

SQUARE DIFFERENCE LABELING OF CIRCULANT NETWORK $G(n; \pm\{1, 2\})$

V. Jude Annie Cynthia¹, P. Poorani²

¹Department of Mathematics
Stella Maris College
Chennai, INDIA

²Department of Mathematics
C.T.T.E College for Women
Chennai, INDIA

Abstract: A function f of a graph $G(P, Q)$ admits *square difference labeling* if there exist a bijection $f : V(G) \rightarrow \{0, 1, 2, \dots, p-1\}$ such that the induced function $f^* : E(G) \rightarrow N$ given by $f^*(uv) = |(f(u))^2 - (f(v))^2|$ for every $uv \in E(G)$ are all distinct. A graph which admits square difference labeling is called *square difference graph*. In this paper we investigate that the circulant network $G(n; \pm\{1, 2\})$, $n \geq 8$ is a square difference graph.

Key Words: square difference labeling, square difference graph, circulant network

1. Introduction

Graph labeling was introduced in the mid 1960's. Graph labeling can be applied in the areas such as communication network, social networking and computer networking, mobile telecommunication, channel assignment and medical field.

A dynamic survey on graph labeling is regularly updated by Gallian [6] and it is published by Electronic Journal of Combinatorics.

Let G be (p, q) graph. Graph G admits *square difference labeling* if there exist a bijection $f : V(G) \rightarrow \{0, 1, 2, \dots, p-1\}$ such that the induced function $f^* : E(G) \rightarrow N$ given by $f^*(uv) = |(f(u))^2 - (f(v))^2|$ for every $uv \in E(G)$ are all distinct.

Any graph which admits *square difference labeling* is called *square difference graph*.

The concept of square difference labeling was introduced by J.Shiamo [13]. J.Shiamo [11] and [12] proved that the following graphs cycle cactus $C_k^{(n)}$, ladder, lattice grids, quadrilateral snakes, wheels, comb, star graphs, crown graphs, central graph of a path P_n , fan graphs, gear graphs, shadow and split graphs has square difference labeling. He has also proved that the path is an odd square difference graph and star graphs are perfect square graphs. Amudha [1] proved double triangular snake, alternative triangular snake, alternative double triangular snake, banana tree, umbrella graph, $P_n(QS_n)$ and $C_n(QS_n)$ are square difference graphs.

Sharon Philomena [10] has proved that key graph admits square difference labeling. Sarasija [8] and [9] proved middle graph, total graph and graphs $P_m \cup P_n$, $P_m \cup C_n$, $P_m \cup S_n$ admits square difference labeling.

2. Circulant Network

The circulant network is a natural generalization of the double loop network, which was first considered by Wong and Coppersmith [15]. Circulant graphs have been used for decades in the design of computer and telecommunication networks due to their optimal fault-tolerance and routing capabilities [4]. It is also used in VLSI design and distributed computation [3], [2], [14]. The term circulant comes from the nature of its adjacency matrix. A matrix is circulant if all its rows are periodic rotations of the first one. Circulant matrices have been employed for designing binary codes [7]. Theoretical properties of circulant graphs have been studied extensively and surveyed by Bermond et al.[3]. Every circulant graph is a vertex transitive graph and a Cayley graph [16]. Most of the earlier research concentrated on using the circulant graphs to build interconnection networks for distributed and parallel systems [3],[4]. Classes of graphs that are circulant graphs include the cycle, complete graphs, complete bipartite graphs, paley graphs of prime order, prism graphs, möbius ladder graph, tetrahedral graph and torus grid graphs.

