

SOME DOMINATION PARAMETERS ON JUMP GRAPH

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Abstract

Let $G = (p, q)$ be a connected graph and $J(G)$ be its corresponding jump graph. A dominating set D is called a Split dominating set if the induced subgraph $V[J(G)] - D$ is disconnected. The minimum cardinality of a split dominating set of $J(G)$ is called the split domination number of $J(G)$ and is denoted by $\gamma_s[J(G)]$. In this paper we obtain several results on Non-split domination number also.

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1 Introduction

All the graphs $G(p, q)$ considered here are simple, finite, non-trivial, undirected and connected with $p = |V|$ and $q = |E|$ vertices and edges respectively [3]. We assume that the graph G under consideration is nonempty and has no isolated vertices. The jump graph $J(G)$ of a graph G is the graph defined on $E(G)$ and in which two vertices are adjacent if and only if they are not adjacent in G . It may be noted that the jump graph $J(G)$ of G , cannot be disconnected [5]. Hence hereafter by a graph we mean a simple graph with $p > 4$ and degree of any edge e in G should be lesser than $q - 1$. A set $D \subseteq V[J(G)]$ is a dominating set of a jump graph, if every vertex not in D is adjacent to a vertex in D . The domination number of a jump graph is the minimum cardinality of a dominating set of a jump graph $J(G)$ [8] and [4]. The concept of split dominating set was introduced by Kulli and Janakiram[7]. The dominating set $D \subseteq V(G)$ is a split dominating set if $V - D$ is a disconnected

subgraph of G . The concept of non-split dominating set { path /cycle non-split dominating set} was introduced by Kulli, Janakiram and Nandargi [6].

Theorem 1. [3] If $G(V, E)$ is a simple graph then, $2|q| \leq |p^2| - |p|$.

Theorem 2. [8] For any Complete graph K_p , with $p \geq 5$, $\gamma[J(K_p)] = 3$.

Theorem 3. [1] For any tree T with diameter greater than 3, $\gamma_c[J(T)] = 2$.

2 Split Domination Number of a Jump Graph.

Definition 4. A dominating set D of $J(G)$ is called a split dominating set if the induced subgraph $V[J(G)] - D$ is disconnected. The minimum cardinality of a split dominating set of $J(G)$ is called the split domination number of $J(G)$ and is denoted by $\gamma_s[J(G)]$.

We study split domination number of a jump graph for few standard graphs.

Theorem 5. For any path P_p ,

$$\gamma_s[J(P_p)] = \begin{cases} p-3 & \text{for } p=5 \\ p-4 & \text{for } p \geq 6. \end{cases}$$

Proof. Proof is obvious for $p=5$. Suppose that $p \geq 6$. Consider any subgraph P_4 of length 3 in P_p . Let E_1 be the set of edges in P_4 . Let V'_1 be the set of vertices in $J(P_4)$ corresponding to the set of edges E_1 of P_p . Clearly the induced subgraph $\langle V'_1 \rangle$ is disconnected and $D = V[J(P_p)] - V'_1$ is a dominating set of $J(P_p)$. Therefore D is a split dominating set of $J(P_p)$. $\square$

Theorem 6. For any wheel W_p ,

$$\gamma_s[J(W_p)] = \begin{cases} 3 & \text{for } p=5 \\ p-3 & \text{for } p \geq 6. \end{cases}$$

Proof. For $p=5$, proof is obvious. Suppose that $p \geq 6$. Consider an edge say e on the spokes of wheel W_p . There exists $p+1$ edges in $N[e]$. Let E_1 be the set of edges containing $N[e]$. Let V'_1 be the set of vertices in $J(W_p)$ corresponding to the edges E_1 of W_p . Clearly the induced subgraph $\langle V'_1 \rangle$ is disconnected and $D = V[J(W_p)] - V'_1$ is a dominating set of $J(W_p)$. Therefore D is a split dominating set of $J(W_p)$. $\square$

