Triangular Approximation of fuzzy numbers - a new approach

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Abstract

In this paper efforts are made to convert any fuzzy number into a triangular fuzzy number. Constructions are made to find the approximate lower r-cut and upper r-cut of a fuzzy number for a given fuzzy number. This approximation helps us to avoid the computational complexity in the process of decision making. An application of this new method is also provided.

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1 Introduction

Fuzzy numbers play an important role in many applications. But the main hurdle in the development of applications is the computational complexity. Hence more attention is needed to simplify arithmetic computation with fuzzy numbers. By restricting fuzzy number to triangular fuzzy numbers, addition and subtraction become simpler. Therefore we need some approximation methods. Recently there have been many research papers investigating on approximation of fuzzy numbers[3, 2, 4]. Chanas[3] have introduced the notion of an approximation interval of a fuzzy number. Grzegorzewski[2] have suggested a new approach to interval approximation of fuzzy numbers. There have been many papers investigating triangular and trapezoidal approximation of fuzzy numbers[1, 5, 6, 7, 8, 9, 10]. Delgado proposed two parameters(value and ambiguity) to obtain canonical representation of fuzzy numbers. Ma et.al[6] have used the concept of the symmetric triangular fuzzy number and they have introduced a new method to defuzzy a general fuzzy quantity. The basic idea of their method is obtaining the nearest symmetric triangular fuzzy number.
Arithmetic operators have been defined for the basic operations of addition, subtraction, multiplication, division and inverse. However for multiplication the arithmetic operators involve complex polynomials that are difficult to evaluate and computationally expensive. Dubois and Prade [11] analyzed these operators and suggested standard approximation to them. Renald E[12] analyzed the error in the standard approximation method used for multiplication of triangular and trapezoidal fuzzy numbers and developed a new approximation method. In this paper, a simple analytical construction is used to approximate a given fuzzy number as a triangular fuzzy number.

2 Preliminaries

Definition 2.1. A fuzzy set \( \tilde{a} \) defined on the set of real numbers \( \mathbb{R} \) is said to be a fuzzy number if its membership function \( \tilde{a}: \mathbb{R} \rightarrow [0,1] \) has the following characteristics:

1. \( \tilde{a} \) is convex
2. \( \tilde{a} \) is normal: There exists a \( x \) in \( \mathbb{R} \) such that \( \tilde{a}(x) = 1 \)
3. \( \tilde{a}(x) \) is piecewise continuous.

Definition 2.2. A fuzzy number \( \tilde{a}(x) \) is said to be a triangular fuzzy number if its membership function \( \mu: \mathbb{R} \rightarrow [0,1] \) satisfies the following characteristics:

\[
\mu(x) = \begin{cases} 
0 & \text{if } x < a \\
\frac{x-a}{b-a} & \text{if } a < x < b \\
\frac{c-x}{c-b} & \text{if } b < x < c 
\end{cases}
\]

3 Construction of triangular Fuzzy number

In this section we construct a triangular Fuzzy number from the given function.

Let \( \tilde{u} \) be a fuzzy number given by a pair of functions \([u(r), \overline{u}(r)]\) where \( u(r), \overline{u}(r): [0,1] \rightarrow \mathbb{R} \) satisfy the following conditions:

1. \( u(r) \) is a bounded left semi-continuous non-decreasing function
2. \( \overline{u}(r) \) is a bounded left semi-continuous non-increasing function.
3. \( u(r) \leq \overline{u}(r) \)

Then we can approximate \( \tilde{u} \) as a triangular fuzzy number \( \tilde{v} \) as a pair of functions \([v(r), \overline{v}(r)]\) where \( v(r), \overline{v}(r): [0,1] \rightarrow \mathbb{R} \) satisfy the following conditions:

1. \( v(r) \) is non-decreasing
2. \( \overline{v}(r) \) is non-increasing.

3. \( \overline{v}(r) \leq \overline{v}(r) \)

Figure 1:

Now \( E \) is the u-coordinate of \( u(r) \) at \( r=1 \). \( F \) is the u-coordinate of \( \overline{u}(r) \) at \( r=1 \). \( J \) is the midpoint of \( E \) and \( F \). \( H \) is the midpoint of \( A \) and \( E \). \( I \) is the midpoint of \( F \) and \( B \). Now we approximate \( u(r) \) as a straight line \( v(r) \) (which is non-decreasing) passing through \( H \) and \( G \). Therefore the equation of \( \overline{v}(r) \) is the line joining the points \( H(\frac{u(0)+u(1)}{2} ,0) \) and \( G(\frac{u(1)+u(1)}{2} ,1) \). Then the equation becomes

\[
\overline{v}(r) = [\frac{u(0)+u(1)}{2}] + r\frac{\overline{u}(1) - u(0)}{2}
\]

