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Somewhat soft fuzzy almost G_δ pre continuous function in soft fuzzy topological space

V.Visalakshi
Department of Mathematics, SRM University
Kattankulathur, Tamil Nadu, India
e-mail : visalsenthil86@gmail.com

Abstract

In this paper the concept somewhat soft fuzzy G_δ pre continuous, soft fuzzy G_δ pre open, somewhat soft fuzzy G_δ pre open, soft fuzzy almost G_δ pre continuous and somewhat soft fuzzy almost G_δ pre continuous functions are discussed and the characterizations are established. Interrelations among them are discussed with counter examples.
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1 Introduction

Zadeh introduced the fundamental concepts of fuzzy sets in his classical paper [1]. Fuzzy sets have applications in many fields such as information [3] and control [4]. In mathematics, topology theoremvided the most natural framework for the concepts of fuzzy sets to flourish. Chang [2] introduced and developed the concept of fuzzy topological spaces. The concept of soft fuzzy topological space is introduced by [7]. Various properties of soft fuzzy topological space was discussed by [9],[8]. [6] introduced the concept of fuzzy $G_\delta$set.

The concept of fuzzy pre open set was introduced by [5]. In this paper somewhat soft fuzzy G_δ pre continuous, somewhat soft fuzzy G_δ pre open, soft fuzzy almost G_δ pre continuous and somewhat soft fuzzy almost G_δ pre continuous functions are discussed and their characterizations are established. Interrelations are discussed with counter examples wherever necessary.

2 Preliminaries

Definition 1. [7] A subset T⊆SF(X) is called an SF-topology on X if

(1) $(0, \phi)$ and $(1, X) \in T$.

(2) $(\mu_j, M_j) \in T, j = 1, 2, 3, ...n \Rightarrow \sqcap_{j=1}^{n}(\mu_j, M_j) \in T$.

(3) $(\mu_j, M_j), j \in J \Rightarrow \sqcup_{j\in J}(\mu_j, M_j) \in T$.

the elements of T are called soft fuzzy open, and those of $T^{'} = \{(\mu, M)/(\mu, M)^{'} \in T\}$ soft fuzzy closed. If T is a SF-topology on X we call the pair (X,T) an SF-topological space(in short, SFTS).

Definition 2. [9] Let (X, T) be a SFTS.Let (λ, N) be any soft fuzzy set. Then (λ, N) is said to be soft fuzzy G_δ pre open set if $(\lambda, N) = (\mu, M) \sqcap (\gamma, L)$,where (μ, M) is soft fuzzy G_δ set and $(\gamma, L)$is soft fuzzy pre open set. The complement of a soft fuzzy G_δ pre open set is soft fuzzy F_σ pre closed.

Theorem 3. *[9] Arbitrary union of soft fuzzy G_δ pre open sets is soft fuzzy G_δ pre open.*

Theorem 4. *[9] Arbitrary intersection of soft fuzzy F_σ pre closed sets is soft fuzzy F_σ pre closed.*

Definition 5. [9] Let (X, T) be any SFTS and let (λ, M) be any soft fuzzy set in (X, T). Then soft fuzzy G_δ pre interior(in short G_δ pre int) of (λ, M) is defined as follows

$$\text{SF } G_\delta \text{ pre int}(\lambda, M) = \sqcup\{(\mu, N)/(\mu, N) \text{ is SF } G_\delta \text{ pre open and } (\mu, N) \sqsubseteq (\lambda, M)\}$$

Definition 6. [9] Let (X,T) be any SFTS and let (λ,M) be any soft fuzzy set in (X,T). Then soft fuzzy F_σ pre closure(in short F_σ pre cl) of (λ,M) is defined as follows

$$\text{SF } F_\sigma \text{ pre cl}(\lambda,M)= \sqcap\{(\mu,N)/(\mu,N) \text{ is SF } F_\sigma \text{ pre closed and } (\mu,N) \sqsupseteq (\lambda,M)\}$$