Theorem 7. For any complete bipartite graph $K_{m,n}$ with $2 \leq m \leq n$,

$$\gamma_s[J(K_{m,n})] = mn - 4.$$

Proof. For $K_{2,3}$ proof is obvious. Let $V_1 = \{v_1, v_2, \dots, v_m\}$ and $V_2 = \{v'_1, v'_2, \dots, v'_n\}$ be the two bipartites of $K_{m,n}$. Choose two edges e_i, e_j in $K_{m,n}$ such that they are incident to a common vertex $v'_j \in V_2$ and the other end vertices of edges chosen are v_i and $v_j \in V_1$. Similarly choose two more edges e_r, e_s in $K_{m,n}$ such that they are incident to a common vertex $v'_k \in V_2$ and $v'_k \neq v'_j$. Also the other end vertices of edges chosen are the same v_i and $v_j \in V_1$. Let $E_1 = \{e_i, e_j, e_r, e_s\}$ be the edge set in $K_{m,n}$. Let V'_1 be the set of vertices in $J(K_{m,n})$ corresponding to the set of edges E_1 of $K_{m,n}$. Clearly the induced subgraph $\langle V'_1 \rangle$ is disconnected and $D = V[J(K_{m,n})] - V'_1$ is a dominating set of $J(K_{m,n})$. Therefore D is a split dominating set of $J(K_{m,n})$. $\square$

Theorem 8. For any complete graph K_p , for $p \geq 5$, $\gamma_s[J(K_p)] = \frac{(p-2)(p-3)}{2}$.

Proof. Consider any edge $e \in K_p$. Let E_1 be the set of edges containing $N[e]$. Let V'_1 be the set of vertices in $J(K_p)$ corresponding to the set of edges of E_1 of K_p . Clearly the induced subgraph $\langle V'_1 \rangle$ is disconnected and $D = V[J(K_p)] - V'_1$ is a dominating set of $J(K_p)$. Therefore D is a split dominating set of $J(K_p)$. Thus

$$\begin{aligned}\gamma_s[J(K_p)] &= |V[J(K_p)]| - |V'_1|, \\ &= \frac{p(p-1)}{2} - 2(p-1) + 1.\end{aligned}$$

□

Theorem 9. If the diameter $\text{diam}(G) \geq 5$, $\gamma_s[J(G)] = q - \Delta'(G) - 1$.

Proof. Let e be an edge with maximum degree in G . Let S be the set of edges in $N[e]$. Let V'_1 be the set of vertices in $J(G)$ corresponding to the set of edges S of G . Since the diameter, $\text{diam}(G) \geq 5$, every vertex in V'_1 is adjacent to some vertex in $D = V[J(G)] - V'_1$. Also the induced subgraph $\langle V'_1 \rangle$ is disconnected. Thus D is a split dominating set of $J(G)$. □

Let e_1 be an edge in G with $d(e_1) = q - 2$. Let e_i be an edge non-adjacent to e_1 in G . Let v'_1 and v'_i be the vertices in $J(G)$ corresponding to the edges e_1 and e_i of G respectively. Then we have the following theorem.

Theorem 10. Let v'_i be a cutvertex of $J(G)$. If there is a block H in $J(G)$ so that v'_i is the cutvertex of H and v'_i is adjacent to all vertices of H , then there is a γ'_s -set of $J(G)$ containing v'_i .

Proof. If there exist at least two blocks in $J(G)$ which are satisfying the given condition then v'_i is in every γ'_s -set of $J(G)$. □

Theorem 11. For any graph G of order $p \geq 5$, $\gamma_s[J(G)] \leq {}^pC_2 - 2d(v) + 1$. Equality holds if and only if G is isomorphic to a complete graph K_p .

Proof. Let v_i and v_j be any two adjacent vertices of G . Since the graph is isomorphic to complete graph K_p , it implies that $d(v_i) = d(v_j) = d(v)$ and there exists an edge e_i common to v_i and v_j . Also by Theorem 1, $q = {}^pC_2$. Suppose E_1 be the set of edges adjacent to v_i and v_j then $|E_1| = d(v_i) + d(v_j) - 1$. Now without loss of generality in $J(G)$, let $V'_1 \subseteq V[J(G)]$ be the set of vertices corresponding to the set of edges E_1 of G . Then the induced subgraph $\langle V'_1 \rangle$ is disconnected and $D = V[J(G)] - V'_1$ is a dominating set of $J(G)$. Thus D forms a minimal split dominating set of $J(G)$. This implies that

$$\begin{aligned}|D| &= |V[J(G)]| - |V'_1| \\ &= |E(G)| - |E_1|.\end{aligned}$$

Therefore $\gamma_s[J(G)] = {}^pC_2 - 2d(v) + 1$. □

Theorem 12. If D is a γ_s set of $J(G)$, then $V[J(G)] - D$ is a dominating set of $J(G)$ and hence $\gamma[J(G)] + \gamma_s[J(G)] \leq q$. 2017