(3.1)

Similarly we approximate \( \overline{u} \) as a straight line \( \overline{u} \) (which is non-increasing) passing through \( I \) and \( G \). Therefore the equation of \( \overline{v} \) is the line joining the points \( I(\frac{\overline{u}(0)+\overline{u}(1)}{2} ,0) \) and \( G(\frac{u(1)+u(1)}{2} ,1) \). Then the equation becomes

\[
\overline{v}(r) = [\frac{\overline{u}(1)+\overline{u}(0)}{2}] - r\frac{\overline{u}(0) - \overline{u}(1)}{2}
\]

(3.2)

If \( u(1) = \overline{u}(1) = m \),

\[
\overline{v}(r) = [\frac{u(0)+m}{2}] + r\frac{m - u(0)}{2}
\]

(3.3)

\[
\overline{v}(r) = [\frac{m + \overline{u}(0)}{2}] - r\frac{\overline{u}(0) - m}{2}
\]

(3.4)

Now we prove the following theorem for \( \tilde{v} \) to be a triangular fuzzy number.
Lemma 3.1. $v(r)$ is non-decreasing and $\overline{v}(r)$ is non-increasing.

Proof. Assume that $r_1 < r_2$.
Then
\[
v(r_1) - v(r_2) = \left[ \frac{u(0) + u(1)}{2} \right] + \frac{(r_1) [\overline{\mu}(1) - u(0)]}{2} - \left[ \frac{u(0) + u(1)}{2} \right] + \frac{r_2^2 [\overline{\mu}(1) - \overline{\mu}(0)]}{2}
\]
\[= \left[ \frac{\overline{\mu}(1) - u(0)}{2} \right] |r_1 - r_2| < 0
\]
Hence $v(r)$ is non-decreasing.

Similarly one can prove that $\overline{v}(r_1) - \overline{v}(r_2) > 0$.

Hence $\overline{v}(r)$ is non-increasing.

4 Numerical Example

4.1 Approximation of a fuzzy number

We approximate the fuzzy number

\[u(x) = \begin{cases} 
4x^2 & 0 < x \leq \frac{1}{2} \\
1 & \frac{1}{2} \leq x \leq \frac{3}{2} \\
4 - 4x & \frac{3}{4} \leq x \leq 1
\end{cases}
\]

$r = 4x^2, u(r) = \frac{\sqrt{r}}{2}$

$r = 4 - 4x, \overline{u}(r) = \frac{4 - r}{4}$

$u(0) = 0, \overline{\mu}(0) = 1u(1) = \frac{1}{2}, \overline{\mu}(1) = \frac{3}{4}$

From (3.1) and (3.2)

\[v(r) = \frac{1}{4} + \frac{3r}{8}
\]

\[\overline{v}(r) = \frac{7}{8} - \frac{r}{4}
\]

Hence the approximate triangular fuzzy number is $(\frac{1}{4}, \frac{5}{8}, \frac{7}{8})$
4.2 Multiplication of two fuzzy numbers

Consider two fuzzy numbers \( \tilde{u}_1 = (1, 2, 3) \) and \( \tilde{u}_2 = (3, 4, 5) \). Then their respective r-cuts will be \( \tilde{u}_1 = (1 + r, 3 - r) \) and \( \tilde{u}_2 = (3 + r, 5 - r) \).

\[
\tilde{u}_1 \ast \tilde{u}_2 = \left( u_1(r) \ast u_2(r), \frac{u_1(r)}{u_2(r)} \right) = \left( (1 + r)(3 + r), (3 - r)(5 - r) \right)
\]

\[
= (r^2 + 4r + 1, r^2 - 8r + 15)
\]

Here \( u(r) = r^2 + 4r + 1 \) and \( \pi(r) = r^2 - 8r + 15 \).

Here \( u(r) \) is a parabola meeting r-axis at \((0,3)\) and \((0,5)\) and meeting u-axis at \((15,0)\).

\( u(r) \) is a parabola meeting the same r-axis at \((0,-1)\) and \((0,-3)\) and meeting u-axis at \((3,0)\). These two curves meet at \((8,1)\). It is obvious that \( u(1) = \pi(1) = 8 \). Now \( m=8 \).

\[
u(r) = 5.5 + 2.5r \quad \pi(r) = 11.5 - 3.5r
\]

Hence the approximate triangular fuzzy from fig.2 is \( \tilde{v} = (5.2, 8, 11.5) \).

Figure 2: R-CUT diagram

5 Conclusion

In the present contribution we have discussed the problem of triangular approximation of fuzzy numbers. It seems that this method is quite simple and gives clear interpretation.
References


