

SOLVING FFLSE WITH CIRCULANT HEXAGONAL FUZZY NUMBER MATRICES BY LU DECOMPOSITION METHOD

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Abstract

In this paper we discuss LU decomposition procedure for solving Fully Fuzzy Linear System of equations by decomposition method of Hexagonal fuzzy number matrices of order 3. L refers to a lower triangular matrix and U refers to an upper triangular matrix. An algorithm have been developed and illustrated with a numerical example.

AMS Subject Classification:

Key Words and Phrases: Hexagonal fuzzy number, LU decomposition, FFLSE, Hexagonal fuzzy number matrices.

1 Introduction

Linear system of equations has wide applications in many areas such as Engineering, Science, Economics and Finance. Fuzzy linear system whose coefficient matrix is crisp and the right hand side column is an arbitrary fuzzy number was first proposed by

Friedman. Different methods with numerical algorithms were used for solving fully fuzzy linear system by different authors.

A linear system is called a fully fuzzy linear system (FFLS) if all coefficients in the system are all fuzzy number. Trapezoidal fuzzy number using embedded approach was investigated by Nasser. Hexagonal fuzzy number using row reduce echelon form was introduced by Amit Kumar. Vijayalakshmi V investigated LU decomposition procedure for circulant triangular and trapezoidal fuzzy number to solve FFLSE.

In this paper we introduce hexagonal number and computing algorithm for $n \times n$ fully fuzzy linear system $\bar{A} \otimes \bar{x} = \bar{b}$ where $\bar{A}$ is a fuzzy matrix $\bar{x}$ and $\bar{b}$ are fuzzy vectors with Hexagonal fuzzy numbers by LU decomposition method.

2 Preliminary

Definition 1. Let X be a nonempty set. A fuzzy set A in X is characterized by its membership function $A : X \rightarrow [0, 1]$, $A(x)$ as the degree of membership of element x in fuzzy set A for each $x \in X$. The value 0 represent complete non membership and the value 1 represent complete membership. The mapping A is also called the membership function of the fuzzy set A .

3 Hexagonal Fuzzy Number

Definition 2. A fuzzy number $\bar{A}_H$ is a hexagonal fuzzy number denoted by $\bar{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6)$ where $a_1, a_2, a_3, a_4, a_5, a_6$ are real numbers and its membership function is given by

$$\mu_{A(x)} = \begin{cases} 0 & \text{for } x < a_1, x > a_6 \\ \frac{1}{2} \left(\frac{x-a_1}{a_2-a_1} \right) & \text{for } a_1 \leq x \leq a_2 \\ \frac{1}{2} + \frac{1}{2} \left(\frac{x-a_2}{a_3-a_2} \right) & \text{for } a_2 \leq x \leq a_3 \\ 1 & \text{for } a_3 \leq x \leq a_4 \\ 1 - \frac{1}{2} \left(\frac{x-a_4}{a_5-a_4} \right) & \text{for } a_4 \leq x \leq a_5 \\ \frac{1}{2} \left(\frac{a_6-x}{a_6-a_5} \right) & \text{for } a_5 \leq x \leq a_6 \end{cases}$$

Definition 3. An hexagonal where fuzzy number denoted by $\bar{A}_H = (P_1(u), Q_1(v), Q_2(v), P_2(w))$ for $u \in [0, 0.5]$ and $v \in [0.5, w]$ where

- i) $P_1(u)$ is a bounded left continuous non decreasing function over $[0, 0.5]$
- ii) $Q_1(v)$ is bounded left continuous non decreasing function over $[0.5, w]$
- iii) $Q_2(v)$ is a bounded continuous non decreasing function over $[w, 0.5]$
- iv) $P_2(u)$ is bounded left continuous non increasing function over $[0.5, 0]$

