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A MATHEMATICAL MODELING OF AYURVEDA DOSHAS: APPLICATION OF FUZZY SOFT TOPOLOGY

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Abstract: We propose a new approach to validate the Vata, Pitta, Kapha test available every where. This paper is a mathematical modeling of Ayurveda 'Doshas'. It is an application of Fuzzy Soft Topology. Different 'Doshas' and the combination are treated as open sets to define the topology. This study proves that all the test available through online with out consulting an expert or a doctor are not valid.

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1. Introduction

There are many uncertainties in most of the engineering, physics, computer sciences, economics, social sciences and medical sciences problems.These may be due to the uncertainties of natural environmental phenomena, of human knowledge about the real world or to the limitations of the means used to measure objects. For example, vagueness or uncertainty in the boundary between states or between urban and rural areas or the exact growth rate of population in a

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country's rural area or making decisions in a machine based environment using database information. Thus classical set theory, which is based on the crisp and exact case may not be fully suitable for handling such problems of uncertainty

Although a number of mathematical tools like probability theory, fuzzy sets [1], rough sets [2] are well known and effective models for dealing with uncertainties. However, each of them has distinguished advantages as well as certain limitations.

The inadequacy of parametrization tools was cleared by Molodtsov [3] by introducing soft set theory. Molodsov proposed a completely new approach for modeling vagueness and uncertainty in soft set theory. Soft set theory is free from the difficulties where as other existing methods viz. Probability Theory, Fuzzy Set Theory [Zadeh, 1965; Zimmerman, 1996], Intuitionistic fuzzy set theory [Atanassov, 1986], Rough Set Theory [Pawlak, 1982] etc. which can be considered as mathematical tools for dealing with uncertainties, have their own difficulties. Probability theory is applicable only for stochastically stable system. There is no limited condition to the description of objects; Many of the established paradigms appear as special cases of Soft Set Theory, so researchers can choose the form of parameters they need, which greatly simplifies the decision making process and make the process more efficient in the absence of partial information.

In 2003, Maji et al. studied the theory of soft sets initiated by Molodstov, they defined equality of two soft sets, subset and super set of a soft set, complement of a soft set, null soft set and absolute soft set with examples. Soft binary operations like AND, OR and also the operations of union, intersection were also defined. In 2005, Pei and Miao and Chen et al. improved the work of Maji et al. In 2009, Ali et al. gave some new notions such as the restricted intersection, the restricted union, the restricted difference and the extended intersection of two soft sets along with a new notion of complement of a soft set. Similarity measures have extensive application in pattern recognition, region extraction, coding theory, image processing and in many other areas. Presently, work on the soft set theory is making progress rapidly. In the standard soft set theory, a situation may be complex in the real world because of the fuzzy nature of the parameters. With this point of view Yang et al. in 2007, expanded this theory to fuzzy soft set theory and discussed some immediate outcomes. To continue the investigation on fuzzy soft sets, Kharal and Ahmad in 2011 introduced the notion of a mapping on the classes of fuzzy soft sets which is a pivotal notion for the advanced development of any new area of mathematical sciences. Similarity of two fuzzy soft sets has been studied by Majumdar and Samanta in 2010, and application of similarity between generalized fuzzy soft sets has been

studied by them. The algebraic structure of soft set theory has been studied increasingly in recent years.

He applied soft set theory in the first work successfully in many fields, such as smoothness of a function, game theory, the Riemann integral, Perron integral and measurement theory. Maji et al. [4] presented an application of soft sets in decision making problems that is based on the reduction of parameters to keep the optimal choice objects. Chen et al. [5] presented a new definition of soft set parametrization reduction Ali et al. presented some new algebraic operations for soft sets. Combining soft sets with fuzzy sets, Maji et al. [4] defined fuzzy soft sets which are rich potentials for solving decision making problem.

The concept of fuzzy topology was first defined in 1968 by Chang. But a Changs fuzzy topology is a crisp subfamily of some family of fuzzy sets and fuzziness in the concept of openness of a fuzzy set has not been considered, which seems to be a drawback in the process of fuzzification of the concept of the topological space. Therefore, in 1985 Sostak, introduced the fundamental concept of a fuzzy topological structure as an extension of both ordinary topology and Changs fuzzy topology, in the sense that not only the object were fuzzifed, but also the axiomatic. Sostak gave some rules and showed how such an extension can be realized. Here we try for a Mathematical modeling through Sostak Fuzzy topology together with soft sets.

