

ANALYSIS OF M/M/1 QUEUEING SYSTEM WITH VARIANT OF WORKING VACATION UNDER SOJOURN TIME

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Abstract: In this, paper we consider an M/M/1 queueing model with server vacations. The customer arrives according to a poisson process and service time follows exponential distribution. We derive the Probability generating function (PGF) of the steady state probabilities and obtain the system sizes when the server is in the different states. We further make comparisons between the mean system sizes under the variant vacation policy and mean system size under sojourn time. Finally, we present some numerical results to show the effects of the results.

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Key words: vacation policy, PGF, service time, server vacation, queueing system, working vacation.

1.Introduction

Queueing systems with server vacations that can be used in modeling numerous real world queueing situations have arisen in systems such as telecommunications, manufacturing and computer science. A queueing system with a temporary unavailable server can describe the service processes of applicants at the cash desk in that break are ordered when the queue empties. The working vacation model as server does not stop the service completely but serves the customers with a lower rate or performs some ancillary task. This type of vacation known as working vacation. Working vacation queueing model is useful for the system where in the server may utilize idle time for different tasks. Vacation queueing model have been studied by many authors in different frame works.

All these papers mentioned above use the single or multiple vacation policy in analysis of the queueing model with impatient customers and vacations[1-3]. Queueing systems with

server vacations literature relating to this topic have been written by [4-5]. [6-8] explained the queue with working vacations and vacation interruption. [9-12] analysed the number of customers in the system and calculating performance measures such as mean system size of server on vacation and server not on vacation. A queue with working vacation was studied by [13-15]. The rest of the paper is organized as follows. The mathematical description of our model is in section 2 and equations governing the model are given in section 3. In section 4, we make comparison between the mean system sizes under the variant vacation policy. Some numerical results in section 5. Conclusions are given section 6.

2. Model description

We consider M/M/1 queueing system with impatient customers and a variant type of vacation policy. Customers arrive according to a poisson process at a rate λ . The server is provided by a single server, who serves the customers on first-come-first served basis. The service time follow an exponential distribution with a service rate μ . When the server finishes serving a customer and finds the system empty, the server leaves for a vacation. If the server finds a customer at a vacation completion instant, the server returns to serve customers immediately. Otherwise the server will take consecutively until the server has taken a maximum number 'n' vacations. The vacation times are assumed to be exponentially distributed with vacation rate η . During the vacation, customers become impatient, whenever a customer arrives at the system during a vacation, it activates an impatience timer 't' which is exponentially distributed with parameter ' β ' and also vacation of the dormant period of the system is D_p , ϵ be the least +ve constant of dominant period.

Generating Function: Let $C(t)$ denote the number of customers in the system at a time t , and $I(t)$ denote the status of the server at a time t , which is defined as follows: $I(t)=i$ denotes that the server is taking the $(i+1)$ vacation at time t for $i=0,1,2,m-1$ while $I(t)=m$ denotes the server is idle or busy at time t . Then the process $C(t), I(t), t \geq 0$ defines a continuous time markov process with state space $\Omega = (n, i) : n \geq 0, i = 0, 1, 2, \dots, m$. Let $P_n i = \lim_{t \rightarrow \infty} P C(t) = n, I(t) = i, n \geq 0, i = 0, 1, 2, \dots, m$ denote the process $C(t), I(t), t \geq 0$.

3. Governing Equation

$$(\lambda + \eta)P_{00} = \beta P_{10} + \mu P_{1m} + D P_{10} + \epsilon_1, \epsilon \geq 0 \quad (1)$$

$$(\lambda + \eta + n\beta)P_{n0} = \lambda P_{(n-1)0} + (n+1)\beta P_{(n+1)0} + D P_{0m} + \epsilon_2, n \geq 1 \quad (2)$$

$$(\lambda + \eta)P_{0i} = \beta P_{1i} + \eta P_{0(i-1)}, i = 1, 2, \dots, m-1 \quad (3)$$

$$(\lambda + \eta + n\beta)P_{ni} = \lambda P_{(n-1)i} + (n+1)\beta P_{(n+1)i} + D P_{im} + \epsilon_j, i = 1, 2, \dots, m-1, n \geq 1 \quad (4)$$

