

Contribution of Con Secondary k-Normal Bimatrices

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Abstract

The concept of conjugate secondary k-normal bimatrices is introduced. Some of characterization on con sk-normal bimatrices are given.

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1 Introduction

The concept of secondary k-normal matrices is introduced [1]. Some equivalent condition on conjugate normal bimatrices were given in [4]. In this paper our intention is to define con.sk-normal bimatrices and prove some characterization on conjugate sk-normal. Also we prove some equivalent condition on con.sk-normal bimatrices [6]. Recently, Hill and waters [3] have developed a theory for k-real and k-hermitian matrices. Anna Lee [1] has initiated the study of secondary symmetric matrices, that is matrices whose entries are symmetric about the (skew) secondary diagonal. Anna Lee has shown that for complex matrix, A_B^S are related as $A_B^S = V_B A_B^T V_B$ where V is the permutation matrix with units in the secondary diagonal.

2 Preliminaries and Notations

Let $C_{n \times n}$ be the space of $n \times n$ complex matrices of order n. For $A_B \in C_{n \times n}$. Let $A_B^T, \overline{A_B}, \overline{A_B^T}, A_B^s, \overline{A_B^s}$, denote the transpose, conjugate, conjugate transpose, secondary transpose, conjugate secondary transpose of matrix A_B respectively. Let 'K' be a fixed product of disjoint transposition in S_n the set of all permutations on $\{1, 2, \dots, n\}$ and K be the associated permutation matrix with units in the secondary diagonal. ' K_B ' and ' V_B ' clearly satisfies the following properties.

$$\begin{aligned} \bar{K}_B &= K_B^T = K_B^S = K_B^* = \overline{K_B^S} = K_B; \quad K_B^2 = 1 \\ \bar{V}_B &= V_B^T = V_B^S = V_B^* = \overline{V_B^S} = V_B; \quad V_B^2 = 1 \end{aligned}$$

Definition 1. A bimatrix A_B is defined as the union of two or square arrangement of number A_1 and A_2 arranged into rows and columns. It is written as follows $A_B = A_1 \cup A_2$, where “ $\cup$ ” just for notation, $A_1 \neq A_2$.

Definition 2. A matrix $A_B \in C_{n \times n}$ is said to be conjugate normal bimatrices if $A_B A_B^* = \overline{A_B^* A_B}$.

Definition 3. A matrix $A_B \in C_{n \times n}$ is said to con. secondary k-normal (s-k normal) bimatrix if

$$\begin{aligned} A_B (K_B V_B A_B^* V_B K_B) &= \overline{(K_B V_B A_B^* V_B K_B) A_B} \\ &= (\overline{K_1 \cup K_2}) (\overline{V_1 \cup V_2}) (\overline{A_1^* \cup A_2^*}) (\overline{V_1 \cup V_2}) (\overline{K_1 \cup K_2}) (\overline{A_1 \cup A_2}) \end{aligned}$$

Where A_B, V_B, K_B be the bimatrices

Example 1.

$$A_1 = \begin{pmatrix} i & i & 0 \\ i & 0 & i \\ 0 & i & i \end{pmatrix} = A_1^* \text{ and } A_2 = \begin{pmatrix} 0 & i & i \\ i & 0 & i \\ i & i & 0 \end{pmatrix} = A_2^*$$

3 Con Sk-Normal Bimatrices

In this section the concept of con Sk-normal bimatrices is introduced for complex square bimatrices as a special case of secondary k-normal matrices characterization of con sk-normal bimatrices analogous to that of secondary k-normal bimatrices are obtained.

Theorem 4. Let $A_B \in C_{n \times n}$

(a). If A_B is con Sk-normal bimatrices then iA_B is con Sk-normal bimatrix.

(b). If A_B is con Sk-normal bimatricesthe $-iA_B$ is con Sk-normal bimatrix.

