Drag on a Fluid Sphere
Embedded in Porous Medium
with Zero Spin Condition

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Abstract

The problem of creeping flow of an incompressible micropolar porous past a fluid sphere. The stream functions are determined by matching the solution of Brinkman equation for the flow outside the fluid sphere and the Stokes equation for inside the fluid sphere. Drag force and drag coefficient are computed numerically. Finally the variation of drag coefficient presented in graphical form.

**Key Words and Phrases:** Drag force, Stokes equation, Porous medium, Modified Bessel's function, Permeability.

1 Introduction

Micropolar fluids are plentiful in engineering science. General examples of micropolar fluids are plasma, human blood, liquid crystals, sediments in rivers, drug suspension in pharmacology, etc [1]. In recent years the micropolar fluids are very development in biomechanical and one of the important part is porous shell. The fluid covered by the porous. The numerous application of flow through the porous is bio-mechanics, physical sciences, chemical engineering, etc [2]. Satya Deo and Pankaj Shukla [3, 5] are evaluated
the fluid sphere embedded in porous. They are found the stream function, drag force and the drag coefficient and they discussed with permeability parameter and viscosity ratio. Pramod Kumar Yadav, Ashish Tiwari [6, 7] are calculated the hydrodynamics permeability of spherical particles enclosed by porous shell using stress jump condition. They discussed the four cell model techniques. Happel, Kvashnin, Mehta Morse and Cunningham and they are evaluated the stress jump condition. It is salient in the case of hydrophobic non filtration membrane.

S.I. Vasin, A.N. Filippov [8, 9] are evaluated the hydrodynamics permeability membrane enclosed with porous shell and non porous shell by a solid particles. They concluded that the thickness of porous shell greater than the Brinkman length, the hydrodynamic conclude to depend on the thickness of porous shell and then the hydrodynamic permeability parameter decreasing when increasing viscosity ratio. Finally they discussed the flow inside the porous shell practically vanishes. S.I.Vasin and A.N.Filippov [10, 11] are found the hydrodynamic permeability of the membrane covered by the porous layer within the cell model. Here they are using internal structure parameters. The hydrodynamic permeability increases when increasing porosity. In next case the permeability parameter increasing is also the porosity decreasing.

In this paper the problem of, creeping flow of an incompressible micropolar porous past a fluid sphere. The porous region for the flow outside the sphere and the Stokes equation for inside the sphere in the stream function formulations are used. Drag force and drag coefficient are computed numerically. Finally the variation of drag coefficient presented in graphical form.

2 Governing Equation of the Problem

We concern the mathematical model the flow of an incompressible viscous fluid with a uniform velocity U directed in the positive Z-direction in a micropolar porous medium. The Permeability Parameter K in which a fluid sphere of s is established. Inside the fluid is viscous fluid and outside the fluid sphere is micropolar fluid. For the region inside the porous sphere, we assume that flow is governed
by Stokes equation
\[ \mu_1 \nabla^2 q^{(1)} = \nabla p^{(1)} \]
\[ \nabla \cdot q^{(1)} = 0 \] (1)

For the flow outside the sphere, we assume that flow is governed by
Brinkman equation,
\[ \frac{\mu}{k} q^{(2)} + \nabla p^{(2)} - k \nabla \times \omega^{(2)} + (\mu + k) \nabla \times \nabla \times q^{(2)} = 0 \] (3)
\[ -2k\omega^{(2)} + k \nabla \times q^{(2)} - \gamma \nabla \times \omega^{(2)} + (\alpha + \beta + \gamma) \nabla (\nabla \cdot \omega^{(2)}) = 0 \] (4)
\[ \nabla \cdot q^{(1)} = 0 \] (5)

Here, \( q^{(1)}, q^{(2)} \) are velocity vector, \( \omega^{(2)} \) is micro rotation vector
\( p^{(1)}, p^{(2)} \) are pressure and \( k \) is the permeability parameter. \( \alpha, \mu, \beta, \gamma \)
are material constants. It satisfies the inequalities,
\[ 3\alpha + \beta + \gamma \geq 0, \quad 2\mu + k \geq 0, \quad \gamma \geq |\beta|, \quad k \geq 0 \] (6)

The flow functions are independent, and the flow is axi-symmetric.
We choose velocity and micro rotation vector for the flow,
\[ q^{(i)} = u^{(i)}(r, \theta)e_r + v^{(i)}(r, \theta)e_\theta, \quad \omega^{(i)} = v^{(i)}_\phi(r, \theta)e_\phi \quad i = 1, 2 \] (7)