$$\lambda P_{0m} = \eta P_{0(m-1)} + D P_{0m} \quad (5)$$

$$(\lambda + \mu)P_{nm} = \lambda P_{(n-1)m} + \mu P_{(n+1)m} + \eta \sum_{i=0}^{m-1} P_{ni}, n \geq 1 \quad (6)$$

$$\sum_{n=0}^{\infty} \sum_{i=0}^m P_{ni} = 1 \quad (7)$$

Define the PGF, $R_i(z) = \sum_{n=0}^{\infty} P_{ni} z^n, 0 \leq z \leq 1, i = 0, 1, 2, \dots, m$. and $R'_i(z) = \frac{d}{dz}[R_i(z)], i = 0, 1, 2, \dots, m$

In this system 'A' customers are present at a service completion epoch, all the existing customers are taken into a service. Customers who arrive while a group of customers are being served cannot join the service even if there is space available. The server takes a vacation of random length 'w' if there are no customers to serve. If no customers arrive and begins to serve the dormant period.

Let us define the following PGF,

$$S(z, n) = \sum_{n=0}^{\infty} S_n(x) z^n \text{ and } \hat{T}(z, n) = \sum_{n=0}^{\infty} T_n(x) z^n$$

The general solution for the system as,

$$S_0(x) = C_1 e^{-\lambda x} [1 - w(n)] = \left[\frac{\lambda I_0(x)}{w^*(\lambda)} \right] [1 - w(x)] e^{-\lambda x} + D P_{10} \quad (8)$$

where $w^*(\lambda) = \int_0^{\infty} e^{-\lambda x} dw(x)$

is the probability that no customers arriving during a vacation and

$I_0(t) = P[C(t) = 0, I(t)]$ then

$$S(z) = \sum_{n=0}^{\infty} S_n z^n = \left\{ \frac{\lambda I_0}{w^*(\lambda)} \right\} \left\{ \frac{1 - w^*(\lambda - \lambda z)}{(\lambda - \lambda z)} \right\}$$

$S(z)$ is the PGF of queue size when the server is on vacation and also

$$\hat{T}(z) = \sum_{n=0}^{\infty} T_n z^n = \frac{\hat{T}(z, 0) [1 - q^* \lambda (1 - z)]}{(\lambda - \lambda z)}$$

note that $\hat{T}(z)$ is the PGF of the queue size when the server is busy in the system. Where q^* be the Laplace-stieltjes transform. We obtain the PGF of the queue size as,

$$P_q(z) = I_0 + S(z) + \hat{T}(z)$$

after some manipulation the PGF of the queue size can be written as

$$P_q(z) = \frac{(1 - w^*(\lambda - \lambda z) + w^*(z)(1 - z)k_0)}{\lambda - \lambda z} + \left\{ \frac{1 - q^*(\lambda - \lambda z)}{\lambda - \lambda z} \right\} \left\{ \frac{w^*(\lambda - \lambda z) + w^*(\lambda)(z^4 - 1) - 1}{z^4 - q^*(\lambda - \lambda z)} \right\} + \left\{ \frac{1 - w^*(\lambda - \lambda z)}{\lambda - \lambda z} \right\} \left\{ \left\{ \frac{\sum_{j=1}^A (z^A - z^j) k_j}{z^A - w^*(\lambda - \lambda z)} \right\} \right\} \text{ where } K_0 = \frac{\lambda I_0}{w^*(\lambda)} \quad (9)$$

4. Mean system size

For $i = 0, 1$. Let f_i and $E[f_i]$ be the number of customers in the system when the server is in state i , then $E[f_i] = S'_i(1) = \sum_{n=1}^{\infty} n P_{in}$ where $i = 0, 1$. Let P_i be the probability that

the server is in state j . Then $P_i = S_i(1) = \sum_{n=0}^{\infty} P_{in}$, $i = 0, 1$. For all possible values of 'n' such that $\eta P_0 = \mu_b P_{11} + \eta P_{00}$. For summing all possible values of 'n' equations (1),(2) and (3) and rearranging terms we get