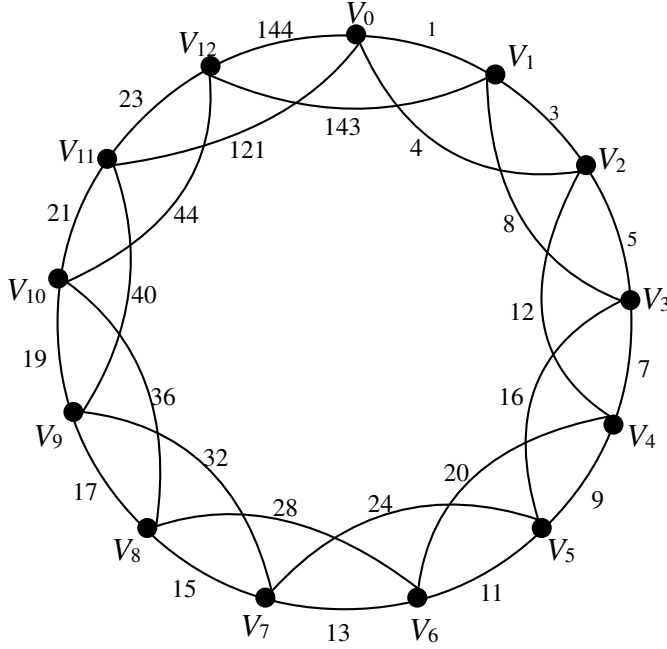


Figure 1: Square Difference Labeling of Circulant Network $G(13, \pm\{1, 2\})$

An undirected circulant graph denoted by $G(n; \pm\{1, 2, \dots, j\})$, $1 \leq j \leq \lfloor \frac{n}{2} \rfloor$, $n \geq 3$ is defined as a graph consisting of the vertex set $V = 0, 1, \dots, n-1$ and the edge set $E = \{(i, j) : |i - j| \equiv s \pmod{n}, s \in 1, 2, \dots, j\}$.

It is clear that $G(n, \pm 1)$ is the undirected cycle C_n and $G(n, \pm\{1, 2, \dots, \lfloor \frac{n}{2} \rfloor\})$ is the complete graph K_n [5].

If n is odd the graph has an outer cycle $v_0v_1v_2\dots v_{n-1}v_0$ of length n and an inner cycle of length n namely $v_0v_2v_4\dots v_{n-1}v_1v_3\dots v_{n-2}v_0$.

If n is even the graph has an outer cycle of length n namely $v_0v_1v_2\dots v_{n-1}$ and two inner cycle each of length $\frac{n}{2}$ namely $v_0v_1v_2\dots v_{n-2}v_0$ and $v_1v_3v_5\dots v_{n-1}v_1$.

3. Square Difference Labeling of Circulant Network

Theorem 1. *The circulant network $G(n; \pm\{1, 2\})$, $n \geq 8$ is a square difference graph.*

Proof. Consider the circulant network $G(n; \pm\{1, 2\})$. Define a bijection $f : V(G) \rightarrow \{0, 1, 2, \dots, n-1\}$ by $f(v_i) = i, 0 \leq i \leq n-1$. Therefore the vertex labeling satisfies the condition for the square difference labeling. Consider the outer cycle of length n :

$$f(v_i v_{i+1}) = |(i)^2 - (i+1)^2| = 2i+1, i = 0, 1, \dots, n-2, f(v_{n-1} v_0) = (n-1)^2$$

Hence for the outer cycle the edges label are given by $2i+1$, where $i = 0, 1, \dots, n-2$. Therefore the labels are distinct satisfying the condition of a square difference labeling.

When n is odd we have one inner cycle of length n and when n is even, we have two inner cycles each of length $\frac{n}{2}$. In either case:

$$f(v_i v_{i+2}) = |(i)^2 - (i+2)^2| = 4(i+1), i = 0, 1, \dots, n-3,$$

$$f(v_{n-2} v_0) = (n-2)^2,$$

$$f(v_{n-1} v_1) = |(n-1)^2 - 1^2| = n(n-2).$$

Then we have the edge label as $4(i+1)$, where $i = 0, 1, \dots, n-3$ and hence the labels are distinct satisfying the condition of a square difference labeling.

Therefore the circulant network $G(n; \pm\{1, 2\}), n \geq 8$ admits square difference labeling and hence is a square difference graph. $\square$

4. Remark

The problem is open for the circulant network $G(n; \pm\{1, 2, \dots, j\}), 1 \leq j \leq \lfloor \frac{n}{2} \rfloor, n \geq 3$.

5. Conclusion

In this paper we have proved that the circulant network $G(n; \pm\{1, 2\}), n \geq 8$ admits square difference labeling. Further we intend to study the square difference labeling of Torus network.

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