3 Solution of the problem

In the stream function \( \psi^{(1)} \) and \( \psi^{(2)} \) both regions are satisfying the
continuity equations are given by
\[ u^{(i)} = -\frac{1}{r^2 \sin \theta} \frac{\partial \psi^{(i)}}{\partial \theta} \quad v^{(i)} = \frac{1}{r \sin \theta} \frac{\partial \psi^{(i)}}{\partial r} \quad i = 1, 2 \] (8)

Solving the equation from (1) to (5), we get the non dimension equation for \( \psi^{(i)} \) for \( i = 1, 2 \)
\[ E^4 \psi^{(1)} = 0 \] (9)
\[ E^2 (E^2 - \alpha^2) (E^2 - \beta^2) \psi^{(2)} = 0 \] (10)

Where the stream function operator \( E^2 \) is given by
\[ E^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} - \frac{\cot \theta}{r^2} \frac{\partial}{\partial \theta} \] (11)
Here
\[
\eta^2 = \frac{\alpha^2}{k}, \\
m^2 = \frac{k(2\mu + k)}{\gamma(\mu + k)}, \\
\]
\[
N = \frac{k}{(\mu + k)} \text{is the coupling number (0} \leq N \leq 1) \text{and } \alpha^2 \text{ is the micropolar parameter and}
\]
\[
\alpha^2 + \beta^2 = \eta^2(1 - N) + m^2, \quad \alpha^2\beta^2 = \frac{2(1 - N)}{2 - N} \eta^2 m^2
\]

4 Boundary Conditions

The continuity of velocity components, continuity of normal stress, and tangential stress at r=1 respectively as,
\[
\psi^{(1)} = 0 \quad \text{on } r = 1 \quad (12) \\
\psi^{(2)} = 0 \quad \text{on } r = 1 \quad (13) \\
\psi^{(1)}_r = \psi^{(2)}_r \quad \text{on } r = 1 \quad (14) \\
\tau^{(1)}_{\theta} = \tau^{(2)}_{\theta} \quad \text{on } r = 1 \quad (15) \\
v_\phi = 0 \quad \text{on } r = 1 \quad (16)
\]

From the equation (9) and (10) are separable equations then using separable of variables method in equation (9) and (10), we get the general solution
\[
\psi^{(1)} = \sum_{n=2}^{\infty} \left[ A_n r^n + B_n r^{-n+1} + C_n r^{-n+3} + D_n r^{n+2} \right] G_n(\zeta) \quad (17)
\]
\[
\psi^{(2)} = \sum_{n=0}^{\infty} \left\{ \left[ A_n r^n + B_n r^{-n+1} + C_n \sqrt{r} K_{n-\frac{1}{2}}(\alpha r) + D_n \sqrt{r} I_{n-\frac{1}{2}}(\alpha r) \right] G_n(\zeta) + \left[ A_n r^n + B_n r^{-n+1} + C_n \sqrt{r} K_{n-\frac{1}{2}}(\beta r) + D_n \sqrt{r} I_{n-\frac{1}{2}}(\beta r) \right] H_n(\zeta) \right\} \quad (18)
\]

Since the modified Bessels functions are
\[
K_{n-\frac{1}{2}}(\alpha r), \quad I_{n-\frac{1}{2}}(\alpha r), \quad K_{n-\frac{1}{2}}(\beta r), \quad I_{n-\frac{1}{2}}(\beta r)
\]
for all $n$, and the entire terms involving $r^{-n+1}$ for $n \geq 2$ are asymmetric at $r = 0$ and we took $C_n = 0, D_n = 0$ for all $n$. $G_n(\zeta), H_n(\zeta)$ are the Gegenbauer functions of first and second kind respectively of order $m$. Here

$$G_2(\zeta) = \frac{1 - \zeta^2}{2}$$

and hence the equation (17) and (18) can be written as,

$$\psi^{(1)} = [B_1 r^2 + D_1 r^4] G_2(\zeta) \quad (19)$$

$$\psi^{(2)} = \left[ r^2 + B_2 r^{-1} + C_2 \sqrt{r} K_{\frac{3}{2}}(ar) + E_2 \sqrt{r} K_{\frac{3}{2}}(\beta r) \right] G_2(\zeta) \quad (20)$$

The micro rotation component of the sphere is given by

$$v^{(2)}_\phi = \frac{1}{2 r \sin \theta} \left[ C_2 A_\alpha \sqrt{r} K_{\frac{3}{2}}(ar) + E_2 A_\beta \sqrt{r} K_{\frac{3}{2}}(\beta r) \right] G_2(\zeta) \quad (21)$$

Using the boundary conditions in the equation (19) to (21), we get the arbitrary constants $B_1, D_1, B_2, C_2, E_2$ using Mathematica software.