Definition 7. [9] Let (X,T) be any SFTS and let $(x_r,\{x\})$ be any soft fuzzy point in (X,T). Then soft fuzzy G_δ pre interior of $(x_r,\{x\})$ is defined as follows

$$\text{SF } G_\delta \text{ pre int}(x_r,\{x\})= \sqcup\{(\mu,N)/(\mu,N) \text{ is SF } G_\delta \text{ pre open and } (\mu,N) \sqsubseteq (x_r,\{x\})\}$$

Definition 8. [9] Let (X,T) and (Y,S) be any two SFTSs. A function $f:(X,T)\to(Y,S)$ is said to be soft fuzzy G_δ-pre continuous, if the inverse image of every soft fuzzy open set in (Y,S) is soft fuzzy G_δ pre open in (X,T).

Theorem 9. *[9] Let (X,T) and (Y,S) be any two SFTSs. For a function $f:(X,T)\to(Y,S)$, the following are equivalent.*

(i) f is soft fuzzy G_δ-pre continuous.

(ii) The inverse image of every soft fuzzy closed set in (Y,S) is soft fuzzy F_σ pre closed in (X,T).

3 Somewhat soft fuzzy G_δ pre continuous function.

Definition 10. Let (X,T) and (Y,S) be any two SFTSs. A function $f:(X,T)\to(Y,S)$ is somewhat soft fuzzy G_δ pre continuous if (λ,N) is soft fuzzy open set and $f^{-1}(\lambda,N)\neq(0,\phi)$ then there exists a soft fuzzy G_δ pre open set (μ,M) such that $(\mu,M)\neq(0,\phi)$ and $(\mu,M)\sqsubseteq f^{-1}(\lambda,N)$.

Definition 11. A soft fuzzy set (λ,N) in (X,T) is called soft fuzzy dense if there exists no soft fuzzy closed set (μ,M) such that $(\lambda,N)\sqsubset(\mu,M)\sqsubset(1,X)$.

Definition 12. A soft fuzzy set (λ, N) in (X, T) is called soft fuzzy G_δ pre dense if there exists no soft fuzzy F_σ pre closed set (μ, M) such that $(\lambda, N) \sqsubset (\mu, M) \sqsubset (1, X)$.

Theorem 13. *Let (X, T) and (Y, S) be any two SFTSs. Let $f : (X, T) \to (Y, S)$ be a bijective function then the following are equivalent:*

(i) f is somewhat soft fuzzy G_δ pre continuous function.

(ii) If (λ, N) is soft fuzzy closed set such that $f^{-1}(\lambda, N) \neq (1, X)$, then there exists a theoremper soft fuzzy F_σ pre closed set (μ, M) such that $(\mu, M) \sqsupset f^{-1}(\lambda, N)$.

(iii) If (λ, N) is a soft fuzzy G_δ pre dense set in (X, T) then $f(\lambda, N)$ is a soft fuzzy dense set in (Y, S).

Theorem 14. *Let (X, T) and (Y, S) be any two SFTSs. Let $f : (X, T) \to (Y, S)$ be somewhat soft fuzzy G_δ pre continuous function. Let $A \subseteq X$ such that $(\chi_A, A) \sqcap (\mu, M) \neq (0, \phi)$, for all soft fuzzy G_δ pre open set. Let $T \mid A$ be the induced SFTS on A. Then $f \mid A : (A, T \mid A) \to (Y, S)$ is somewhat soft fuzzy G_δ pre continuous.*

The following example shows that the above theorem is invalid if the hypothesis fails.