Proof. (a). Given A_B is con Sk-normal

$$\begin{aligned} A_B (K_B V_B A_B^* V_B K_B) &= \overline{(K_B V_B iA_B^* V_B K_B) A_B} \\ i^2 A_B (K_B V_B A_B^* V_B K_B) &= i^2 \overline{(K_B V_B A_B^* V_B K_B) A_B} \\ -(iA_B) (K_B V_B iA_B^* V_B K_B) &= -\overline{(K_B V_B iA_B^* V_B K_B) iA_B} \end{aligned}$$

$$\begin{aligned} (iA_1 \cup iA_2) [(k_1 \cup k_2) (V_1 \cup V_2) (iA_1^* \cup iA_2^*) (V_1 \cup V_2) (k_1 \cup k_2)] \\ = \overline{(k_1 \cup k_2) (V_1 \cup V_2) (iA_1^* \cup iA_2^*) (V_1 \cup V_2) (k_1 \cup k_2) ((iA_1 \cup iA_2))} \end{aligned}$$

$$(iA_1 \cup iA_2) [(k_1 V_1 iA_1^* V_1 k_1) \cup (k_2 V_2 iA_2^* V_2 k_2)] = \overline{(k_1 V_1 iA_1^* V_1 k_1) \cup (k_2 V_2 iA_2^* V_2 k_2)} (\overline{iA_2 \cup iA_1})$$

Therefore $iA_B iA_B^* = \overline{(iA_B^*) (iA_B)}$. iA_B is con Sk-normal bimatrix

(b).

$$\begin{aligned}
A_B (k_B V_B A_B^* V_B k_B) &= \overline{(k_B V_B A_B^* V_B k_B)} A_B \\
i^2 (A_B K_B V_B A_B^* V_B k_B) &= i^2 \overline{(k_B V_B A_B^* V_B k_B)} \overline{A_B} \\
- (i A_B) k_B V_B (-i A_B^*) (V_B k_B) &= - \overline{(k_B V_B (-i A_B) V_B k_B)} \overline{(i A_B)} \\
(-i A_1 \cup -i A_2) [(k_1 \cup_1 (-i A_1^*) V_1 k_1) \cup (k_2 V_2 (-i A_2^*) V_2 k_2)] \\
&= \overline{(k_1 V_1 (-i A_1) V_2 k_1)} \cup \overline{(k_2 V_2 (-i A_2) V_2 k_2)} (-i A_1 \cup -i A_2)
\end{aligned}$$

Therefore $-i A_B$ is con Sk-normal bimatrix

□

Theorem 5. Let $A_B D_B \in C_{n \times n}$, A_B and D_B are Con Sk-normal bimatrices $A_B D_B^* = \overline{D_B^* A_B}$ and $D_B A_B^* = \overline{A_B^* D_B}$ then $A_B + D_B$ is con Sk-normal bimatrix.

Proof. Assume that A_B and D_B are con Sk-normal bimatrix $A_B A_B^* = \overline{A_B^* A_B}$ and $D_B D_B^* = \overline{D_B^* D_B}$. To Prove