$$B_2 = -1 \frac{3(1 + \alpha)(1 + \beta)(3 + 2\lambda)(A_\alpha - A_\beta)}{-(1 + \alpha)\beta^2(3 + (3 + \beta)\lambda)A_\alpha + \alpha^2(3 + (3 + \alpha)\lambda)(1 + \beta)A_\beta}$$

$$C_2 = \frac{3 e^\alpha \sqrt{\frac{\alpha}{2}(1 + \beta)(3 + 2\lambda)A_\beta}}{-(1 + \alpha)\beta^2(3 + (3 + \beta)\lambda)A_\alpha + \alpha^2(3 + (3 + \alpha)\lambda)(1 + \beta)A_\beta}$$

$$E_2 = \frac{3 e^\beta \sqrt{\frac{\beta}{2} \beta^2(1 + \alpha)(3 + 2\lambda)A_\alpha}}{(1 + \alpha)\beta^2(3 + (3 + \beta)\lambda)A_\alpha - \alpha^2(3 + (3 + \alpha)\lambda)(1 + \beta)A_\beta}$$

$$B_1 = \frac{3(1 + \alpha)(1 + \beta)(-\beta^2A_\alpha + \alpha^2A_\beta)}{2((1 + \alpha)\beta^2(3 + (3 + \beta)\lambda)A_\alpha - \alpha^2(3 + (3 + \alpha)\lambda)(1 + \beta)A_\beta)}$$

$$D_1 = \frac{3(1 + \alpha)(1 + \beta)(\beta^2A_\alpha - \alpha^2A_\beta)}{2((1 + \alpha)\beta^2(3 + (3 + \beta)\lambda)A_\alpha - \alpha^2(3 + (3 + \alpha)\lambda)(1 + \beta)A_\beta)}$$

5 Evaluation of Drag Force

Drag force is calculated by the interior spherical particle,

$$F = \mu \pi u a \int_0^\pi \omega^3 \frac{\partial}{\partial r} \left( \frac{E^2 \Psi}{\omega^2} \right) r \, d\theta. \quad (22)$$
Here inserting the values $\omega = rsin\theta$ and

$$E^2(\Psi) = \frac{\partial^2\Psi}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2\Psi}{\partial \theta^2} - \frac{\cot\theta}{r^2}$$

in equation (22), and integrating, we get the drag force,

$$D = 2\pi \mu U a \ B_2$$

(23)

where

$$B_2 = -1 + \frac{3(1 + \alpha)(1 + \beta)(3 + 2\lambda)(A_\alpha - A_\beta)}{-(1 + \alpha)\beta^2(3 + (3 + \beta)\lambda)A_\alpha + \alpha^2(3 + (3 + \alpha)\lambda)(1 + \beta)A_\beta}$$

(24)

The drag coefficient is given by

$$D_N = \frac{D}{\frac{1}{2} \rho U^2 \pi a^2}$$

(25)

$$D_N = \frac{16 \ B_2}{Re}$$

where the Reynolds number is $\frac{2aU}{v}$ and the Kinematic viscosity is $\frac{\upsilon}{\rho}$.

6 Deduction of Known Results:

**Case:** 1 In the case $A_\alpha \rightarrow \frac{\eta^2}{2}$ and $\beta \rightarrow 0$ the micropolar fluid is change in to clear fluid. After the limiting case, we get the fluid sphere

$$F = -6 \pi \mu U a \ \frac{(1 + \frac{2}{3} \gamma)}{1 + \gamma}$$

The result was reported from the book of Happel and Brenner.

**Case:** 2 If $\gamma \rightarrow 0$ then the fluid sphere change in to solid sphere then drag force comes out as $F = -6 \pi \mu U a$.

7 Result and Discussion

The oscillation of the drag coefficient with permeability parameter $\eta$ for the different values of micropolar parameter $m$ and fixed coupling number $N$ is shown in the figure 1. In figure 1 the oscillation of drag coefficient increases with increasing permeability parameter $\eta$. That is decreases with micropolar parameter for the values $m = 2, 4, 6$ and decrease the drag coefficient. The oscillation of drag coefficient in figure 2. for different value of Coupling number $N = 10, 20, 30$ increases the drag coefficient is also increases.
8 Conclusion

We obtained the problem of an incompressible micropolar porous sphere past a viscous fluid is calculated analytically. From this paper assuming zero spin condition for the micro rotation component. We calculated the drag force and drag coefficient. The variation of drag coefficient increases by the increasing or decreasing depends upon the permeability parameter, coupling numbers and the micropolar parameter.
References


