

A REVIEW ON CLASSICAL BULK ARRIVAL AND BATCH SERVICE QUEUEING MODEL

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Abstract: This article reviews about research work done in the area of bulk arrival and batch service queueing models. The concept of bulk arrivals and batch service has gained a tremendous significance in real life situations. To avoid congestion problem researches have to focus their attention to develop models and procedures to deal with the problems. In many of the industries particularly in production line, mathematical modelling of queueing model is essential to reduce congestion problems. This survey attempts to review the work done in the area of bulk queues with vacations, modelling various phenomenons. This paper aims to provide enough information to analysts, researchers and industry people to model congestion problems and need to derive various performance measures to optimize the queueing model.

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1. Introduction

Queueing theory deals with one of the most unpleasant experiences of life, waiting. Queueing is quite common in many fields, for example, in telephone exchange, in a supermarket, at a petrol station, at computer systems, etc. The first problems of queueing theory were raised by calls and Erlang was the first who treated congestion problems in the beginning of 20th century. His

works inspired engineers, mathematicians to deal with queueing problems using probabilistic methods. Queueing theory became a field of applied probability and many of its results have been used in operations research, computer science, telecommunication, traffic engineering, and reliability theory, just to mention some.

2. Bulk Queues

A classical queueing system can be described as customers arriving for service, if not immediately provided and if having waited for service, leaving the system after being served. The arrival is described as flow of arrivals in batches. In a given time instant if the arrival of customer is more than one, then the queueing model is considered as bulk queues. Bulk queues have wide range of applications in production line and manufacturing. Letters arriving at a post office, ships arriving at a port in convoy, people going to a theatre, and so on are some of the examples for bulk queues.

Mathematical modelling of Bulk arrival and batch service queues yields different performance measures to analyse production line systems. By the method of optimization one can fix the minimum and maximum capacity of the server.

Kendall [18] developed theory of queues within the framework of theory of stochastic processes. His analysis is based on method of embedded Markov chains. Bulk queues have been analysed by many authors in past. Neuts [27] classified bulk queues with Poisson input. Takagi [34] has explained bulk queueing systems in detail with and without vacations. Queueing systems with server vacations with several combinations have been analyzed by various authors. Initially Doshi [6] have made comprehensive survey of queueing systems with vacations. Lee [22] et al analysed a $M^X/G/1$ queueing system with N policy and multiple vacations, using supplementary variable technique. A batch arrival queue with threshold was discussed by Lee et al. [23]. Krishnareddy et al. [20] have analysed a bulk queueing model and multiple vacations with setup time. They derived probability generating function of a system size at an arbitrary time epoch and derived its various performance measures. Krishna Reddy and Anitha [21] studied a $M/G(a, b)/1$ queue with different vacation policies and obtained Laplace transform of the joint distribution of the queue length and the remaining service time and the remaining vacation time depending on the state of the server.

Bulk arrival and batch service queueing models have been discussed by

many authors. In some of the systems the server has to fix some threshold to start the service that threshold value should be greater than the minimum capacity of the server that is determined by N policy in batch service. Lee [11] et al. introduced fixed size batch service in a batch service queueing model with vacations. They derived queue length distributions for both single and multiple vacation. Lee et al. [32] concentrates probabilistic measures of the single vacation model with threshold. They developed procedure to optimize the long run average cost of the system. Arumuganathan and Jeyakumar [2] derived steady state condition for bulk arrival and batch service queueing model with N policy multiple vacations, set-up time, close down times. They used supplementary variable technique for analysing the model. Various performance measures such as expected queue length, expected length of busy and idle period, probability that the server is busy and the server is on vacation of the queueing system are derived with suitable numerical illustration. Optimum cost analysis also undergone in this paper explicitly.

Al-khedhairi [1] analysed choice of service and re-service in a bulk service queue under Bernoulli schedule. For the proposed queueing model results are derived in both discrete and continuous time. Also, real time scenario concerns the choice of service and re-service types explained explicitly. Suganya [33] used supplementary variable technique for deriving steady condition for bulk arrival and batch service queueing model with vacation interruption, optional re-service and balking. For the above model expected queue length also derived.

Jeyakumar and Arumuganathan [16] presented control policy on request for re-service in an $M^X/G(a, b)/1$ queueing model with multiple vacations. In this model re-service provided only if queue length is less than a minimum threshold value of the server. In this work they obtained steady state condition and various performance measures by the method of supplementary variable technique. Cost analysis is also presented in the paper. Haridass and Arumuganathan [10] derived various performance measures of Batch service queueing model with multiple vacations and restricted admissibility of arriving batches. In this paper results are justified with numerical example. The above queueing model is also extended with Optimum cost analysis which is used to fix the minimum and maximum capacity of the server. Maurya [38] given performance analysis under transient state of Markovian two state batch arrival and batch service queue with multiple vacations. Some valuable conclusive observations are presented in the paper.

