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Modified Fuzzy Sequential Linear Goal Programming Method for Solving Fuzzy Multi Objectives Linear Programming Problems

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Abstract

In this paper, fuzzy sequential linear goal programming method can be used to solve multi objective decision making problems, which involve the trade-off decisions. This paper proposes some modifications in traditional Fuzzy SLGP methods for multi objective decision making problems. It uses existing single objective Fuzzy Linear Programming (FLP) techniques. The information required in each iteration is taken from the previous iteration. The proposed method is illustrated by some numerical examples.

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1 Introduction

The fuzzy sequential linear goal programming method is a quite different approach to the solution of the lexicographic linear goal programming method was developed in 1967 by Ignizco [1]. This approach denoted as fuzzy sequential linear goal programming was based on the observations by Paul Haus that one could solve an lexicographic linear goal programming model [2] via a sequential of conventional linear programming models. Based upon the solution of the previous Linear programming problem an augmented constraints is added to the sequential model. The FGP model is one in which all objectives are converted into goals. This conversion is accomplished by an '*aspiration level*' to the right hand side of each objective. Then one seeks

the solution that minimizes the 'distances' between that solution and the aspired solution. This fundamental ideal leads to the following analytical framework. The general structure of the i$^{\text{th}}$ goal can be expressed as: $\tilde{g}_i = \tilde{f}_i(x) + \tilde{\eta}_i - \tilde{\rho}_i;\ \ i = 1, 2, \ldots, K$ where K = total number of goals. After the formulation of all K number of goals, the next aim is to detect the unwanted deviation variables. The variables are unwanted in the sense that these are the variables that DM wants to minimize. The concept of fuzzy numbers and arithmetic operations with these numbers was first introduced and investigated by Zadeh [3], etc. One of the major applications of fuzzy number arithmetic is treating linear systems and their parameters that are all partially respected by fuzzy number. Friedman etal [4] introduced a general model for solving a fuzzy $n \times n$ linear system whose coefficient matrix is crisp and the right-hand side column is an arbitrary fuzzy number vector. They used the parametric form of fuzzy numbers and replaced the original fuzzy $n \times n$ linear system by a crisp $2n \times 2n$ linear system. In this paper, all the variables are considered as fuzzy variables.

2 Preliminaries

Definition 1 (Fuzzy set)**.** A fuzzy set $\tilde{A}$ is defined by $\tilde{A} = \{(x,\ \mu_A(x)) : x \in A,\ \mu_A(x) \in [0,\ 1]\}$. In the pair $(x,\ \mu_A(x))$, the first element x belongs to the classical set A and the second element $\mu_A(x)$ belongs to the interval [0, 1] called *membership function.*

Definition 2 (Fuzzy number)**.** A fuzzy set $\tilde{A}$ on R must possess at least the following three properties to qualify as a fuzzy number:

(i). $\tilde{A}$ must be a normal fuzzy set;

(ii). $\alpha_{\tilde{A}}$ must be a closed interval for every $\alpha \in [0,\ 1]$; and

(iii). the support of $\tilde{A}$ must be bounded.

Definition 3 (Hexagonal Fuzzy Number)**.** A fuzzy number $\tilde{A}_H$ is a hexagonal fuzzy numbers [5, 6] denoted by $\tilde{A}_H = (a_1,\ a_2,\ a_3,\ a_4,\ a_5,\ a_6)$, where $(a_1,\ a_2,\ a_3,\ a_4,\ a_5,\ a_6)$ are real numbers and its membership function $\mu_{\tilde{A}_H}(x)$ is given below

$$\mu_{\tilde{A}_H}(x) = \begin{cases} 0, & x < a_1 \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x-a_2}{a_2-a_1}\right), & a_1 \le x \le a_2 \\ 1 + \frac{1}{2}\left(\frac{x-a_3}{a_3-a_2}\right), & a_2 \le x \le a_3 \\ 1, & a_3 \le x \le a_4 \\ 1 - \frac{1}{2}\left(\frac{x-a_4}{a_4-a_5}\right), & a_4 \le x \le a_5 \\ \frac{1}{2} - \frac{1}{2}\left(\frac{x-a_6}{a_6-a_5}\right), & a_5 \le x \le a_6 \\ 0, & x > a_6 \end{cases}$$

Following are the three operations that can be performed on hexagonal fuzzy numbers.

