1, 2 \pmod 4; 3 \leq i \leq n \end{cases} \qquad f(v_1) = 1 \quad f(v_2) = -1$$

The induced edge labeling $f^* : E(G) \to \{1, -1\}$ is defined as follows

$$f^*(v_i v_{i+1}) = \begin{cases} 1, & \text{if } f(v_i) \text{ and } f(v_{i+1}) \text{ have same sign} \\ -1 & \text{if } f(v_i) \text{ and } f(v_{i+1}) \text{ have oppositesigns}; 3 \leq i \leq n-1 \end{cases}$$

$$f^*(v_1 v_{i+2}) = \begin{cases} 1, & \text{if } f(v_1) \text{ and } f(v_{i+2}) \text{ have same sign} \\ -1 & \text{if } f(v_1) \text{ and } f(v_{i+2}) \text{ have oppositesigns}; 1 \leq i \leq n-2 \end{cases}$$

$$f^*(v_2 v_{i+2}) = \begin{cases} 1, & \text{if } f(v_2) \text{ and } f(v_{i+2}) \text{ have same sign} \\ -1 & \text{if } f(v_2) \text{ and } f(v_{i+2}) \text{ have oppositesigns}; 2 \leq i \leq n-2 \end{cases}$$

$$f^*(v_i v_{i+1}) = \begin{cases} 1, & \text{if } f(v_i) \text{ and } f(v_{i+1}) \text{ have same sign} \\ -1 & \text{if } f(v_i) \text{ and } f(v_{i+1}) \text{ have oppositesigns}; 1 \leq i \leq 2 \end{cases}$$

In view of the above labeling pattern, the vertex and the edge labeling conditions are as follows

Table 2: Vertex and Edge Conditions of a Planar Graph

n	$v_f(1)$	$v_f(-1)$	$\lvert v_f(1) - v_f(-1) \rvert$
$n \equiv 2 \pmod 4$	$n/2$	$n/2$	0
$n \equiv 1 \pmod 2$	$(n+1)/2$	$(n-1)/2$	1
n	$e_{f^*}(1)$	$e_{f^*}(-1)$	$\lvert e_{f^*}(1) - e_{f^*}(-1) \rvert$
$n \equiv 2 \pmod 4$	$(3n-6)/2$	$(3n-6)/2$	0
$n \equiv 1 \pmod 2$	$(3n-7)/2$	$(3n-5)/2$	1

From the above table, we can conclude that the planar graph Pl_n, $n > 4$ admits signed product cordial labeling. □

Theorem 7. *The graph $Go(P_n)$, $n > 4$ is a signed product cordial graph.*

Proof. Consider the graph $Go(P_n)$, $n > 4$. The vertex set of the graph $Go(P_n)$ is defined as $V = \{v_1, v_2, \ldots, v_n\}$ and the edge set is defined as follows; when n is even,
$E(G) = \{v_i v_{2j}/1 \leq j \leq n/2; i \text{ is odd and } i < 2j\} \cup \{v_i v_{2j-1}/1 \leq j \leq n/2; i \text{ is even and } i < 2j-1\}$ and when n is odd
$E(G) = \{v_i v_{2j}/1 \leq j \leq (n-1)/2; i \text{ is odd and } i < 2j\} \cup \{v_i v_{2j-1}/1 \leq j \leq (n+1)/2; i \text{ is even and } i < 2j-1\}$
Define the vertex labeling $f : V \to \{1, -1\}$ as follows:
Case (i) $n \equiv 1 \pmod 2$

$$f(v_i) = f(v_{i+1}) = \begin{cases} 1 & i \equiv 1 \pmod 4 \\ -1 & i \equiv 3 \pmod 4; 1 \leq i \leq n-1; i \equiv 1 \bmod 2 \end{cases}$$

Case (ii) $n \equiv 0 \pmod 2$
Subcase (i) $n \equiv 0 \pmod 4$

$$f(v_i) = f(v_{i+1}) = \begin{cases} 1 & i \equiv 1 \pmod 4 \\ -1 & i \equiv 3 \pmod 4; 1 \leq i \leq n-1; i \equiv 1 \bmod 2 \end{cases}$$