Proof. Suppose $V[J(G)] - D$ is not a dominating set of $J(G)$ then there exists a vertex v in D which is not adjacent to any of the vertices in $V[J(G)] - D$. Thus $D - \{v\}$ is a split dominating set of $J(G)$, a contradiction to the minimality of D . Further $V[J(G)] - D$ is a dominating set of $J(G)$ and so $|V[J(G)] - D| \geq \gamma[J(G)]$. This implies that inequality stated above is true. $\square$

Theorem 13. For any connected graph G , $\gamma_s[J(G)] \leq q - \Delta(G)$.

Proof. Let $V = \{v_1, v_2, \dots, v_p\}$ be the set of vertices in G and let $V_1 = V - v_1$ where v_1 is one of the vertex of maximum degree in G . Choose $E_1 = \{e_1, e_2, \dots, e_k\}$ as the set of edges adjacent to v_1 in G . Let $H \subseteq V[J(G)]$ be the set of vertices corresponding to the set of edges of E_1 . Then $I \subseteq V[J(G)] - H$ forms a minimally dominating set. Clearly the induced subgraph $\langle H \rangle$ is disconnected. Therefore $\gamma_s[J(G)] \leq |V[J(G)]| - |H|$. $\square$

3 Non-Split Domination Number of a Jump graph.

Definition 14. A dominating set D of $J(G)$ is called a non-split dominating set if the induced subgraph $V[J(G)] - D$ is connected. The minimum cardinality of a non-split dominating set of $J(G)$ is called the non-split domination number of $J(G)$ and is denoted by $\gamma_{ns}[J(G)]$.

Theorem 15. 1. For any path P_p with $p \geq 5$, $\gamma_{ns}[J(P_p)] = 2$.

2. For any cycle C_p ,

$$\gamma_{ns}[J(C_p)] = \begin{cases} 3 & \text{for } p = 5 \\ 2 & \text{for } p \geq 6. \end{cases}$$

3. For any complete graph K_p with $p \geq 5$, $\gamma_{ns}[J(K_p)] = 3$.

4. For any complete bipartite graph $K_{m,n}$,

$$\gamma_{ns}[J(K_{m,n})] = \begin{cases} 2 & \text{for } K_{2,n}, n \geq 4, \\ 3 & \text{for } K_{2,3} \text{ and } 3 \leq m \leq n. \end{cases}$$

5. For any wheel W_p ,

$$\gamma_{ns}[J(W_p)] = \begin{cases} 3 & \text{for } p = 5, 6 \\ 2 & \text{for } p \geq 7. \end{cases}$$

Theorem 16. For any connected graph G , $\gamma_{ns} \leq q - \beta_1(G) + 1$ where $\beta_1(G)$ is edge independent number of G .

Proof. Let $V_1 = \{v_1, v_2, \dots, v_n\}$ be the set of vertices in $J(G)$ corresponding to the set of independent edges $\{e_1, e_2, \dots, e_n\}$ of G . By the definition of $J(G)$, the induced subgraph $\langle V_1 \rangle$ forms a complete graph K_n . Choose any vertex $v \in V_1$, then $D = S \cup \{v\}$ where $S = V[J(G)] - V_1$ forms a dominating set. Further the induced

subgraph $\langle V[J(G)] - D \rangle$ is a complete graph with $(n - 1)$ vertices. Thus D forms a non-split dominating set of $J(G)$. Therefore

$$D = \{V[J(G)] - V_1\} \cup \{v\}$$

$$\gamma_{ns}[J(G)] \leq q - \beta_1(G) + 1.$$

□

Theorem 17. For any graph G with diameter, $\text{diam}(G) > 2$, $\gamma_{ns}[J(G)] = 2$.