Definition 4 (Operations of Hexagonal Fuzzy Numbers [7, 8])**.**
Suppose $\tilde{A}_H = (a_1,\ a_2,\ a_3,\ a_4,\ a_5,\ a_6)$ and $\tilde{B}_H = (b_1,\ b_2,\ b_3,\ b_4,\ b_5,\ b_6)$ are two hexagonal fuzzy numbers then

Addition: $\tilde{A}_H + \tilde{B}_H = (a_1 + b_1,\ a_2 + b_2,\ a_3 + b_3,\ a_4 + b_4,\ a_5 + b_5,\ a_6 + b_6)$.
Subtraction: $\tilde{A}_H - \tilde{B}_H = (a_1 - b_1,\ a_2 - b_2,\ a_3 - b_3,\ a_4 - b_4,\ a_5 - b_5,\ a_6 - b_6)$.
Multiplication: $\tilde{A}_H * \tilde{B}_H = (a_1 * b_1,\ a_2 * b_2,\ a_3 * b_3,\ a_4 * b_4,\ a_5 * b_5,\ a_6 * b_6)$.

Definition 5 (Ranking Function)**.**

$$R(\tilde{A}) = \left(\frac{a_1 + 2a_2 + a_3 + a_4 + 2a_5 + a_6}{4}\right)$$

Where $(a_1,\ a_2,\ a_3,\ a_4,\ a_5,\ a_6)$ are hexagonal fuzzy numbers.

3 Multi-Objective Fuzzy Linear Programming Model

A multi objective fuzzy linear programming models as

Find $\tilde{x} = (\tilde{x}_1,\ \tilde{x}_2, \ldots .\tilde{x}_n)^T$

$$\begin{aligned} &Max\ \tilde{Z}_k(x) = \sum_{j=1}^{n} \tilde{c}_j^k \tilde{x}_j, \quad k = 1, 2, 3, \ldots, p \\ &\textit{Subject to} \\ &\qquad \sum \tilde{a}_{ij}\tilde{x}_j \ \leq \tilde{b}_i, \quad i = 1, 2, 3, \ldots, m \\ &\qquad\quad \tilde{x}_j \geq 0, \qquad j = 1, 2, 3, \ldots, n \end{aligned}$$

where $\tilde{c}_j^k$, $\tilde{a}_{ij}$ and $\tilde{b}_i$ are fuzzy numbers. The general fuzzy goal programming model for the above multi objective fuzzy linear programming problem as

Find $\tilde{x} = (\tilde{x}_1,\ \tilde{x}_2, \ldots, \tilde{x}_n)^T$

$$\begin{aligned} &\textit{Lexicographically}\ \min : \tilde{a} = (\tilde{a}_1,\ \tilde{a}_2, \ldots, \tilde{a}_p) \\ &\quad = \{\tilde{g}_1(n, \rho),\ \tilde{g}_2(n, \rho), \ldots, \tilde{g}_p(n, \rho)\} \\ &\textit{Subject to} \\ &\qquad \sum_{j=1}^{n} \tilde{c}_j^k \tilde{x}_j + \tilde{\eta}_k - \tilde{\rho}_k = \tilde{g}_k \quad k = 1, 2, 3, \ldots, k \\ &\qquad \sum_{j=1}^{n} \tilde{a}_{ij}\tilde{x}_j + \tilde{\eta}_{k+1} - \tilde{\rho}_{k+1} = \tilde{b}_i \ \ i = 1, 2, 3, \ldots, m \\ &\qquad \tilde{x}_j,\ \tilde{\eta}_k \text{ and } \tilde{\rho}_k \geq 0;\ j = 1, 2, ..., n \text{ and } i = 1, 2, ..., k + m \end{aligned}$$

4 Modified Fuzzy Sequential Linear Goal Programming Algorithm [9]

Step 1: Set k = 1 (where k is used to represent the priority level under consideration and P is the total number of priorities).