1.1. Preliminaries

Definition 1. [3] Let U be an initial universe set and E be set of parameters. Let $P(U)$ denotes the power set of U and $A \subseteq U$. A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$.

Definition 2. [4] Let U be an initial universal set and let E be a set of parameters. Let I^U denote the power set of all fuzzy subsets of U. Let $A \subseteq E$. A pair (F, E) is called a fuzzy soft set over U, where F is a mapping given by $F : A \to I^U$

Definition 3. [4] Let τ be the family of fuzzy soft set over (U, E). Then τ is said to be a Fuzzy Soft set Topology on (U, E) if

1. $\phi, U \in \tau$

2. The union of any number of fuzzy soft sets in τ belongs to τ.

3. The intersection of any two fuzzy soft sets in τ belongs to τ.

The triple (U, E, τ) is called a fuzzy soft topological space over X. This Fuzzy Topology is said to be Fuzzy Topology on Chang's view

Definition 4. [6] Let X be a non-empty set and $\tau : I^X \to$ I be a mapping satisfies the following conditions:

1. $\tau(\phi) = \tau(X) = 1$.
2. If $A, B \in I^X$, then $\tau(A \wedge B) \geq \tau(A) \wedge \tau(B)$.
3. For any $\{A_i : i \in \Delta\} \subset I^X$, we have that $\tau(\vee_{i \in I} A_i) \geq \wedge_{i \in I} \tau(A_i)$.

Then τ is called a Sostak fuzzy topology on X (or gradation of openness on X, or smooth topology).

2. Vata, Pitta and Kapha

In this section we go through the Ayurvedic Doshas. Here we connect the mathematical abstract concept like fuzzy topology with these doshas. Later we connect it with Sostak Fuzzy Topology and we show that the better way of mathematical modeling of Ayurvedic Doshas is to connect it with Sostak Fuzzy topology.At last we show that the online test available through online are not valid and its better to consult a doctor to know the doshas and there by enjoy the better treatment for any decease.

According to Ayurveda, the five elements (fire, earth, water, air & space) in their biological form combine to form three basic energies in the body. These three basic energies are the primary life forces or biological humors, called doshas in Ayurveda. These three primary doshas are vata, pitta and kapha. They help regulate physical functionings within our body, besides providing us with our individual physical characteristics.

The constitution of a person is primarily determined by the dominant dosha. Your predominant dosha could be any one of the three, a combination of any two or all the three in a balanced form: Vata, Kapha, Vata-Kapha, Pitta, Vata-Pitta, Pitta-Kapha.

The following is the question and options which are available through online test to know which type we are? To find out your Ayurvedic constitution, select one quality from each row that best describes your physical qualities. The category with the maximum score is supposedly your predominant dosha or Ayurvedic constitution. Very often there are two (equal or almost equal) predominant doshas. This indicates that you have a dual constitution. Rarely

does one have a tridoshic constitution, which is a combination of all the three Ayurvedic doshas.

Vada, Pitha, Kafa test (VPK test) are available in many form. We took a test which is mostly used by many experts and doctors. Here we try to take the test valid. For that we did a statistical and mathematical frame work on this Ayurvedic study. A person is taken to find out in which category he belongs to. The table shows some questions to check which category he belongs to.

While going through the questions a person may feel that some questions have more relevance in his life and some questions have no such relevance in his life. That may lead to incorrect result. So the use of soft sets, that is giving different weightage to each question and this weightage or giving different parameters to each question is termed as soft sets.

Let the questions be $U = \{1, 2, 3, \ldots, 32\}$, and let $A = \{e_1, e_2, e_3\}$ where set of parameters where e_1 = very good question (ie. very relevant to me), e_2 = good question (ie relevant to me), and e_3 = average question (ie. not relevant to me). The soft set (F, A) describes the 'question is relevant to him'. That is if a person selects one of the options of first question there is a chance that he may belong to Vada (V), Pitha (P) or Kafa (K) is a fuzzy value $\frac{1}{32}$. Also if very relevant the fuzziness of a person belong to a category will greatly increase, if it is relevant the fuzziness of a person belong to a category will increase and if it is not relevant the fuzziness of a person belong to a category will slightly increase.