Example 15. Let $X = \{a, b, c\}$ and $T = \{(0, \phi),\ (1, X),\ (\lambda_1, M_1),\ (\lambda_2, M_2),\ (\lambda_3, M_3)\}$ where $(\lambda_1, M_1),\ (\lambda_2, M_2),\ (\lambda_3, M_3)$ are defined as follows $\lambda_1(a) = 1, \lambda_1(b) = 0, \lambda_1(c) = 0, M_1 = \{c\}$; $\lambda_2(a) = 0, \lambda_2(b) = 0, \lambda_2(c) = 1, M_2 = \{c\}$, $\lambda_3(a) = 1, \lambda_3(b) = 0, \lambda_3(c) = 1, M_3 = \{c\}$. (X, T) is a SFTS. Let $Y = \{p, q, r\}$ and $S = \{(0, \phi), (1, Y), (\gamma, L)\}$ where (γ, L) is defined as follows $\gamma(p) = 0, \gamma(q) = 1, \gamma(r) = 1, L = \{p, r\}$. (Y, S) is a SFTS. Let $f : (X, T) \to (Y, S)$ defined as follows $f(a) = p, f(b) = q, f(c) = r$. f is somewhat soft fuzzy G_δ pre continuous. Let $A = \{a, b\}$. Since $\{\chi_A, A\} \sqcap (\lambda_2, M_2) = (0, \phi)$, A does not satisfy the hypothesis of the above theorem. Consider $f \mid A : (A, T \mid A) \to (Y, S)$ defined as follows $f \mid A(a) = p$, $f \mid A(b) = q$. $T \mid A = \{(0, \phi), (1, A), (\lambda_1, M_1) \mid A, (\lambda_2, M_2) \mid A, (\lambda_3, M_3) \mid A\}$. Since $f^{-1}(\gamma, L)$ is not soft fuzzy G_δ pre open in $(A, T \mid A)$. Thus $f \mid A$ is not somewhat soft fuzzy G_δ pre continuous.

4 Somewhat soft fuzzy G_δ pre open function.

Definition 16. Let (X,T) and (Y,S) be any two SFTSs. A function $f:(X,T)\to(Y,S)$ is soft fuzzy G_δ pre open function if (λ,N) is soft fuzzy open set in (X,T), $f(\lambda,N)$ is soft fuzzy G_δ pre open set.

Definition 17. Let (X,T) and (Y,S) be any two SFTSs. A function $f:(X,T)\to(Y,S)$ is somewhat soft fuzzy G_δ pre open function iff (λ,N) is soft fuzzy open set and $(\lambda,N)\neq(0,\phi)$ then there exists a soft fuzzy G_δ pre open set (μ,M) such that $(\mu,M)\neq(0,\phi)$ and $(\mu,M)\sqsubseteq f(\lambda,N)$.

Clearly every soft fuzzy G_δ pre open function is somewhat soft fuzzy G_δ pre open function but the converse need not be true as shown in the following example.

Example 18. Let $X=\{a,b,c\}$ and $T=\{(0,\phi),(1,X),(\lambda_1,M_1)\}$ where (λ_1,M_1) is defined as follows $\lambda_1(a)=0.5,\lambda_1(b)=0.4,\lambda_1(c)=0.4$, $M_1=\{a,b\}$; Thus (X,T) is a SFTS. Let $Y=\{p,q,r\}$ and $S=\{(0,\phi),\ (1,Y),\ (\gamma_1,L_1),\ (\gamma_2,L_2),\ (\gamma_3,L_3),\ (\gamma_4,L_4)\}$ where (γ_1,L_1), (γ_2,L_2), (γ_3,L_3), (γ_4,L_4) are defined as follows $\gamma_1(p)=0.4,\gamma_1(q)=0.4,\gamma_1(r)=0.4$, $L_1=\{\phi\}$; $\gamma_2(p)=0.4,\gamma_2(q)=0.4,\gamma_2(r)=0.4$, $L_2=\{r\}$; $\gamma_3(p)=0.4,\gamma_3(q)=0.4,\gamma_3(r)=0.4$, $L_3=\{p,q\}$; $\gamma_4(p)=0.4,\gamma_4(q)=0.4,\gamma_4(r)=0.4$, $L_4=Y=\{p,q,r\}$. Thus (Y,S) is a SFTS. Let $f:(X,T)\to(Y,S)$ defined as follows $f(a)=p,f(b)=q,f(c)=r$. f is somewhat soft fuzzy G_δ pre open function. Since $f(\lambda_1,M_1)$ is not soft fuzzy G_δ pre open in (Y,S). Thus f is not soft fuzzy G_δ pre open function.

Theorem 19. *Suppose (X,T) and (Y,S) be any two SFTSs. Let $f:(X,T)\to(Y,S)$ be bijective function. Then the following conditions are equivalent.*

(i) f is somewhat soft fuzzy G_δ pre open function.