Subcase (ii) $n \equiv 2 \pmod 4$

$$f(v_i) = f(v_{i+1}) = \begin{cases} 1 & i \equiv 1 \pmod 4 \\ -1 & i \equiv 3 \pmod 4; 1 \leq i \leq n-2; i \equiv 1 \bmod 2 \end{cases}$$

$$f(v_n) = -1 \qquad f(v_{n-1}) = 1$$

The induced edge labeling $f^* : E(G) \to \{1, -1\}$ is defined as follows

$$f^*(v_i v_{2j}) = \begin{cases} 1, & \text{if } f(v_i) \text{ and } f(v_{2j}) \text{ have same sign} \\ -1 & \text{if } f(v_i) \text{ and } f(v_{2j}) \text{ have oppositesigns;} \\ & 1 \leq j \leq n/2, n \text{ even}, i < 2j, i \text{ odd} \end{cases}$$

$$f^*(v_i v_{2j-1}) = \begin{cases} 1, & \text{if } f(v_i) \text{ and } f(v_{2j-1}) \text{ have same sign} \\ -1 & \text{if } f(v_i) \text{ and } f(v_{2j-1}) \text{ have oppositesigns;} \\ & 1 \leq j \leq n/2, n \text{ even}, i < 2j, i \text{ even} \end{cases}$$

$$f^*(v_iv_{2j}) = \begin{cases} 1, & \text{if } f(v_i) \text{ and } f(v_{2j}) \text{ have same sign} \\ -1 & \text{if } f(v_i) \text{ and } f(v_{2j}) \text{ have oppositesigns;} \\ & 1 \leq j \leq (n-1)/2, n \text{ odd}, i < 2j, i \text{ odd} \end{cases}$$

$$f^*(v_iv_{2j-1}) = \begin{cases} 1, & \text{if } f(v_i) \text{ and } f(v_{2j-1}) \text{ have same sign} \\ -1 & \text{if } f(v_i) \text{ and } f(v_{2j-1}) \text{ have oppositesigns;} \\ & 1 \leq j \leq (n+1)/2, n \text{ odd}, i < 2j-1, i \text{ even} \end{cases}$$

The vertex and edge labeling conditions of the graph $Go(P_n)$ are as follows:

Table 3: Vertex and Edge Conditions of a $Go(P_n)$

n	$v_f(1)$	$v_f(-1)$	$\lvert v_f(1) - v_f(-1)\rvert$
$n \equiv 1 \pmod 2$	$(n+1)/2$	$(n-1)/2$	1
$n \equiv 0 \pmod 4$	$n/2$	$n/2$	0
$n \equiv 2 \pmod 4$	$n/2$	$n/2$	0
n	$e_{f^*}(1)$	$e_{f^*}(-1)$	$\lvert e_{f^*}(1) - e_{f^*}(-1)\rvert$
$n \equiv 1 \pmod 2$	$(n^2-1)/8$	$(n^2-1)/8$	0
$n \equiv 0 \pmod 4$	$n^2/8$	$n^2/8$	0
$n \equiv 2 \pmod 4$	$(n^2-4)/8$	$(n^2+4)/8$	1

Hence the resultant graph follows signed product cordial labeling. □

Theorem 8. *If a graph G is a signed product cordial graph then $G\hat{o}e$ is also a signed product cordial graph except for few vertex and edge labeling conditions.*

Proof. Let G be a signed product cordial graph with $\{v_1, v_2, v_3 \dots v_n\}$ vertices and $\{e_1, e_2, e_3 \dots e_m\}$ edges. Consider an edge $e = \{v_{n+1}v_{n+2}\}$. Then $G\hat{o}e$ is the graph obtained by superimposing an edge e with any vertex v_i of G. Denote this graph by G' with $|V'| = n+1$ and $|E'| = m+1$. Define the vertex labeling as $f : V'(G') \to \{1, -1\}$ with an induced edge labeling $f^* : E'(G') \to \{1, -1\}$ then the following four different cases are to be considered.

Case (i) $|v_f(1) - v_f(-1)| = 0$; $|e_{f^*}(1) - e_{f^*}(-1)| = 0$

subcase (i) If v_i and v_{n+1} have the same label $+1$, then the edge e will receive the same label $+1$. Therefore $|v'_f(1) - v'_f(-1)| = 1$ and