$$\beta z(1-z)S'_0(z) + f(z)S_0(z) + Bz - C(1-z) = 0$$

where $f(z) = \lambda z^2 - (\lambda + \eta + \mu_v)z + \mu_v$, $B = \mu_b P_{11} + \eta P_{00}$, $C = \mu_v P_{00}$
we derive the formula,

$$S_1(1) = \frac{\eta}{\mu_b - \lambda} S'_0(1) \text{ where } S'_0 = E(f_0) \quad (A)$$

defined as the mean number of customers when the server is on a working vacation. Since $S_1(1) = P_i = 1 - P_0$. From the equation (A) we get $E[f_0] = \frac{\mu_b - \lambda}{\eta}(1 - P_0)$. In order to obtain $E[f_0]$, we need to derive P_0 , that being the proportion of time the server is on a working vacation.

We derive the probability and the mean system sizes when the server is in different states.

$$P_{00} = \left[\frac{\mu\eta + (\mu - \lambda)\beta}{(\mu - \lambda)(\eta + \beta)} \cdot \frac{1 - A^k}{A(1 - A)} + \frac{\mu\eta}{\lambda(\mu - \lambda)} A^{m-1} \right]^{-1}$$

$$\text{where } A = \frac{\eta}{\beta}c \text{ and } c = \int_0^1 e^{-\frac{\lambda}{\beta}x} (1-x)^{\frac{\eta}{\beta}-1} dx \quad (10)$$

multiplying each equations for 'n' in equations (1),(2),(3) and (4) by z^n summing all possible values 'n' and re-arranging terms, we get

$$\beta(1-z)S'_0(z) - [\lambda(1-z) + \eta]S_0(z) = -\mu P_{1m} + \psi P_{0m} \quad (11)$$

$$\text{and } \beta(1-z)S'_i(z) - [\lambda(1-z) + \eta]S_i(z) = -\eta P_{0i-1} + \psi_{im}, i = 1, 2, \dots, m-1 \quad (12)$$

generally $P_{0m} = \frac{\eta}{\lambda} A^{m-1} P_{00}$ we obtain the mean system size when the server is on vacation as,

$$E[C_v] = \frac{\lambda^2(\mu - \lambda)}{\mu\lambda[\lambda + (\eta + \beta)G(m)] + \lambda\beta(\mu - \lambda)} + D_{P_{im}} + \epsilon \quad (13)$$

$$\text{where } G(m) = \frac{A^m(1-A)}{1-A^m} \quad (13a)$$

and ϵ be the last dominant period of the every vacation of $D_{P_{im}}$ and also derive mean system size when the server is not on vacation as ,

$$E[C_m] = \frac{\eta}{\mu - \lambda} \left(\frac{\mu}{\mu - \lambda} + \frac{\lambda}{\eta + 2\beta} \right) E(C_v) + \frac{\lambda\mu}{(\mu - \lambda)^2} P_{0m} + D_{P_{im}} \quad (14)$$

where, $i = 0, 1, 2, \dots, m$

and the probability P_{0m} that the server is idle is calculated by,

$$P_{0m} = \frac{\eta(\mu - \lambda)(\eta + \beta)G(m)}{\mu\eta[\lambda + (\eta + \beta)G(m)] + \lambda\beta(\mu - \lambda)}$$

$$\text{where, } G(m) = \frac{A^m(1-A)}{1-A^m} = \frac{A^m - A^{m+1}}{1-A^m} \quad (15)$$

and let C be the number of customer in the system. Then the mean system size is $E(C) = E(C_v) + E(C_m)$

Sojourn Times: Let be the total sojourn time of a customer in the system, measured from the moment of arrival until departure, that departure either being due to completion of service or as a result of abandonment then by Little's law we have,
 $E(S) = \frac{1}{\lambda} E(C_v) + E(C_m)$. The probability when the system is empty and the server is on vacations given by, $P_e = \sum_{i=0}^{m-1} P_{0i} = \frac{1-A^m}{1-A} P_{00}$