Step 2: Establish the mathematical model for priority level 1 (P1) only. That is,

$$\begin{aligned}
&\min : \tilde{a}_1 = \tilde{g}_1(n,\rho)\\
&Subject\ to\\
&\qquad \sum_{j=1}^{n} \tilde{c}_j^k \tilde{x}_j + \tilde{\eta}_k - \tilde{\rho}_k = \tilde{b}_i \quad i \in P_1\\
&\qquad \sum_{j=1}^{n} \tilde{a}_{ij} \tilde{x}_j + \tilde{\eta}_{k+1} - \tilde{\rho}_{k+1} = \tilde{b}_i \quad i = 1,2,3,\ldots,m\\
&\qquad \tilde{x}_j,\ \tilde{\eta}_k \text{ and } \tilde{\rho}_k \geq 0
\end{aligned}$$

The resultant problem is simply a single-objective linear programming problem and may be solved by fuzzy simplex method.

Step3: Solve the single-objective problem associated with the priority level k via fuzzy simplex algorithm. (Before solving the problem expresses the basic variables in terms of non-basic variables if it is there in a goal). Let the optimal solution to this problem be given as a $*$, where a$*$ is the optimal value of $\tilde{g}_k(n,\rho)$

Step4: Set k = k + 1. If k > P, go to Step 7.

Step5: Establish the equivalent, single-objective model for the next priority level (P_k). This model is given by:

$$\begin{aligned}
&\text{Lexicographically } \min : \tilde{a}_k = \tilde{g}_k(n,\rho)\\
&Subject\ to\\
&\qquad f_i(\tilde{x}) + \tilde{\eta}_k - \tilde{\rho}_k = \tilde{b}_i\\
&\qquad g_s(\tilde{\eta},\tilde{\rho}) = \tilde{a}_s^*\\
&\qquad \tilde{x},\tilde{\eta}\ and\ \tilde{\rho} \geq 0
\end{aligned}$$

where s = 1,. . . , k-1, t = set of subscripts associated with those 'goals 'or constraints included in priority levels 1, 2,. . . , k.

Step 6: Express the basic variables in terms of non-basic variables if it is there in a goal. Go to Step 3.

Step 7: The solution vector x$*$associated with the last single-objective model solved, is the optimal vector for the original goal programming model.

This is an algorithm used to solve multiple objective fuzzy linear programming problems when there is a priority among the objectives. That is, if there is an order such that one goal is more important than the other and next to this goal if there is another goal which is more important than the rest of the other.In such away if all goals are ordered one can use modified fuzzy sequential linear goal programming algorithm to solve multiple objective linear programming problems.

5 Revised Multi-Phase Fuzzy Simplex Algorithm for MFSLGP Algorithm [10]

Find $\tilde{x} = (\tilde{x}_1,\ \tilde{x}_2, \ldots, \tilde{x}_n)^T$

$$
\begin{aligned}
&\textit{Lexicographically } \min : \tilde{a} = (\tilde{a}_1,\ \tilde{a}_2, \ldots, \tilde{a}_p) \\
&= \{\tilde{g}_1(n,\rho), \tilde{g}_2(n,\rho), ...\tilde{g}_p(n,\rho)\} \\
&\textit{Subject to} \\
&\qquad \textstyle\sum_{j=1}^{n} \tilde{c}_j^k \tilde{x}_j + \tilde{\eta}_k - \tilde{\rho}_k = \tilde{g}_k \quad k = 1, 2, 3, \ldots, k \\
&\qquad \textstyle\sum_{j=1}^{n} \tilde{a}_{ij} \tilde{x}_j + \tilde{\eta}_{k+1} - \tilde{\rho}_{k+1} = \tilde{b}_i \ \ i = 1, 2, 3, \ldots, m \\
&\qquad \tilde{x}_j, \tilde{\eta}_k \ and\, \tilde{\rho}_k \geq 0;\ j = 1, 2, \ldots, n \text{ and } i = 1, 2, \ldots, k+m
\end{aligned}
$$

Where P is the number of priorities in the achievement function A. We have the following corresponding simplex algorithm:

Step 1: Set K=1 and have an initial simplex table for the problem under Step 2 of MFSLGP algorithm in format of the following, where the corresponding spaces of $\tilde{Z}_j - \tilde{C}_j$ are empty. Then, got to step 2.