$|e'_{f*}(1) - e'_{f*}(-1)| = 1$.
subcase (ii) If v_i has the label +1and v_{n+1} has the label -1, then the edge e will receive the label -1. Therefore $|v'_f(1) - v'_f(-1)| = 1$ and $|e'_{f*}(1) - e'_{f*}(-1)| = 1$.
subcase (iii) If v_i has the label -1and v_{n+1} has the label +1, then the edge e will receive the label -1. Therefore $|v'_f(1) - v'_f(-1)| = 1$ and $|e'_{f*}(1) - e'_{f*}(-1)| = 1$.
subcase (iv) If v_i and v_{n+1} have the same label -1, then the edge e will receive the same label +1. Therefore $|v'_f(1) - v'_f(-1)| = 1$ and $|e'_{f*}(1) - e'_{f*}(-1)| = 1$.
Case (ii) $|v_f(1) - v_f(-1)| = 1$; $|e_{f^*}(1) - e_{f^*}(-1)| = 1$.
subcase (i) If v_i has the label +1and v_{n+1} has the label -1 with $|v_f(1)| > |v_f(-1)|$ and $|e_{f^*}(1)| > |e_{f|*}(-1)|$, then the edge e will receive the label -1. Therefore $|v'_f(1) - v'_f(-1)| = 0$ and $|e'_{f^*}(1) - e'_{f^*}(-1)| = 0$.
subcase (ii) If v_i has the label -1and v_{n+1} has the label +1 with $|v_f(-1)| > |v_f(1)|$ and $|e_{f^*}(1)| > |e_{f|*}(-1)|$, then the edge e will receive the label -1. Therefore $|v'_f(1) - v'_f(-1)| = 0$ and $|e'_{f^*}(1) - e'_{f^*}(-1)| = 0$.
subcase (iii) If v_i has the label 1and v_{n+1} has the label 1 with $|v_f(-1)| > |v_f(1)|$ and $|e_{f*}(-1)| > |e_{f*}(1)|$, then the edge e will receive the label 1. Therefore $|v'_f(1) - v'_f(-1)| = 0$ and $|e'_{f^*}(1) - e'_{f^*}(-1)| = 0$.
subcase (iv) If v_i has the label -1and v_{n+1} has the label -1 with $|v_f(1)| > |v_f(-1)|$ and $|e_{f*}(-1)| > |e_{f*}(1)|$, then the edge e will receive the label 1. Therefore $|v'_f(1) - v'_f(-1)| = 0$ and $|e'_{f^*}(1) - e'_{f^*}(-1)| = 0$.
Case (iii) $|v_f(1) - v_f(-1)| = 1$; $|e_{f^*}(1) - e_{f^*}(-1)| = 1$.
subcase (i) If v_i has the label +1and v_{n+1} has the label 1 with $|e_{f*}(-1)| > |e_{f*}(1)|$, then the edge e will receive the label 1. Therefore $|v'_f(1) - v'_f(-1)| = 1$ and $|e'_{f^*}(1) - e'_{f^*}(-1)| = 0$.
subcase (ii) If v_i has the label -1and v_{n+1} has the label -1 with $|e_{f*}(-1)| > |e_{f*}(1)|$, then the edge e will receive the label 1. Therefore $|v'_f(1) - v'_f(-1)| = 1$ and $|e'_{f^*}(1) - e'_{f^*}(-1)| = 0$.
subcase (iii) If v_i has the label +1and v_{n+1} has the label -1 with $|e_{f*}(1)| > |e_{f*}(-1)|$, then the edge e will receive the label -1. Therefore $|v'_f(1) - v'_f(-1)| = 1$ and $|e'_{f^*}(1) - e'_{f^*}(-1)| = 0$.
subcase (iv) If v_i has the label -1and v_{n+1} has the label +1 with $|e_{f*}(1)| > |e_{f*}(-1)|$, then the edge e will receive the label -1.

Therefore $|v'_f(1) - v'_f(-1)| = 0$ and $|e'_{f^*}(1) - e'_{f^*}(-1)| = 0$.
Case (iv) $|v_f(1) - v_f(-1)| = 1$ and $|e_{f^*}(1) - e_{f^*}(-1)| = 0$
subcase (i) If v_i has the label 1and v_{n+1} has the label 1 with $|v_f(-1)| > |v_f(1)|$, then the edge e will receive the label 1. Therefore $|v'_f(1) - v'_f(-1)| = 0$ and $|e'_{f^*}(1) - e'_{f^*}(-1)| = 1$.
subcase (ii) If v_i has the label -1and v_{n+1} has the label -1 with $|v_f(1)| > |v_f(-1)|$, then the edge e will receive the label 1. Therefore $|v'_f(1) - v'_f(-1)| = 0$ and $|e'_{f^*}(1) - e'_{f^*}(-1)| = 1$.
subcase (iii) If v_i has the label 1and v_{n+1} has the label -1 with $|v_f(1)| > |v_f(-1)|$, then the edge e will receive the label -1. Therefore $|v'_f(1) - v'_f(-1)| = 0$ and $|e'_{f^*}(1) - e'_{f^*}(-1)| = 1$.
subcase (iv) If v_i has the label -1and v_{n+1} has the label 1 with $|v_f(-1)| > |v_f(1)|$, then the edge e will receive the label -1. Therefore $|v'_f(1) - v'_f(-1)| = 0$ and $|e'_{f^*}(1) - e'_{f^*}(-1)| = 1$. Hence from the above cases, we infer that the graph $G\hat{o}e$ admits signed product cordial labeling. □