4 LU Decomposition Method

In this section we are going to obtain a solution of FFLS $\bar{A} \otimes \bar{x} = \bar{b}$ where $\bar{A} = (A, B, C, D, E, F) > 0$, $\bar{b} = (b, h, g, l, m, n) > 0$ and $\bar{x} = (x, y, z, u, v, w)$

$$(A, B, C, D, E, F) \otimes (x, y, z, u, v, w) = (b, h, g, l, m, n)$$

$$Ax = b$$

$$Ay + Bx = h$$

$$AZ + Cx = g$$

$$Au + Dx = l$$

$$Av + Ex = m$$

$$Aw + Fx = n \text{ Let us assume that } A \text{ be an nonsingular matrix}$$

$$x = A^{-1}b$$

$$y = A^{-1}(h - Bx)$$

$$z = A^{-1}(g - cx)$$

$$u = A^{-1}(l - Dx)$$

$$v = A^{-1}(m - Ex)$$

$$w = A^{-1}(n - Fx)$$

Let A be an $n \times n$ matrix with all non-zero leading principal minors. A has a unique factorization $A = LU$ where L is a unit lower triangular matrix, U is an upper triangular matrix. A, B, C, D, E, F is a full rank crisp matrix.

Let $(L_1, 0, 0, 0, 0, 0) \otimes (U_1, U_2, U_3, U_4, U_5, U_6) = (A, B, C, D, E, F)$

$$L_1 U_1 = A, \Rightarrow U_1 = L_1^{-1} A$$

$$L_1 U_2 = B, \Rightarrow U_2 = L_1^{-1} B$$

$$L_1 U_3 = C, \Rightarrow U_3 = L_1^{-1} C$$

$$L_1 U_4 = D, \Rightarrow U_4 = L_1^{-1} D$$

$$L_1 U_5 = E, \Rightarrow U_5 = L_1^{-1} E$$

$$L_1 U_6 = F, \Rightarrow U_6 = L_1^{-1} F$$

Where L_1 is a lower triangular crisp matrix

U_1 is an upper triangular matrix.

Consider a fully fuzzy linear system $\bar{A} \otimes \bar{x} = \bar{b}$ where

$\bar{A} = (A, B, C, D, E, F) > 0$, $\bar{b} = (b, h, g, l, m, n) > 0$ and $\bar{x} = (x, y, z, u, v, w)$

$$(A, B, C, D, E, F) \otimes (x, y, z, u, v, w) = (b, h, g, l, m, n)$$

$$(L_1 U_1, L_1 U_2, L_1 U_3, L_1 U_4, L_1 U_5, L_1 U_6) \otimes (x, y, z, u, v, w) = (b, h, g, l, m, n)$$

On comparing,

$$L_1 U_1 x = b$$

$$L_1 U_2 x + L_1 U_1 y = h$$

$$L_1 U_3 x + L_1 U_1 z = g$$

$$L_1 U_4 x + L_1 U_1 u = l$$

$$L_1 U_5 x + L_1 U_1 v = m$$

$$L_1 U_6 x + L_1 U_1 w = n$$

$$x = U_1^{-1} L_1^{-1} b$$

$$y = U_1^{-1} L_1^{-1} (h - L_1 U_2 x)$$

$$z = U_1^{-1} L_1^{-1} (g - L_1 U_3 x)$$

$$u = U_1^{-1} L_1^{-1} (l - L_1 U_4 x)$$

$$v = U_1^{-1} L_1^{-1} (m - L_1 U_5 x)$$

$$w = U_1^{-1} L_1^{-1} (n - L_1 U_6 x)$$

Algorithms

STEP I

Assume that $\bar{A} = (A, B, C, D, E, F)$ where A is a full rank crisp matrix.