Step 2: Compute $\tilde{Z}_j - \tilde{C}_j$ for all non-basic variables. Then go to Step 3.

Step 3: Optimality condition: If all $\tilde{Z}_j - \tilde{C}_j \leq 0$ for all non-basic variables, then the current table is optimal table. Then go to Step 4. Otherwise goto Step 5.

Step 4: Set $K = K+1$. If $k < p$, the drop all columns with $\tilde{Z}_j - \tilde{C}_j < 0$ from the optimal table of k$^{\text{th}}$ iteration and express basic variables in terms of non-basic variable from the optimal table of k$^{\text{th}}$ iteration if it there in a goal. Then add this constraint correspondingly in the bottom of optimal table of k$^{\text{th}}$ iteration after the removal of all columns with $\tilde{Z}_j - \tilde{C}_j < 0$. Then go to Step 2. Otherwise, the current optimal solution is the optimal solution for the original problem of MFSLGP algorithm.

Step 5: Select Entering Variable. The entering variable is the non-basic variable with the most positive $\tilde{Z}_j - \tilde{C}_j$. Then by having this at hand go to Step 6.

Step 6: Determine Leaving Variable. The leaving variable is determined as usual in fuzzy primal simplex algorithm by minimum ratio test, and all condition like unboundedness is the same as in that of simplex method. Then if it is bounded go to Step 7.

Step 7: Since from Step 6 we have that pivot element say ' and pivot column, then do the pivot operations we have in usual simplex method. Then rename this table by table before. Multiply the elements pivot column of table we have obtained from Step 6 by $-\frac{1}{a}$ the pivot element, and also replace the pivot element of this pivot column by $\frac{1}{a}$ except Then replace this pivot column to the table before, and rename this table to be table after. This table is final table of this step. Then by having this final table at hand go to Step 3.

6 Numerical example

$$
\begin{aligned}
&Min : \tilde{Z}_1 = 3\tilde{x}_2 + 2\tilde{x}_4 \\
&Min : \tilde{Z}_2 = 10\tilde{x}_1 + 9\tilde{x}_2 + 8\tilde{x}_3 + 7\tilde{x}_4 \\
&Subject\,to \\
&\tilde{x}_1 + \tilde{x}_2 \geq (10, 12, 14, 16, 18, 20) \\
&\tilde{x}_3 + \tilde{x}_4 \geq (10, 12, 14, 16, 18, 20) \\
&3\tilde{x}_1 + 2\tilde{x}_3 \leq (55, 57, 59, 61, 63, 65) \\
&\tilde{x}_i \geq 0, i = 1:4
\end{aligned}
$$

Then the resultant lexicographic linear goal programming having five goals, three priority levels and four decision variables can be Represented a

$$
\begin{aligned}
&Lexicographically\, Min\, \tilde{a} = \{\tilde{\eta}_1 + \tilde{\eta}_2 + \tilde{\rho}_3, \tilde{\rho}_4, \tilde{\eta}_5\} \\
&Subject\,to \\
&\tilde{x}_1 + \tilde{x}_2 + \tilde{\eta}_1 - \tilde{\rho}_1 = (10, 12, 14, 16, 18, 20) \\
&\tilde{x}_3 + \tilde{x}_4 + \tilde{\eta}_2 - \tilde{\rho}_2 = (10, 12, 14, 16, 18, 20) \\
&3\tilde{x}_1 + 2\tilde{x}_3 + \tilde{\eta}_3 - \tilde{\rho}_3 = (55, 57, 59, 61, 63, 65) \\
&3\tilde{x}_2 + 2\tilde{x}_4 + \tilde{\eta}_4 - \tilde{\rho}_4 = (5, 7, 9, 11, 13, 15,) \\
&10\tilde{x}_1 + 9\tilde{x}_2 + 8\tilde{x}_3 + 7\tilde{x}_4 + \tilde{\eta}_5 - \tilde{\rho}_5 = (395, 397, 399, 401, 403, 405) \\
&\tilde{x}_j, \tilde{\eta}_i \text{ and } \tilde{\rho}_i \geq 0, i = 1:5\ ,\ j = 1:4
\end{aligned}
$$