Theorem 9. *If a graph G is a signed product cordial graph then $G-e$ is also a signed product cordial graph except for the cases $|v_f(1)-v_f(-1)| = 1$; $|e_{f*}(1)-e_{f*}(-1)| = 1$ and $|v_f(1)-v_f(-1)| = 0$; $|e_{f*}(1) - e_{f*}(-1)| = 1$ with $f^*(e) = 1$, $|e_{f*}(-1)| > |e_{f*}(1)|$ and $f^*(e) = -1$, $|e_{f*}(1)| > |e_{f*}(-1)|$.*

Proof. Let G be a signed product cordial graph with $\{v_1, v_2, v_3 \dots v_n\}$ vertices and $\{e_1, e_2, e_3 \dots e_m\}$ edges. Delete an edge e from the graph G. Then $G - e$ is the graph obtained by deleting any edge e from G. Denote this graph by G' with $|V'| = n$ and $|E'| = m-1$. Define the vertex labeling as $f : V'(G') \to \{1, -1\}$ with an induced edge labeling $f^* : E'(G') \to \{1, -1\}$ then the following four different cases are to be considered.
Case (i) $|v_f(1) - v_f(-1)| = 1$; $|e_{f*}(1) - e_{f*}(-1)| = 1$
If the edge e is labeled as 1 and $|e_{f*}(1)| > |e_{f*}(-1)|$ then $|e'_{f*}(1) - e'_{f*}(-1)| = 0$. If the edge e is labeled as -1 and $|e_f(-1)| > |e_f(1)|$ then $|e'_{f*}(1) - e'_{f*}(-1)| = 0$.
Case (ii) $|v_f(1) - v_f(-1)| = 0$; $|e_{f*}(1) - e_{f*}(-1)| = 0$
If the edge e is labeled as 1or -1 we have $|e'_{f*}(1) - e'_{f*}(-1)| = 1$.
Case (iii) $|v_f(1) - v_f(-1)| = 1$; $|e_{f*}(1) - e_{f*}(-1)| = 0$
If the edge e is labeled as 1 or -1 , we have $|e'_{f*}(1) - e'_{f*}(-1)| = 1$.
Case (iv) $|v_f(1) - v_f(-1)| = 0$; $|e_{f*}(1) - e_{f*}(-1)| = 1$

If the edge e is assigned the label 1 and $|e_f(1)| > |e_f(-1)|$ then we have; $|e_{f*}(1) - e_{f*}(-1)| = 0$. If the edge e is assigned the label -1 and $|e_f(1)| > |e_f(-1)|$ then $|e'_{f*}(1) - e'_{f*}(-1)| = 0$. From the above cases we infer that the graph $G-e$ is a signed product cordial graph. □

3 Conclusion

In this paper, an existence of signed product cordial labeling has been examined for some special graphs. To apply the signed product cordial labeling in any of the real time applications is our future work.

References

[1] J. Baskar Babujee, Planar graphs with maximum edges-antimagic property, *The Mathematics Education*, **37**(4) (2003), 194-198.

[2] J. Baskar Babujee, L. Shobana, On Signed Product Cordial Labeling, *Applied Mathematics*, **12** (2011), 1525-1530.

[3] J. Baskar Babujee, S. Babitha, On 1-vertex bimagic vertex labeling, *Tamkang journal of Mathematics*, **45**, No. 3 (2014), 259-273.

[4] S. Babitha, J. Baskar Babujee, Prime cordial labeling on Graphs, *International journal of Mathematical, Computational, Physical, Electrical and Computer engineering*, **7**, No. 5 (2013).

[5] J.A. Gallian, A Dynamic Survey of Graph Labeling, *Electronic Journal of Combinatorics*, #**DS6** (2015), 1-389.

[6] L. Shobana, J. Baskar Babujee, Signed Product Cordial Labeling on Special Graphs, *Global Journal of Pure and Applied Mathematics*, **12** (2016), 376-380.