STEP II

Compute LU decomposition for matrix A as follows $L_2 = L_3 = L_4 = L_5 = L_6 = 0$

STEP III

Compute

$$\begin{aligned}U_2 &= L_1^{-1}B \\U_3 &= L_1^{-1}C \\U_4 &= L_1^{-1}D \\U_5 &= L_1^{-1}E \\U_6 &= L_1^{-1}F\end{aligned}$$

STEP IV

Compute the solutions of fully fuzzy linear system of circulant hexagonal fuzzy matrices

$$(A, B, C, D, E, F) \otimes (x, y, z, u, v, w) = (b, h, g, l, m, n)$$

$$\begin{aligned}x &= U_1^{-1}L_1^{-1}b \\y &= U_1^{-1}L_1^{-1}(h - L_1U_2x) \\z &= U_1^{-1}L_1^{-1}(g - L_1U_3x) \\u &= U_1^{-1}L_1^{-1}(l - L_1U_4x) \\v &= U_1^{-1}L_1^{-1}(m - L_1U_5x) \\w &= U_1^{-1}L_1^{-1}(n - L_1U_6x)\end{aligned}$$

5 Numerical Example

Solve the following FFLS by LU decomposition method

$$\begin{aligned}&(2, 3, 4, 6, 8, 9) \otimes (x_1, y_1, z_1, u_1, v_1, w_1) \oplus (3, 4, 6, 8, 9, 10) \otimes (x_2, y_2, z_2, u_2, v_2, w_2) \\&\quad \oplus (1, 2, 3, 5, 6, 7) \otimes (x_3, y_3, z_3, u_3, v_3, w_3) = (5, 6, 8, 9, 10, 11) \\&(1, 2, 3, 5, 6, 7) \otimes (x_1, y_1, z_1, u_1, v_1, w_1) \oplus (2, 3, 4, 6, 8, 9) \otimes (x_2, y_2, z_2, u_2, v_2, w_2) \\&\quad \oplus (3, 4, 6, 8, 9, 10) \otimes (x_3, y_3, z_3, u_3, v_3, w_3) = (2, 4, 6, 7, 8, 9) \\&(3, 4, 6, 8, 9, 10) \otimes (x_1, y_1, z_1, u_1, v_1, w_1) \oplus (1, 2, 3, 5, 6, 7) \otimes (x_2, y_2, z_2, u_2, v_2, w_2) \\&\quad \oplus (2, 3, 4, 6, 8, 9) \otimes (x_3, y_3, z_3, u_3, v_3, w_3) = (1, 3, 4, 9, 10, 11)\end{aligned}$$

Solution:

Above equation can be expressed as

$$\begin{bmatrix} (2, 3, 4, 6, 8, 9) & (3, 4, 6, 8, 9, 10) & (1, 2, 3, 5, 6, 7) \\ (1, 2, 3, 5, 6, 7) & (2, 3, 4, 6, 8, 9) & (3, 4, 6, 8, 9, 10) \\ (3, 4, 6, 8, 9, 10) & (1, 2, 3, 5, 6, 7) & (2, 3, 4, 6, 8, 9) \end{bmatrix} \begin{bmatrix} (x_1, y_1, z_1, u_1, v_1, w_1) \\ (x_2, y_2, z_2, u_2, v_2, w_2) \\ (x_3, y_3, z_3, u_3, v_3, w_3) \end{bmatrix} \\ = \begin{bmatrix} (5, 6, 8, 9, 10, 11) \\ (2, 4, 6, 7, 8, 9) \\ (1, 3, 4, 9, 10, 11) \end{bmatrix}$$

$$\text{Let } A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}, B = \begin{bmatrix} 3 & 4 & 2 \\ 2 & 3 & 4 \\ 4 & 2 & 3 \end{bmatrix}, C = \begin{bmatrix} 4 & 6 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 4 \end{bmatrix}, \\ D = \begin{bmatrix} 6 & 8 & 5 \\ 5 & 6 & 8 \\ 8 & 5 & 6 \end{bmatrix}, E = \begin{bmatrix} 8 & 9 & 6 \\ 6 & 8 & 9 \\ 9 & 6 & 8 \end{bmatrix}, F = \begin{bmatrix} 9 & 10 & 7 \\ 9 & 9 & 10 \\ 10 & 7 & 9 \end{bmatrix}, \\ b = \begin{bmatrix} 5 \\ 2 \\ 1 \end{bmatrix}, h = \begin{bmatrix} 6 \\ 4 \\ 3 \end{bmatrix}, g = \begin{bmatrix} 8 \\ 6 \\ 4 \end{bmatrix}, l = \begin{bmatrix} 9 \\ 7 \\ 9 \end{bmatrix}, m = \begin{bmatrix} 10 \\ 8 \\ 10 \end{bmatrix}, n = \begin{bmatrix} 11 \\ 9 \\ 11 \end{bmatrix},$$