Queueing system with vacation interruption. Li and Tian [24] analyzed the discrete-time GI/Geo/1 queue with working vacation and vacation interruption. Li et al [15] analysed GI/M/1 queue with working vacations and vacation

interruption. Zhang and Hou [26] studied an M/G/1 queue with working vacations and vacation interruption. Haridass and Arumuganathan [9] studied $M^X/G(a, b)/1$ queueing model with vacation interruption. The derived various characteristics of queueing system with real time application.

Only few authors have analyzed about queue with server breakdown which includes). Takine and Sengupta [36] discussed a single server queue with service interruptions. They characterized the queue-length distribution as well as the waiting time distribution of a single-server queue which is subject to service interruptions. Wang et al. [19] analysed the optimal control of the N policy M/G/1 queueing system with server breakdowns and general start-up times. In their system, the server is immediately turned on but is temporarily unavailable to serve the waiting customers. The server needs a start-up time before providing service until there are no customers in the system. Wang et al. [38] obtained the various system performance measures for the T policy M/G/1 queue with server breakdowns and general start up times.

Jain and Agrawal [12] have analysed the optimal policy for bulk queue with multiple types of server breakdown. They consider an unreliable server $M^X/M/1$ queueing system with multiple types of server breakdowns under N-policy. Breakdowns occur only when the server is in busy state and each type of breakdown requires a random number of finite stages of repair. Ke [13] studied the operating characteristics of a batch arrival queues under vacation policies with server breakdowns and start up/closedown times. In his analysis, there are two vacation policies namely multiple vacation and single vacation. If a customer arrives during a closedown times, the server is immediately started without a start-up time. Madan et al. [25] studied steady state analysis of two $M^X/M(a, b)/1$ queue models with random breakdowns. They considered that the repair time is exponential for one model and deterministic for another one. In the literature of queueing models with server breakdown, it is observed that all the authors except Madan et al. [25], who deal the server breakdown in the bulk service queueing model, deal with the server which can serve only one customer at a time. It is clear that if breakdown occurs the server is allowed to interrupt immediately. But in most of the situations it is not possible to disturb the server before completing its batch of service. This simulates Arumuganathan and Malliga [3] to model bulk arrival and batch service system with breakdown without service interruption. In the above model they described as when the server got breakdown there is no need to stop its service it will be continued for some time by doing some technical arrangements. After completed its batch service Server will be repaired before starting another service that period is called renovation time. Breakdown without service in-

interruption in a bulk arrival and batch service queueing model also studied by Jeyakumar and senthilmathan [17]. They modelled with closedown time and derived probability generating function of service completion epoch, vacation completion epoch and renovation completion epoch.

The M/G/1 queueing model with controllable service rate has several applications and has received considerable attention in the literature. Nishimura and Jiang [28] analysed a two level switch over rule for an M/G/1 queue with two service modes and switch over times. They obtained a full and tractable solution for the number of customers present in the system. Nobel [29] studied an M/G/1 model with bulk arrival with two service modes and changing the service mode based on the queue length. He developed an algorithm using Markov decision theorem that minimizes the long run average number of customers in the system. From the literature on queueing models with state queueing models with state dependent service, it is also observed that the service rate varies depending on the queue length. However, there are some manufacturing systems in which the server has to choose either single service mode or a batch service mode depending on the number of units available. State dependent service in a bulk arrival non Markovian queueing model extensively analysed by Arumuganathan and Jeyakumar [24]. In this paper service provided in two manners namely single or in bulk depending upon the queue size. Switch over time from single service is not considered in this paper.

Discrete time queues has been analysed extensively by many of the researchers, because this systems are more appropriate than continuous time in modelling computer and communication systems. The batch arrival discrete-time $Geo^X/G/1$ queue under multiple vacations governed by a geometrically distributed timer was analyzed by Fiems and Bruneel [7]. Chang and Choi [5] analysed a single-server batch arrival bulk-service queue where customers are served in batches of random size and the server takes multiple vacations whenever the queue is empty. Samanta [30] et al. presented the discrete-time $Geo^X/Geo/1/N$ queues with batch arrival and bulk-service under single and multiple vacation policies. In all these references, it is assumed that the server cannot take service during the vacations.

Goswami et al. [8] considered bulk service queues with accessible and non-accessible batches at discrete time, in this model steady state probabilities are derived for both finite buffer and infinite buffer. Also, the effectiveness of the model parameters are presented with numerical results. Li et al. [14] provided steady state condition for Discrete time batch arrival and single service queueing model with working vacations. In this paper at the departure epoch they formulated the system as an embedded Markov chain and probability

generating function of M/G/1 type stationary queue length obtained by using matrix analytic approach. For the above model stochastic decomposition also used to give another equivalent probability generating function of the queue length.

3. Conclusions

This survey reviews the work done in the area of bulk queues. The idea related to Bulk arrival and batch service queues with vacations discussed in various papers have been synthesized. It can help researchers, engineers, statisticians for using these models. Proper references have been cited for wide range of literature in bulk queues.

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