(ii) If (λ,N) is a soft fuzzy G_δ pre dense set in (Y,S), then $f^{-1}(\lambda,N)$ is a soft fuzzy dense set in (X,T).

Theorem 20. *Suppose (X,T) and (Y,S) be any two SFTSs.*

Let $f : (X,T) \to (Y,S)$ be bijective function. Then the following conditions are equivalent.

(i) f is somewhat soft fuzzy G_δ pre open function.

(ii) If (λ, N) is a soft fuzzy closed set such that $f(\lambda, N) \neq (1, Y)$, then there exists soft fuzzy F_σ pre closed set (μ, M) in (Y,S) such that $(\mu, M) \neq (1, Y)$ and $(\mu, M) \sqsupset f(\lambda, N)$.

5 Somewhat and soft fuzzy almost G_δ pre continuous function.

Definition 21. Let (X,T) be a SFTS. A soft fuzzy set (λ, N) is said to be soft fuzzy regular open set in (X,T) if $(\lambda, N) = \text{int}(\text{cl}(\lambda, N))$. The complement of a soft fuzzy regular open set is soft fuzzy regular closed set.

Definition 22. Let (X,T) and (Y,S) be any two SFTSs. A function $f : (X,T) \to (Y,S)$ is soft fuzzy almost G_δ pre continuous function if the inverse image of each soft fuzzy regular open set in (Y,S) is a soft fuzzy G_δ pre open set in (X,T).

Definition 23. Let (X,T) and (Y,S) be any two SFTSs. A function $f : (X,T) \to (Y,S)$ is somewhat soft fuzzy almost G_δ pre open function iff (λ, N) is soft fuzzy regular open set and $(\lambda, N) \neq (0, \phi)$ then there exists a soft fuzzy G_δ pre open set (μ, M) such that $(\mu, M) \neq (0, \phi)$ and $(\mu, M) \sqsubseteq f(\lambda, N)$.

Theorem 24. *Suppose (X,T) and (Y,S) be any two SFTSs. Let $f : (X,T) \to (Y,S)$ be a function. Then the following conditions are equivalent.*

(i) f is soft fuzzy almost G_δ pre continuous function.

(ii) $f^{-1}(\lambda, N) \sqsubseteq G_\delta$ pre int$(f^{-1}(\text{int}(\text{cl}(\lambda, N))))$ for each soft fuzzy open set (λ, N) in (Y,S).

(iii) F_σ pre cl$(f^{-1}(\text{cl}(\text{int}(\mu, M)))) \sqsubseteq f^{-1}(\mu, M)$ for each soft fuzzy closed set (μ, M) in (Y,S).

(iv) $f^{-1}(\lambda, N)$ is a soft fuzzy F_σ pre closed set , for each soft fuzzy regular closed set (λ, N) in (Y,S).

Definition 25. A soft fuzzy set (λ, N) in (X, T) is called soft fuzzy regular dense if there exists no soft fuzzy regular closed set (μ, M) such that $(\lambda, N) \sqsubset (\mu, M) \sqsubset (1, X)$.

Definition 26. Let (X, T) be a SFTS. Then the regular interior of a soft fuzzy set (λ, M) in (X, T) is defined as follows

$R - Int(\lambda, M) = \sqcup\{(\mu, N) : (\mu, N) \sqsubseteq (\lambda, M)$ and (μ, N) is soft fuzzy regular open in $(X, T)\}$

The regular closure of a soft fuzzy set (μ, M) in (X, T) is defined as follows

$R - Cl(\mu, M) = \sqcap\{(\lambda, N) : (\mu, M) \sqsubseteq (\lambda, N)$ and (λ, N) is soft fuzzy regular closed in $(X, T)\}$.

Theorem 27. *Suppose (X, T) and (Y, S) be any two SFTSs. Let $f : (X, T) \to (Y, S)$ be a bijective function. Then the following conditions are equivalent.*

(i) f is somewhat soft fuzzy almost G_δ pre continuous function.