To start with find LU decomposition for the matrix A

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix} = L_1 U_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 1.5 & -7 & 1 \end{bmatrix} \\ U_2 = L_1^{-1} B$$

$$\text{Where } L_1^{-1} \text{ is the inverse of } L_1, L_1^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -0.5 & 1 & 0 \\ -5 & 7 & 1 \end{bmatrix}$$

$$U_2 = L_1^{-1} B = \begin{bmatrix} 1 & 0 & 0 \\ -0.5 & 1 & 0 \\ -5 & 7 & 1 \end{bmatrix} \begin{bmatrix} 3 & 4 & 2 \\ 2 & 3 & 4 \\ 4 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 4 & 2 \\ 0.5 & 1 & 3 \\ 3 & 3 & 21 \end{bmatrix}$$

$$U_3 = L_1^{-1} C = \begin{bmatrix} 1 & 0 & 0 \\ -0.5 & 1 & 0 \\ -5 & 7 & 1 \end{bmatrix} \begin{bmatrix} 4 & 6 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 6 & 2 \\ 0 & 1 & 5 \\ 0 & 0 & 36 \end{bmatrix}$$

$$U_4 = L_1^{-1} D = \begin{bmatrix} 1 & 0 & 0 \\ -0.5 & 1 & 0 \\ -5 & 7 & 1 \end{bmatrix} \begin{bmatrix} 6 & 8 & 5 \\ 5 & 6 & 8 \\ 8 & 5 & 6 \end{bmatrix} = \begin{bmatrix} 6 & 8 & 5 \\ 2 & 2 & 5.5 \\ 13 & 7 & 37 \end{bmatrix}$$

$$U_5 = L_1^{-1} E = \begin{bmatrix} 1 & 0 & 0 \\ -0.5 & 1 & 0 \\ -5 & 7 & 1 \end{bmatrix} \begin{bmatrix} 8 & 9 & 6 \\ 6 & 8 & 9 \\ 9 & 6 & 8 \end{bmatrix} = \begin{bmatrix} 8 & 9 & 6 \\ 2 & 3.5 & 6 \\ 11 & 17 & 41 \end{bmatrix}$$