From Step 1 and Step 2 of MFSLGP

$$
\begin{aligned}
&Lexicographically\, Min\, \tilde{a}_1 = \tilde{\eta}_1 + \tilde{\eta}_2 + \tilde{\rho}_3 \\
&Subject\,to \\
&\tilde{x}_1 + \tilde{x}_2 + \tilde{\eta}_1 - \tilde{\rho}_1 = (10, 12, 14, 16, 18, 20) \\
&\tilde{x}_3 + \tilde{x}_4 + \tilde{\eta}_2 - \tilde{\rho}_2 = (10, 12, 14, 16, 18, 20) \\
&3\tilde{x}_1 + 2\tilde{x}_3 + \tilde{\eta}_3 - \tilde{\rho}_3 = (55, 57, 59, 61, 63, 65) \\
&\tilde{x}_j, \tilde{\eta}_i \text{ and } \tilde{\rho}_i \geq 0, i = 1:3\ ,\ j = 1:4
\end{aligned}
$$

Now also from Step 3 of the MFSLGP algorithm and the corresponding simplex algorithm

		C_j	0	0	0	0	1	1	0	0	0	1	
C_B	Y_B	X_B	x_1	x_2	x_3	x_4	η_1	η_2	η_3	ρ_1	ρ_2	ρ_3	θ
1	η_1	$(10, 12, 14, 16, 18, 20)$	$(\mathbf{1})$	1	0	0	1	0	0	-1	0	0	30
1	η_2	$(10, 12, 14, 16, 18, 20)$	0	0	1	1	0	1	0	0	-1	0	-
0	η_3	$(55, 57, 59, 61, 63, 65)$	3	0	2	0	0	0	1	0	0	-1	40
		$Z_j - C_j$	1	1	1	1	0	0	0	-1	-1	-1	

Table 1: MFSLGP: Initial table

x_1 is enter the table and η_1 is leave the table

		C_j	0	0	0	1	1	0	0	0	1	
C_B	Y_B	X_B	x_2	x_3	x_4	η_1	η_2	η_3	ρ_1	ρ_2	ρ_3	θ
0	x_1	$(10, 12, 14, 16, 18, 20)$	1	0	0	1	0	0	-1	0	0	-
1	η_2	$(10, 12, 14, 16, 18, 20)$	0	1	1	0	1	0	0	-1	0	30
0	η_3	$(25, 21, 17, 13, 9, 5)$	-3	$(\mathbf{2})$	0	-3	0	1	3	0	-1	15
		$Z_j - C_j$	0	1	1	-1	0	0	0	-1	-1	

Table 2: MFSLGP: First table

x_3 is enter the table and η_3 is leave the table

		C_j	0	0	1	1	0	0	0	1	
C_B	Y_B	X_B	x_2	x_4	η_1	η_2	η_3	ρ_1	ρ_2	ρ_3	θ
0	x_1	$(10, 12, 14, 16, 18, 20)$	1	0	1	0	0	-1	0	0	30
1	η_2	$(\frac{-5}{2}, \frac{3}{2}, \frac{11}{2}, \frac{19}{2}, \frac{27}{2}, \frac{35}{2})$	$(\mathbf{\frac{3}{2}})$	1	$\frac{3}{2}$	1	$-\frac{1}{2}$	$-\frac{3}{2}$	-1	$\frac{1}{2}$	10
0	x_3	$(\frac{25}{2}, \frac{21}{2}, \frac{17}{2}, \frac{13}{2}, \frac{9}{2}, \frac{5}{2})$	$-\frac{3}{2}$	0	$-\frac{3}{2}$	0	$\frac{1}{2}$	$\frac{3}{2}$	0	$-\frac{1}{2}$	-
		$Z_j - C_j$	$\frac{3}{2}$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	