(ii) If (λ, N) is a soft fuzzy regular closed set such that $f^{-1}(\lambda, N) \neq (1, X)$, then there exists a theoremper soft fuzzy F_σ pre closed set (μ, M) such that $(\mu, M) \sqsupseteq f^{-1}(\lambda, N)$.

(iii) If (λ, N) is a soft fuzzy G_δ pre dense set in (X, T), then $f(\lambda, N)$ is a soft fuzzy regular dense set in (Y, S).

6 Interrelations between above discussed continuous functions.

From the above defined continuous functions the following interrelation obtained.

Soft fuzzy	$\Rightarrow$	Somewhat soft fuzzy
$G_\delta$pre continuous function	$\not\Leftarrow$	G_δ pre continuous function
$\Downarrow\not\Uparrow$		$\Downarrow\not\Uparrow$
Soft fuzzy almost	$\Rightarrow$	Somewhat soft fuzzy almost
$G_\delta$pre continuous function	$\not\Leftarrow$	$G_\delta$pre continuous function

The converse of the implication need not be true shown in the following the examples.

Example 28. Let $X = \{a, b, c\}$ and $T = \{(0, \phi),\ (1, X),\ (\lambda_1, M_1),\ (\lambda_2, M_2), (\lambda_3, M_3), (\lambda_4, M_4)\}$ is a soft fuzzy topology on

X where (λ_1, M_1), (λ_2, M_2), (λ_3, M_3), (λ_4, M_4) are defined as follows $\lambda_1(a) = 0.4, \lambda_1(b) = 0.4, \lambda_1(c) = 0.4$, $M_1 = \{\phi\}$; $\lambda_2(a) = 0.4, \lambda_2(b) = 0.4, \lambda_2(c) = 0.4$, $M_2 = \{c\}$; $\lambda_3(a) = 0.4, \lambda_3(b) = 0.4, \lambda_3(c) = 0.4$,$M_3 = \{a, b\}$; $\lambda_4(a) = 0.4, \lambda_4(b) = 0.4, \lambda_4(c) = 0.4$,$M_4 = X$. Thus (X, T) is a SFTS. Let $Y = \{p, q, r\}$ and $S = \{(0, \phi), (1, Y), (\gamma, L)\}$ is a soft fuzzy topology on Y where (γ, L) is defined as follows $\gamma(p) = 0.5, \gamma(q) = 0.4, \gamma(r) = 0.4$, $L = \{p, q\}$. Thus (Y, S) is a SFTS. Let $f : (X, T) \to (Y, S)$ defined as follows $f(a) = p, f(b) = q, f(c) = r$. f is somewhat soft fuzzy G_δ pre continuous function. Since $f^{-1}(\gamma, L)$ is not soft fuzzy G_δ pre open in (X, T). Thus f is not soft fuzzy G_δ pre continuous function.

Example 29. Let $X = \{a, b, c\}$ and $T = \{(0, \phi), (1, X), (\lambda_1, M_1)\}$ is a soft fuzzy topology on X where (λ_1, M_1) is defined as follows $\lambda_1(a) = 0.6, \lambda_1(b) = 0.7, \lambda_1(c) = 0.8$, $M_1 = \{a, b\}$. Thus (X, T) is a SFTS. Let $Y = \{p, q, r\}$ and $S = \{(0, \phi), (1, Y), (\gamma_1, L_1), (\gamma_2, L_2), (\gamma_3, L_3), (\gamma_4, L_4)\}$ is a soft fuzzy topology on Y where $(\gamma_1, L_1), (\gamma_2, L_2), (\gamma_3, L_3), (\gamma_4, L_4)$ are defined as follows $\gamma_1(p) = 0.4$, $\gamma_1(q) = 0.3$, $\gamma_1(r) = 0.2$, $L_1 = \{\phi\}$; $\gamma_2(p) = 0.4, \gamma_2(q) = 0.3, \gamma_2(r) = 0.2$, $L_2 = \{r\}$; $\gamma_3(p) = 0.5, \gamma_3(q) = 0.4, \gamma_3(r) = 0.4$, $L_3 = \{p, q\}$; $\gamma_4(p) = 0.5, \gamma_4(q) = 0.4, \gamma_4(r) = 0.4$, $L_4 = Y$. Thus (Y, S) is a SFTS. Let $f : (X, T) \to (Y, S)$ defined as follows $f(a) = p, f(b) = q, f(c) = r$. f is soft fuzzy almost G_δ pre continuous function. Since $f^{-1}(\gamma_1, L_1)$ is not soft fuzzy G_δ pre open in (X, T). Thus f is not soft fuzzy G_δ pre continuous function.