Proof. Let uv be a path of maximum distance in G . Then $d(u, v) = \text{diam}(G)$. Let e_1 be any edge adjacent to u and e_2 be any edge adjacent to v . Let $V'_1 = \{v'_1, v'_2\} \subseteq V[J(G)]$ be a vertex set corresponding to the edge set $\{e_1, e_2\} \in E(G)$. Then V'_1 forms a dominating set in a jump graph. Also since the diameter of the graph G is greater than 2 it follows that $\langle V[J(G)] - V'_1 \rangle$ forms a connected subgraph. Therefore V'_1 forms a minimum non-split dominating set of $J(G)$. Hence $\gamma_{ns}[J(G)] = 2$. □

Theorem 18. For any tree T with $\text{diam}(G) > 3$, $\gamma_{ns}[J(T)] = \gamma_c[J(T)]$.

Proof. Let $u_1 - u_i$ be the longest path of T , labelled as $u_1 e_1 u_2 e_2 \cdots u_{i-2} e_{i-2} u_{i-1} e_{i-1} u_i$. Let e_1, e_{i-1} be the two pendant edges of the $u_1 - u_i$ path. Let v'_1, v'_{i-1} be the vertices in $J(T)$ corresponding to the edges e_1, e_{i-1} respectively of G . Then by Theorem 3 $D = \{v'_1, v'_{i-1}\}$ be the minimal dominating set of $J(T)$. Also the induced subgraph $\langle D \rangle$ is connected. Further, suppose e_2, e_{i-2} be the two edges of $N(e_1)$ and $N(e_{i-1})$ respectively of $u_1 - u_i$ path. Let v'_2 and v'_{i-2} be the vertices in $J(T)$ corresponding to the edges e_2, e_{i-2} of G respectively. Then $D_1 = \{v'_2, v'_{i-2}\}$ forms the minimal dominating set of $J(T)$. The induced subgraph $\langle V[J(G)] - D_1 \rangle$ is also a non-split dominating set of $J(T)$. □

4 Path Non-Split Domination Number of a Jump graph.

Definition 19. A dominating set D of $J(G)$ is called a path non-split dominating set if the induced subgraph $V[J(G)] - D$ is a path. The minimum cardinality of a path non-split dominating set of $J(G)$ is called the path non-split domination number of $J(G)$ and is denoted by $\gamma_{pns}[J(G)]$.

Theorem 20. For any path of length greater than or equal to 4,

$$\gamma_{pns}[J(P_p)] = \begin{cases} p - 3 & \text{for } p = 5, \\ p - 4 & \text{for } p = 6, \\ p - 5 & \text{for } p \geq 7. \end{cases}$$

Proof. For $p = 5, 6$ proof is obvious. Suppose that $p \geq 7$. Consider any subgraph P_5 of length 4 in P_p . Let E_1 be the set of edges in P_5 . Let V'_1 be the set of vertices in $J(P_p)$ corresponding to the set of edges $E_1 \in G_1$. Clearly the induced subgraph $\langle V'_1 \rangle$ is a path of length 3 and $D = V[J(P_p)] - V'_1$ is a dominating set of $J(P_p)$.

If the path considered is greater than P_5 then the induced subgraph $\langle V'_1 \rangle$ is connected but it is not a path. Therefore, any set other than D cannot be a path non-split dominating set of $J(G)$. This implies that D is the path non-split dominating set with minimum cardinality. □

Theorem 21. For a complete bipartite graph $K_{m,n}$,

$$\gamma_{pns}[J(K_{m,n})] = \begin{cases} mn - 2 & \text{for } K_{2,3} \\ mn - 5 & \text{otherwise.} \end{cases}$$

Proof. Proof for $K_{2,3}$ is obvious. Let $V_1 = \{v_1, v_2, \dots, v_m\}$ and $V_2 = \{v'_1, v'_2, \dots, v'_n\}$ be the two bipartites of $K_{m,n}$. Choose two edges e_i, e_j in $K_{m,n}$ such that they are incident to a common vertex $v'_j \in V_2$ and the other end vertices of edges chosen are v_i and $v_j \in V_1$. Similarly choose two more edges e_r, e_s in $K_{m,n}$ such that they are incident to a common vertex $v'_k \in V_2$ and $v'_k \neq v'_j$ and the other end vertices of edges chosen are the same v_i and $v_j \in V_1$. Suppose $S_1 = S \cup \{e\}$ be a subgraph of $K_{m,n}$ where $S = \{e_i, e_j, e_r, e_s\}$ and $e = \{uv\}$ such that $u \in V_1$, $u \neq v_i$ and $u \neq v_j$ and $v \in V_2$, $v = v'_j$ or v'_k . Let V'_1 be the set of vertices in $J(K_{m,n})$ corresponding to the set of edges in S_1 . Clearly the induced subgraph $\langle V'_1 \rangle$ is a path of length 4 and $D = V[J(K_{m,n})] - V'_1$ is a dominating set of $J(K_{m,n})$.