$$\begin{aligned}
(A_B + D_B) (k_B V_B (A_B + D_B)^* V_B k_B) &= \overline{(k_B V_B (A_B + D_B)^* V_B k_B)} (A_B + D_B) \\
(A_B + D_B) (k_B V_B (A_B + D_B)^* V_B k_B) &= \overline{(A_B + D_B)} \overline{(k_B V_B (A_B^* + D_B^* V_B k_B))} \\
&= \overline{A_B k_B V_B A_B^*} + \overline{A_B D_B^* V_B k_B} + \overline{D_B k_B V_B A_B^*} + \overline{D_B V_B k_B D_B^*} \\
&= \overline{(A_1 \cup A_2) (k_1 \cup k_2) (V_1 \cup V_2) (A_1^* \cup A_2^*)} + \overline{(A_1 \cup A_2) (D_1^* \cup D_2^*)} \\
&\quad \overline{(V_1 \cup V_2) (k_1 \cup k_2)} + \overline{(D_1 \cup D_2) (k_1 \cup k_2) (V_1 \cup V_2) (A_1^* \cup A_2^*)} \\
&\quad + \overline{(D_1 \cup D_2) (D_1^* \cup D_2^*) (V_1 \cup V_2) (k_1 \cup k_2)} \\
(k_B V_B (A_B + D_B)^* V_B k_B) (A_B + D_B) &= \overline{(k_B V_B (A_B^* + D_B^* V_B k_B))} (\overline{A_B} + \overline{D_B}) \\
&= \overline{k_B V_B A_B^* A_B} + \overline{D_B^* V_B k_B A_B} + \overline{k_B V_B A_B^* D_B} + \overline{D_B^* D_B V_B k_B} \\
&= \overline{(A_1 \cup A_2) (k_1 \cup k_2) (V_1 \cup V_2) (A_1^* \cup A_2^*)} + \overline{(A_1 \cup A_2) (D_1^* \cup D_2^*)} \\
&\quad \overline{(V_1 \cup V_2) (k_1 \cup k_2)} + \overline{(D_1 \cup D_2) (k_1 \cup k_2) (V_1 \cup V_2) (A_1^* \cup A_2^*)} \\
&\quad + \overline{(D_1 \cup D_2) (D_1^* \cup D_2^*) (V_1 \cup V_2) (k_1 \cup k_2)}
\end{aligned}$$

Therefore $\overline{A_B} + \overline{D_B}$ is con Sk+normal.

□

Theorem 6. Let $A_B D_B \in C_{n \times n}$, A_B and D_B are Con Sk-normal bimatrices $A_B D_B^* = \overline{D_B^* A_B}$ and $D_B A_B^* = \overline{A_B^* D_B}$ then $A_B - D_B$ is con Sk-normal bimatrix.

Proof. Assume that A_B and D_B are con Sk-normal bimatrix $A_B A_B^* = \overline{A_B^* A_B}$ and $D_B D_B^* = \overline{D_B^* D_B}$. To Prove

$$(A_B - D_B) (k_B V_B (A_B - D_B)^* V_B k_B) = \overline{(k_B V_B (A_B - D_B)^* V_B k_B)} (A_B - D_B)$$

$$\begin{aligned}
(A_B - D_B)(k_B V_B (A_B - D_B)^* V_B k_B) &= \overline{(A_B - D_B)(k_B V_B (A_B^* - D_B^* V_B k_B))} \\
&= \overline{A_B k_B V_B A_B^* - A_B D_B^* V_B k_B - D_B k_B V_B A_B^* - D_B V_B k_B D_B^*} \\
&= \overline{(A_1 \cup A_2)(k_1 \cup k_2)(V_1 \cup V_2)(A_1^* \cup A_2^*) - (A_1 \cup A_2)(D_1^* \cup D_2^*)} \\
&\quad \overline{(V_1 \cup V_2)(k_1 \cup k_2) - (D_1 \cup D_2)(k_1 \cup k_2)(V_1 \cup V_2)(A_1^* \cup A_2^*)} \\
&\quad - \overline{(D_1 \cup D_2)(D_1^* \cup D_2^*)(V_1 \cup V_2)(k_1 \cup k_2)} \\
(k_B V_B (A_B - D_B)^* V_B k_B)(A_B - D_B) &= \overline{(k_B V_B (A_B^* - D_B^* V_B k_B))} \overline{(A_B - D_B)} \\
&= \overline{k_B V_B A_B^* A_B - D_B^* V_B k_B A_B - k_B V_B A_B^* D_B - D_B^* D_B V_B k_B} \\
&= \overline{(A_1 \cup A_2)(k_1 \cup k_2)(V_1 \cup V_2)(A_1^* \cup A_2^*) - (A_1 \cup A_2)(D_1^* \cup D_2^*)} \\
&\quad \overline{(V_1 \cup V_2)(k_1 \cup k_2) - (D_1 \cup D_2)(k_1 \cup k_2)(V_1 \cup V_2)(A_1^* \cup A_2^*)} \\
&\quad - \overline{(D_1 \cup D_2)(D_1^* \cup D_2^*)(V_1 \cup V_2)(k_1 \cup k_2)}
\end{aligned}$$