$$\begin{aligned}
U_6 &= L_1^{-1}F = \begin{bmatrix} 1 & 0 & 0 \\ -0.5 & 1 & 0 \\ -5 & 7 & 1 \end{bmatrix} \begin{bmatrix} 9 & 10 & 7 \\ 9 & 9 & 10 \\ 10 & 7 & 9 \end{bmatrix} = \begin{bmatrix} 9 & 10 & 7 \\ 2.5 & 4 & 6.5 \\ 14 & 20 & 44 \end{bmatrix} \\
U_1 &= \begin{bmatrix} 2 & 3 & 1 \\ 0 & 0.5 & 2.5 \\ 0 & 0 & 18 \end{bmatrix} \quad U_1^{-1} = \begin{bmatrix} 0.5 & -3 & -.388 \\ 0 & 2 & -0.277 \\ 0 & 0 & 0.0555 \end{bmatrix} \\
U_1^{-1}L_1^{-1} &= \begin{bmatrix} 3.94 & 5.716 & -0.388 \\ 0.3885 & 0.0561 & -0.2777 \\ -0.2775 & 0.3885 & 0.0555 \end{bmatrix} \\
x &= U_1^{-1}L_1^{-1} = \begin{bmatrix} 3.94 & 5.716 & -0.388 \\ 0.3885 & 0.0561 & -0.2777 \\ -0.2775 & 0.3885 & 0.0555 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 11.832 \\ 1.7745 \\ -0.5525 \end{bmatrix} \\
y &= U_1^{-1}L_1^{-1}(h - L_1U_2x) \\
L_1U_2x &= \begin{bmatrix} 3 & 4 & 2 \\ 2 & 3 & 4 \\ 4 & 2 & 3 \end{bmatrix} \begin{bmatrix} 11.832 \\ 1.7745 \\ -0.5525 \end{bmatrix} = \begin{bmatrix} 41.489 \\ 26.7775 \\ 49.2195 \end{bmatrix} \\
y &= \begin{bmatrix} -242.089 \\ -2.2301 \\ -1.5589 \end{bmatrix} \\
z &= U_1^{-1}L_1^{-1}(g - L_1U_3x) \\
L_1U_3x &= \begin{bmatrix} 4 & 6 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 4 \end{bmatrix} \begin{bmatrix} 11.832 \\ 1.7745 \\ -0.5525 \end{bmatrix} = \begin{bmatrix} 56.87 \\ 27.447 \\ 72.231 \end{bmatrix} \\
z &= \begin{bmatrix} -288.665 \\ -1.2647 \\ 1.4521 \end{bmatrix} \\
u &= U_1^{-1}L_1^{-1}(l - L_1U_4x) \\
L_1U_4x &= \begin{bmatrix} 6 & 8 & 5 \\ 5 & 6 & 8 \\ 8 & 5 & 6 \end{bmatrix} \begin{bmatrix} 11.832 \\ 1.7745 \\ -0.5525 \end{bmatrix} = \begin{bmatrix} 82.4255 \\ 65.387 \\ 100.2135 \end{bmatrix} \\
u &= \begin{bmatrix} -587.645 \\ -6.4984 \\ -7.3612 \end{bmatrix} \\
v &= U_1^{-1}L_1^{-1}(m - L_1U_5x) \\
L_1U_5x &= \begin{bmatrix} 8 & 9 & 6 \\ 6 & 8 & 9 \\ 9 & 6 & 8 \end{bmatrix} \begin{bmatrix} 11.832 \\ 1.7745 \\ -0.5525 \end{bmatrix} = \begin{bmatrix} 107.3115 \\ 80.2155 \\ 112.715 \end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
v &= \begin{bmatrix} 69.2299 \\ -13.3560 \\ -6.7497 \end{bmatrix} \\
w &= U_1^{-1} L_1^{-1} (n - L_1 U_6 x) \\
L_1 U_6 x &= \begin{bmatrix} 9 & 10 & 7 \\ 7 & 9 & 10 \\ 10 & 7 & 9 \end{bmatrix} \begin{bmatrix} 11.832 \\ 1.7745 \\ -0.5525 \end{bmatrix} = \begin{bmatrix} 120.3655 \\ 93.2695 \\ 125.769 \end{bmatrix} \\
w &= \begin{bmatrix} -868.0541 \\ -15.3703 \\ -8.7567 \end{bmatrix} \\
\text{thus } \bar{x}_i &= (x_i, y_i, z_i, u_i, v_i, w_i) \text{ for } i = 1, 2, 3 \\
\bar{x}_1 &= (11.832, -252.089, -288.665, -587.645, 69.2299, -868.054) \\
\bar{x}_2 &= (1.7745, -2.2301, -1.2646, -6.4984, -13.3560, -15.3703) \\
\bar{x}_3 &= (-0.5525, -1.5589, 1.4521, -7.3612, -6.7497, -8.7567)
\end{aligned}$$

6 Conclusion

In this paper solution of FFLS $\bar{A} \otimes \bar{x} = \bar{b}$ is obtained by LU decomposition method in the form of hexagonal fuzzy number matrices. An algorithm has been developed and verified with a numerical example.

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