If the subgraph considered is other than S_1 then the induced subgraph $\langle V'_1 \rangle$ is connected but not a path. Therefore, any set other than D cannot be a path non-split dominating set of $J(K_{m,n})$. This implies that D is the path non-split dominating set with minimum cardinality. $\square$

Theorem 22. For any wheel W_p ,

$$\gamma_{pns}[J(W_p)] = \begin{cases} 2p - 5 & \text{for } p = 5 \\ 2p - 7 & \text{for } p \geq 6. \end{cases}$$

Proof. Proof for W_5 is obvious. Let C_{p-1} be a cycle labelled as $u_1e_1u_2e_2u_3 \dots u_{p-1}e_{p-1}u_1$. We know that Wheel is a graph obtained by joining K_1 to each vertex of C_{p-1} . Consider an edge say $e = u_iv$ where $u_i \in C_{p-1}$ and $v = K_1$ on the spokes of wheel W_p . Let $S_1 = \{u_{i-1}u_i, u_iu_{i+1}\}$ be the set of edges adjacent to e on the cycle C_{p-1} , $S_2 = \{u_{i-1}v, u_{i+1}v\}$ be the set of spokes on the wheel and e_k be a spoke on W_p non-adjacent to the edges of S_1 . Suppose $S = S_1 \cup S_2 \cup \{e_k\}$ be a subgraph in W_p . Let V'_1 be the set of vertices in $J(W_p)$ corresponding to the set of edges of S . Clearly the induced subgraph $\langle V'_1 \rangle$ is a path of length 4 in $J(G)$ and $D = V[J(W_p)] - V'_1$ is a path non-split dominating set of $J(W_p)$.

If the subgraph considered is other than S then the induced subgraph $\langle V'_1 \rangle$ is connected but not a path. Therefore, any set other than D cannot be a path non-split dominating set of $J(W_p)$. This implies that D is the path non-split dominating set with minimum cardinality. $\square$

Theorem 23. For a complete graph K_p , $\gamma_{pns}[J(K_p)] = \gamma[J(K_p)] + S$ where S is the number of edges in the induced subgraph K_{p-3} of K_p .

Proof. Choose any three vertices $V_1 = \{v_i, v_j, v_k\}$ in K_p such that the induced subgraph $\langle v_i, v_j, v_k \rangle$ forms a K_3 . Let $\{e_i, e_j, e_k\}$ be the edges in K_3 respectively. Let $V'_1 = \{v'_i, v'_j, v'_k\}$ be the set of vertices in $J(K_p)$ corresponding to the edges $\{e_i, e_j, e_k\}$ of K_p . Then by Theorem 2 $V'_1 = \{v'_i, v'_j, v'_k\}$ is a dominating set in $J(K_p)$. Choose $V_2 = V(G) \setminus V_1$ as a vertex set. Let E_2 be the set of edges in the induced subgraph of V_2 . Let V'_2 be the set of vertices in $J(K_p)$ corresponding to the edge set E_2 of V_2 . Let $|V'_2| = S$ also $D = V'_1 \cup V'_2$ forms a path non-split dominating set in $J(K_p)$. $\square$

If the subgraph considered is other than $V_1' \cup V_2'$ then the induced subgraph $\langle V_1' \cup V_2' \rangle$ is connected but not a path. Therefore, any set other than D cannot be a path non-split dominating set of $J(K_p)$. This implies that D is the path non-split dominating set with minimum cardinality.

$$\begin{aligned}\gamma_{pns}[J(K_p)] &= |V_1'| + |V_2'| \\ &= \gamma[J(K_p)] + \mathcal{S}.\end{aligned}$$

□

5 Cycle Non-Split Domination Number of a Jump graph.

Theorem 24. For a path ($p \geq 7$), $\gamma_{cns}[J(P_p)] = p - 3$.