Therefore $\overline{A_B} - \overline{D_B}$ is con Sk-normal. $\square$

Note 1. Let $A_B D_B \in C_{n \times n}$, A_B and D_B are con Sk-unitary matrices if $A_B (k_B V_B D_B^* V_B k_B) = (k_B V_B D_B^* V_B k_B) A_B = I_B$ then $A_B \pm D_B$ is con Sk-unitary matrices are also con secondary k normal matrices.

Theorem 7. Let $A_B D_B \in C_{n \times n}$, if A_B and D_B are con. secondary K -unitary the $A_B D_B$ is con secondary k -normal.

Proof. Given A_B and D_B are con Secondary K -unitary

$$\begin{aligned}
A_B (k_B V_B A_B^* V_B k_B) &= (k_B V_B A_B^* V_B k_B) A_B = I_B \\
D_B (k_B V_B D_B^* V_B k_B) &= (k_B V_B D_B^* V_B k_B) D_B = I_B
\end{aligned}$$

To Prove:

$$\begin{aligned}
(k_B V_B A_B D_B)(A_B D_B k_B V_B)^* &= (A_B D_B k_B V_B)^* (k_B V_B A_B D_B) \\
&= (K_B V_B A_B (D_B D_B^*) A_B^* V_B^* K_B^*) \\
&= k_B V_B A_B I_B A_B^* k_B^* V_B^* \\
&= I_B \\
&= I_1 \cup I_2 \\
(A_B D_B k_B V_B)(k_B V_B A_B D_B) &= \overline{(k_B V_B A_B D_B)(A_B^* D_B^* k_B^* V_B^*)} \\
&= \overline{(k_B V_B A_B D_B)(A_B^* D_B^* k_B^* V_B^*)} \\
&= \overline{k_B k_B^* V_B V_B^* A_B A_B^* D_B D_B^*} \\
&= I_B \\
&= I_1 \cup I_2
\end{aligned}$$

Therefore $A_B D_B$ is con Sk-unitary $\square$

Theorem 8. The transpose of an S-K normal bimatrix is con S-K normal bimatrix.

Proof. A_B is con s-k normal

$$\begin{aligned}
 k_B V_B A_B A_B^* k_B V_B &= \overline{(k_B V_B A_B^* A_B k_B V_B)} \\
 (k_B V_B A_B A_B^* k_B V_B)^T &= \overline{(k_B V_B A_B^* A_B k_B V_B)^T} \\
 (k_B)^T (V_B)^T (A_B)^T (A_B^*)^T (k_B)^T &= \overline{(k_B)^T (V_B)^T (A_B^*)^T (A_B)^T (k_B)^T (V_B)^T} \\
 k_B V_B A_B^T (A_B^T)^* k_B V_B &= \overline{k_B V_B (A_B^T)^* (A_B)^T k_B V_B} \\
 k_1 V_1 A_1^T (A_1^T)^* k_1 V_1 \cup (k_2 V_2 A_2^T (A_2^T)^* k_2 V_2) &= \overline{[k_1 V_1 A_1^T (A_1^T)^* k_1 V_1] \cup (k_2 V_2 A_2^T (A_2^T)^* k_2 V_2)} \\
 &= A_B^T \text{ is con S-k normal}
 \end{aligned}$$

A_B is con S-k normal bimatrix. □

Theorem 9. The secondary transpose of an con SK normal Bimatrix is con SK normal bimatrix

Proof.