5. Numerical illustration

In this section, we investigate numerically the effects of the parameter β and η of the mean system size $E(C_v)$ when the server is on vacation and the mean system size $E(C_m)$ when the server is not on vacation and also both of them including Dominant periods. We choose parameters: $\lambda = 4, \mu = 5$ and $\eta = 2$ the vacations of some performance measures with 'n' are presented for various β . In Table 1, the values of β are chosen to be smaller than that in Table 2. (i.e) Table 1: $\beta = 0.5, 1.0, 1.5$, Table 2: $\beta = 2.5, 3.0, 3.5$. It's observed from Table 1 and Table 2 that $E(C_v)$ and $E(C)$ decreases with for finite 'n'. Therefore, more customers may abandon the system during the server's vacation with the increasing of parameter β , which results in the decreasing of the mean system size $E(C_v)$ and $E(C)$ and also decreasing for the sojourn time.

Table 1

n	β	$E(C_v)$	$E(C_m)$	$E(C)$	$E(S)$
1	0.5	0.5147	4.8818	5.3965	1.3491
1	1.0	0.4892	4.4892	4.9784	1.2446
1	1.5	0.4660	4.2266	4.6926	1.7232
2	0.5	0.6611	5.0823	5.7434	1.4359
2	1.0	0.6397	4.6397	5.2794	1.3199
2	1.5	0.6169	4.3320	4.9516	1.2379
3	0.5	0.7743	5.2109	5.9852	1.4963
3	1.0	0.7551	4.7551	5.5102	1.3776
3	1.5	0.7370	4.4337	5.1707	1.2927
4	0.5	0.8786	5.3204	6.1990	1.5498
4	1.0	0.8605	4.5344	5.3780	1.3445
4	1.5	0.8436	4.6397	5.2794	1.3199
5	0.5	0.9801	5.4236	6.4037	1.6009
5	1.0	0.9624	4.9624	5.9248	1.4812
5	1.5	0.9461	4.6346	5.5807	1.3952

Table 1:performance measures with variations of β and η for $\beta=0.5,1.0,1.5$ **Table 2**

n	β	$E(C_v)$	$E(C_m)$	$E(C)$	$E(S)$
1	2.5	0.4225	3.8949	4.3174	1.0794
1	3.0	0.3989	3.8022	4.2011	1.0503
1	3.5	0.3596	3.7833	4.1429	1.0357
2	2.5	0.5818	3.9175	4.4993	1.1248
2	3.0	0.5568	3.7864	4.3432	1.0858
2	3.5	0.5130	3.7439	4.2569	1.0642
3	2.5	0.7032	3.9885	4.6917	1.1729
3	3.0	0.6814	3.8372	4.5186	1.1297
3	3.5	0.6394	3.7749	4.4143	1.1036
4	2.5	0.8164	4.0705	4.8345	1.2586
4	3.0	0.7990	3.9003	4.6993	1.1748
4	3.5	0.7651	3.8077	4.5728	1.1432
5	2.5	0.9192	4.1668	5.0860	1.2715
5	3.0	0.9052	3.9896	4.8948	1.2237
5	3.5	0.8773	3.8760	4.7533	1.1883

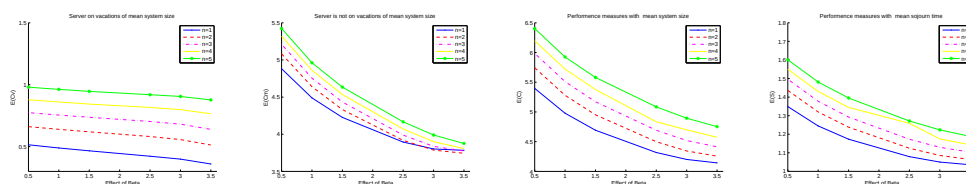
Table 2:performance measures with variations of β and η for $\beta=2.5,3.0,3.5$ 

Figure 1:

6.Conclusion:

In this paper, we have studied an M/M/1 queueing system with variant server vacation policy. We have derived PGF of the system sizes when the server is on vacation and not in vacation respectively. We have made comparison between the working vacation policy for the mean system size and mean sojourn time. The numerical examples illustrate that as impatience parameter β then $E(C_v), E(C_m), E(C), E(s)$ as n . It shows the efficiency of the results.

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