Proof. Let S be an edge set containing any three independent edges of P_p . Let V_1' be the set of vertices in $J(P_p)$ corresponding to the edge set S of P_p . Clearly the induced subgraph $\langle V_1' \rangle$ is a cycle of length 3 and $D = V[J(P_p)] - V_1'$ is a dominating set of $J(P_p)$. If the set S contains more than 3 independent edges then $\langle V_1' \rangle$ induced is not a cycle. This implies that any set other than D cannot be a cycle non-split dominating set of $J(P_p)$. Therefore D is a cycle non-split dominating set of P_p with minimum cardinality. □

Theorem 25. For any wheel W_p with $p \geq 6$, $\gamma_{cns}[J(W_p)] = 2(p - 3)$.

Proof. Let C_{p-1} be a cycle labeled as $u_1e_1u_2e_2u_3 \cdots u_{p-1}e_{p-1}u_1$. We know that Wheel is a graph obtained by joining K_1 to each vertex of C_{p-1} . Consider an edge say $e = u_i v$ where $u_i \in C_{p-1}$ and $v = K_1$ on the spokes of wheel W_p . There exist p edges in $N(e)$. Let $S_1 = \{u_{i-1}u_i, u_iu_{i+1}\}$ be the set of edges adjacent to e on the cycle C_{p-1} and S_2 be any two spokes which are non-adjacent to the edges of S_1 . Suppose $S = S_1 \cup S_2$ be a subgraph in W_p . Let V_1' be the set of vertices in $J(W_p)$ corresponding to the set of edges of S . Clearly the induced subgraph $\langle V_1' \rangle$ is a cycle of length 4 in $J(G)$ and $D = V[J(W_p)] - V_1'$ is a cycle non-split dominating set of $J(W_p)$.

If the subgraph considered is other than S then the induced subgraph $\langle V_1' \rangle$ is connected but not a cycle. Therefore, any set other than D cannot be a cycle non-split dominating set of $J(W_p)$. This implies that D is the cycle non-split dominating set with minimum cardinality. □

Theorem 26. For a complete graph with $p \geq 5$, $\gamma_{cns}[J(K_p)] = {}^pC_2 - 6$.

Proof. Choose S as a subgraph of K_p such that $\langle S \rangle$ induced is a complete bipartite graph $K_{2,3}$. Let E_1 be an edge set on S . Let V_1' be the set of vertices in $J(K_p)$ corresponding to the edge set E_1 of K_p . Clearly the induced subgraph $\langle V_1' \rangle$ is a cycle of length 6 and $D = V[J(K_p)] - V_1'$ is a cycle non-split dominating set of $J(K_p)$.

$$\gamma_{cns}[J(K_p)] = |V[J(K_p)]| - |V_1'|$$

If the subgraph considered is other than $K_{2,3}$ then the induced subgraph $\langle V_1' \rangle$ is connected but not a cycle. Therefore, any set other than D cannot be a cycle non-split dominating set of $J(K_p)$. This implies that D is the cycle non-split dominating set with minimum cardinality. □

Theorem 27. For any complete bipartite graph $K_{m,n}$ with $3 \leq m \leq n$,

$$\gamma_{cns}[J(K_{m,n})] = mn - 4.$$

Proof. Let $V_1 = \{v_1, v_2, \dots, v_m\}$ and $V_2 = \{v'_1, v'_2, \dots, v'_n\}$ be the two bipartites of $K_{m,n}$. Choose two edges e_i, e_j in $K_{m,n}$ such that they are incident to a common vertex $v_j \in V_1$ and the other end vertices of edges chosen are v'_i and $v'_j \in V_2$. Similarly choose two more edges e_r, e_s in $K_{m,n}$ such that they are incident to a common vertex $v'_k \in V_2$ where $v'_k \neq v'_i$ and $v'_k \neq v'_j$ and also the other end vertices of edges chosen are v_r and $v_s \in V_1$ where v_r and $v_s \neq v_j$. Let $S = \{e_i, e_j, e_r, e_s\}$ be the edge set in $K_{m,n}$. Let V'_1 be the set of vertices in $J(K_{m,n})$ corresponding to the set of edges in S . Clearly the induced subgraph $\langle V'_1 \rangle$ is a cycle of length 4 and $D = V[J(K_{m,n})] - V'_1$ is a dominating set of $J(K_{m,n})$.

If the subgraph considered is other than S then the induced subgraph $\langle V'_1 \rangle$ is connected but not a cycle. Therefore, any set other than D cannot be a cycle non-split dominating set of $J(K_{m,n})$. This implies that D is the cycle non-split dominating set with minimum cardinality. $\square$

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