Example 30. In the same example above defined f is somewhat soft fuzzy almost G_δ pre continuous function. Since $f^{-1}(\gamma_1, L_1)$ is not soft fuzzy G_δ pre open in (X, T) and also there exists no soft fuzzy G_δ pre open set $(\mu, M) \sqsubseteq f^{-1}(\gamma_1, L_1)$. Thus f is not somewhat soft fuzzy G_δ pre continuous function.

Example 31. Let $X = \{a, b, c\}$ and $T = \{(0, \phi), (1, X), (\lambda_1, M_1)\}$ is a soft fuzzy topology on X where (λ_1, M_1) is defined as follows $\lambda_1(a) = 0.4, \lambda_1(b) = 0.3, \lambda_1(c) = 0.2$, $M_1 = \{\phi\}$. Thus (X, T) is a SFTS. Let $Y = \{p, q, r\}$ and $S = \{(0, \phi), (1, Y), (\gamma_1, L_1), (\gamma_2, L_2), (\gamma_3, L_3), (\gamma_4, L_4)\}$ is a soft fuzzy topology on Y where $(\gamma_1, L_1), (\gamma_2, L_2), (\gamma_3, L_3), (\gamma_4, L_4)$ are defined as follows $\gamma_1(p) = 0.4$, $\gamma_1(q) = 0.3$, $\gamma_1(r) = 0.2$, $L_1 = \{\phi\}$; $\gamma_2(p) = 0.4, \gamma_2(q) = 0.3, \gamma_2(r) = 0.2$, $L_2 = \{q\}$; $\gamma_3(p) = 0.5, \gamma_3(q) = 0.6, \gamma_3(r) = 0.4$, $L_3 = \{r\}$;

$\gamma_4(p) = 0.5, \gamma_4(q) = 0.6, \gamma_4(r) = 0.4$, $L_4 = \{q, r\}$. Thus (Y, S) is a SFTS. Let $f : (X, T) \to (Y, S)$ defined as follows $f(a) = p, f(b) = q, f(c) = r$. f is somewhat soft fuzzy almost G_δ pre continuous function but f is not soft fuzzy almost G_δ pre continuous function.

References

[1] L. A. Zadeh, Fuzzy sets, *Inform and control,* **8** (1965), 338-353.

[2] C. L. Chang, Fuzzy topological spaces, *J. Math. Anal. Appl.,* **24** (1968), 182-190.

[3] P. Smets, The degree of belief in a fuzzy event, *Information Sciences,* **25** (1981), 1-19.

[4] M. Sugeno, An introductory Survey of fuzzy control, *Information Sciences,* **36** (1985), 59-83.

[5] A. S. Bin shahna, On fuzzy strong semi-continuity and fuzzy pre-continuity, *Fuzzy sets and systems,* **44** (1991), 303-408.

[6] G. Balasubramanian, Maximal fuzzy topologies, *KYBERNETIKA,* **31** (1995), 459-465.

[7] Ismail U. Tiryaki, Fuzzy sets over the poset I, *Hacettepe Journal of Mathematics and Statistics,* **37(2)** (2008), 143-166.

[8] T. Yogalakshmi, E. Roja, M. K. Uma, A view on soft fuzzy topological spaces, *Journal of Fuzzy Mathematics,* **21** (2013), 349 - 370.

[9] V. Visalakshi, M. K. Uma, E. Roja, On soft fuzzy G_δ pre continuity in soft fuzzy topological space, *Annals of Fuzzy Mathematics and Informatics,* **8(6)** (2014), 921-939.