$$\begin{aligned}
 A_B (K_B V_B A_B^* V_B K_B) &= \overline{(K_B V_B A_B^* V_B K_B)} \overline{A_B} \\
 (A_B (K_B V_B A_B^* V_B K_B))^S &= \overline{((K_B V_B A_B^* V_B K_B) \overline{A_B})^S} \\
 (K_B V_B (A_B^*)^S V_B K_B) A_B^S &= \overline{A_B^S (K_B V_B (A_B^*)^S V_B K_B)} \\
 (K_1 \cup K_2) (V_1 \cup V_2) (A_1^S)^* \cup (A_2^S)^* (V_1 \cup V_2) A_1^S \cup A_2^S \\
 &= \overline{A_1^S \cup A_2^S (\overline{K_1} \cup \overline{K_2}) (\overline{V_1} \cup \overline{V_2}) (\overline{A_1^S})^* \cup (\overline{A_2^S})^* (\overline{V_1} \cup \overline{V_2}) (\overline{K_1} \cup \overline{K_2})} \\
 (A_1^S \cup A_2^S) (K_1 V_1 (A_1^S)^* V_1 K_1) \cup (K_2 V_2 (A_2^S)^* V_2 K_2) \\
 &= \overline{(K_1 V_1 (A_1^S)^* V_1 K_1) \cup (K_2 V_2 (A_2^S)^* V_2 K_2)} \overline{A_1^S \cup A_2^S}
 \end{aligned}$$

Thus A_B^S is con SK normal Bimatrix. □

Theorem 10. The conjugate of an SK normal Bimatrix is con SK normal Bimatrix.

Proof.

$$\overline{A_B (\overline{K_B V_B A_B^* V_B K_B})} = (\overline{K_B V_B A_B^* V_B K_B}) \overline{A_B}$$

$$\begin{aligned}
& \overline{(A_1 \cup A_2)} (K_1 \cup K_2) (V_1 \cup V_2) \overline{(A_1^* \cup A_2^*)} (V_1 \cup V_2) (K_1 \cup K_2) \\
&= \overline{(K_1 \cup K_2) (V_1 \cup V_2) \overline{(A_1^* \cup A_2^*)} (V_1 \cup V_2) (K_1 \cup K_2)} \\
& (\overline{A_1} \cup \overline{A_2}) (K_1 \cup K_2) (V_1 \cup V_2) (\overline{A_1})^* \cup (\overline{A_2})^* (V_1 \cup V_2) (K_1 \cup K_2) \\
&= \overline{(K_1 \cup K_2) (\overline{V_1} \cup \overline{V_2}) (\overline{A_1})^* \cup (\overline{A_2})^* (\overline{V_1} \cup \overline{V_2}) (\overline{K_1} \cup \overline{K_2}) (\overline{A_1} \cup \overline{A_2})} \\
\Rightarrow & (\overline{A_1} \cup \overline{A_2}) (K_1 \cup K_2) (V_1 \cup V_2) (A_1^* \cup A_2^*) (V_1 \cup V_2) (K_1 \cup K_2) \\
&= \overline{(K_1 \cup K_2) (\overline{V_1} \cup \overline{V_2}) (\overline{A_1} \cup \overline{A_2})^* (\overline{V_1} \cup \overline{V_2}) (\overline{K_1} \cup \overline{K_2})} \\
& (\overline{A_1} \cup \overline{A_2}) \overline{A_1} \cup \overline{A_2} (K_1 V_1 A_1^* V_1 K_1) \cup (K_2 V_2 A_2^* V_2 K_2) \\
&= \overline{(K_1 V_1 A_1^* V_1 K_1)} \cup \overline{(K_2 V_2 A_2^* V_2 K_2)}
\end{aligned}$$

$$\overline{A_1} \cup \overline{A_2} \overline{(K_1 V_1 A_1^* V_1 K_1)} \cup \overline{(K_2 V_2 A_2^* V_2 K_2)} = \overline{(K_1 V_1 A_1^* V_1 K_1)} \cup \overline{(K_2 V_2 A_2^* V_2 K_2)}$$

Hence $\overline{A_B}$ is con SK normal Bimatrices. □

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