Table 3: MFSLGP: Second table

x_2 is enter the table and η_2 is leave the table

		C_j	0	1	1	0	0	0	1	
C_B	Y_B	X_B	x_4	η_1	η_2	η_3	ρ_1	ρ_2	ρ_3	θ
0	x_1	$(\frac{35}{3}, \frac{33}{3}, \frac{31}{3}, \frac{29}{3}, \frac{27}{3}, \frac{25}{3})$	$-\frac{2}{3}$	0	$-\frac{2}{3}$	$\frac{1}{3}$	0	$\frac{2}{3}$	$\frac{1}{3}$	
0	x_2	$(\frac{-5}{3}, \frac{3}{3}, \frac{11}{3}, \frac{19}{3}, \frac{27}{3}, \frac{35}{3})$	$\frac{2}{3}$	1	$\frac{2}{3}$	$-\frac{1}{3}$	-1	$-\frac{2}{3}$	$-\frac{1}{3}$	
0	x_3	$(\frac{20}{2}, \frac{24}{2}, \frac{28}{2}, \frac{32}{2}, \frac{36}{2}, \frac{40}{2})$	1	0	1	0	0	-1	0	
		$Z_j - C_j$	0	-1	-1	0	0	0	0	

Table 4: MFSLGP: Third table

Therefore the Table 3 is optimal tableau, the η_1,η_1 are negative columns may be dropped. Step 4 and Step 5 of the MFSLGP algorithm

$$
\begin{aligned}
&Lexicographically\, Min\, \tilde{a}_2 = \tilde{\rho}_4 \\
&Subject\, to \\
&\tilde{x}_1 + \tilde{x}_2 + \tilde{\eta}_1 - \tilde{\rho}_1 = (10, 12, 14, 16, 18, 20) \\
&\tilde{x}_3 + \tilde{x}_4 + \tilde{\eta}_2 - \tilde{\rho}_2 = (10, 12, 14, 16, 18, 20) \\
&3\tilde{x}_1 + 2\tilde{x}_3 + \tilde{\eta}_3 - \tilde{\rho}_3 = (55, 57, 59, 61, 63, 65) \\
&3\tilde{x}_2 + 2\tilde{x}_4 + \tilde{\eta}_4 - \tilde{\rho}_4 = (5, 7, 9, 11, 13, 15,) \\
&\tilde{x}_j, \tilde{\eta}_i\, and \tilde{\rho}_i \geq 0, i = 1:4\ ,\ j = 1:4
\end{aligned}
$$

Then we express basic variable in terms of non-basic variables, from the above table. Then we obtain

$$-\tilde{\eta}_3 - \tilde{\eta}_4 - 3\rho_1 - 2\rho_2 + \tilde{\rho}_4 = (-10, -4, 2, 8, 14, 20)$$

		C_j	0	0	0	0	0	
C_B	Y_B	X_B	x_4	η_3	ρ_1	ρ_2	η_4	θ
0	x_1	$(\frac{35}{3}, \frac{33}{3}, \frac{31}{3}, \frac{29}{3}, \frac{27}{3}, \frac{25}{3})$	$-\frac{2}{3}$	$\frac{1}{3}$	0	$\frac{2}{3}$	0	
0	x_2	$(\frac{-5}{3}, \frac{3}{3}, \frac{11}{3}, \frac{19}{3}, \frac{27}{3}, \frac{35}{3})$	$\frac{2}{3}$	$-\frac{1}{3}$	-1	$-\frac{2}{3}$	0	
0	x_3	$(\frac{20}{2}, \frac{24}{2}, \frac{28}{2}, \frac{32}{2}, \frac{36}{2}, \frac{40}{2})$	1	0	0	-1	0	
1	ρ_4	$(-10, -4, 2, 8, 14, 20)$	0	-1	-3	-2	-1	
		$Z_j - C_j$	0	-1	-3	-2	-1	

Table 5: MFSLGP fourth table:

Then we solve the problem using Step 3

Therefore the Table 4 is optimal tableau, the η_3, ρ_1, ρ_2, η_4 are negative columns may be dropped. Using Step 4 and Step 5 of MFSLGP

$$
\begin{aligned}
&Lexicographically\, Min\, \tilde{a}_3 = \tilde{\eta}_5\\
&Subject\, to\\
&\tilde{x}_1 + \tilde{x}_2 + \tilde{\eta}_1 - \tilde{\rho}_1 = (10, 12, 14, 16, 18, 20)\\
&\tilde{x}_3 + \tilde{x}_4 + \tilde{\eta}_2 - \tilde{\rho}_2 = (10, 12, 14, 16, 18, 20)\\
&3\tilde{x}_1 + 2\tilde{x}_3 + \tilde{\eta}_3 - \tilde{\rho}_3 = (55, 57, 59, 61, 63, 65)\\
&3\tilde{x}_2 + 2\tilde{x}_4 + \tilde{\eta}_4 - \tilde{\rho}_4 = (5, 7, 9, 11, 13, 15,)\\
&10\tilde{x}_1 + 9\tilde{x}_2 + 8\tilde{x}_3 + 7\tilde{x}_4 + \tilde{\eta}_5 - \tilde{\rho}_5 = (395, 397, 399, 401, 403, 405)\\
&\tilde{x}_j, \tilde{\eta}_i \;\text{ and }\; \tilde{\rho}_i \geq 0, i = 1:5\ ,\ j = 1:4
\end{aligned}
$$

From the above table, we obtain $-\frac{1}{3}x_4 + \tilde{\eta}_5 - \tilde{\rho}_5 = (213, 182, 150, 120, 88, 57)$.

		C_j	0	0	
C_B	Y_B	X_B	x_4	ρ_5	θ
0	x_1	$(\frac{35}{3}, \frac{33}{3}, \frac{31}{3}, \frac{29}{3}, \frac{27}{3}, \frac{25}{3})$	$-\frac{2}{3}$	0	
0	x_2	$(\frac{-5}{3}, \frac{3}{3}, \frac{11}{3}, \frac{19}{3}, \frac{27}{3}, \frac{35}{3})$	$\frac{2}{3}$	0	
0	x_3	$(\frac{20}{2}, \frac{24}{2}, \frac{28}{2}, \frac{32}{2}, \frac{36}{2}, \frac{40}{2})$	1	0	
0	ρ_4	$(-10, -4, 2, 8, 14, 20)$	0		
1	η_5	$(213, 182, 150, 120, 88, 57)$	$-\frac{1}{3}$	-1	
		$Z_j - C_j$	0	-1	

Table 6: MFSLGP fifth table:

From Step 3 of MFSLGP algorithm the obtained solution is the following

$\tilde{x}_1 = \left(\frac{35}{3}, \frac{33}{3}, \frac{31}{3}, \frac{29}{3}, \frac{27}{3}, \frac{25}{3}\right)$, $\tilde{x}_2 = \left(\frac{-5}{3}, \frac{3}{3}, \frac{11}{3}, \frac{19}{3}, \frac{27}{3}, \frac{35}{3}\right)$, $\tilde{x}_3 = \left(\frac{20}{2}, \frac{24}{2}, \frac{28}{2}, \frac{32}{2}, \frac{36}{2}, \frac{40}{2}\right)$ and $\tilde{x}_4 = (0, 0, 0, 0, 0, 0)$, $\tilde{\rho}_4 = (-10, -4, 2, 8, 14, 20)$, $\tilde{\eta}_5 = (213, 182, 150, 120, 88, 57)$. The achievement function is $\tilde{A} = \{(0, 0, 0, 0, 0, 0), (-10, -4, 2, 8, 14, 20), (213, 182, 150, 120, 88, 57)\}$.

7 Conclusion

The MFSLGP algorithm uses a series of conventional LP models. This is accomplished by partitioning the FGP model according to priority levels. It also involves the solution to an

interrelated sequence of conventional FLP models. It may not be very easy to solve man- ally a FGP problem. Unless we express the basic variable in terms of non-basic variables a problem cannot be solved manually. The MFSLGP algorithm also assures best-compromise solution like other FGP algorithm.

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