A person having a balanced form of VPK are considered as full set. Or here we define a soft topology with elements in the set $\{VPK, VP, VK, PK, V, P, K, \phi\}$. That is $\{\frac{VPK}{\text{all in balanced form}}$,

$\frac{VP}{\text{balanced form of Vada and pitha}}$, $\frac{VK}{\text{balanced form of Vada and kapha}}$,
$\frac{PK}{\text{balanced form of Kapha and pitha}}$, $\frac{V}{\text{balanced form of Vada only}}$,
$\frac{P}{\text{balanced form of pitha only}}$, $\frac{K}{\text{balanced form of Kapha only}}\}$.

The fuzziness can be applied to each question as the most of a person react to a question in different way in different time, in this case fuzzy soft topology is essential.

Each question is marked by a quantity ≤ 32, that is if a person select one option in question number 1, there is a chance that he may belong to V, P or K is $\leq$32 and the sum of each column is noted and can see the fuzziness of each category. We define the topology like $\{\tau, \phi, V, P, K\}$, $\{\tau, \phi, V\}$ etc. This means that a person is now dominating vada. Here we take the balanced form as full set. Similarly we define $\{\tau, \phi, P\}$, $\{\tau, \phi, K\}$, $\{\tau, \phi, V, P\}$, $\{\tau, \phi, V, K\}$, $\{\tau, \phi, P, K\}$. We can see that all these are topologies.

1. Body Frame	Thin,bony,tallorshort	Medium, balanced	Large, broad
2. Weight	Low	Moderate	Heavy
3. Skin	Dry,roughcool,dull	Soft,oily,warm	Thick,moist,cold,pale
4. Hair	Dark, dry, curly	Soft,oily,fair/red	Thick,oily,wavy
5. Teeth	Protruding,big	Moderate, yellowish	Strong, even, white
6. Nails	Rough,dry,bitten	Soft,pink,strong	Soft,large,white
7. Eyes	Small,dull,dry,nervous	Sharp,penetrating	Big,thick lashes
8. Appetite	Variable,small	Good,regular	Slow and steady
9. Thirst	Variable	Excessive	Minimal
10. Elimination	Dry,hard,constipated	Soft,oily,loose	Thick,heavy,slow
11. Urine	Frequent but sparse	Yellow,copious	Infrequent
12. Sweat	Minimal	Profuse,pungent	Slow to begin
13. Pulse	Weak,erratic	Stable,strong	Slow,smooth
14. Blood	Circulation	Variable,poor	Good,Moderatee
15. Sleep	Light,disturbed	Sound,moderate	Heavy,excessive
16. Speech	Rapid,highORhoarse	Sharp,cutting,loud	Slow,harmonious
17. Libido	Varies,fantasy	Passionate,excessive	Slow but strong
18. Immunity	Variable,poor	Moderate	High
19. Activity	High,restless,mobile	Moderate,directed	Minimal,slow
20. Endurance	Minimal	Moderate	Excellent
21. Mind	Restless, curious	Aggressive,clever	Calm,slow
22. Memory	Short term	Sharp, good	Long term
23. Routine	Dislikes	Enjoys planning	Adaptable
24. Faith	Erratic, changeable	Fanatical	Steady,devoted
25. Moods	Variable,fluctuate	Expresses forcefully	Changes slowly
26. Finances	Poor,spends rapidly	Moderate,luxuries	Rich,thrifty
27. Hobbies	Travel,art philosophy	Sports,luxuries	Serene leisurely
28. Food	Simple,sparse snacks	Regular meals	Gourmet,fatty
29. Creativity	Original,fertile	Technical,scientific	Entrepreneurial
30. Sensitivities	Cold,wind,dryness	Heat,sun,fires	Cold,humidity
31. Temperament	Nervous,insecure,shy	Determine,motivate	Conservative
32. Dreams	Frequent,fearful	Fiery,violent,vivid	Romantic,calm

As we have seen that some questions are relevant and some are not relevant, the final value to say whether he belongs to Vada, Pitha, Kapha or a combined form of these three (balanced form) or two will not be accurate. Suppose a person who knows that some questions are not relevant to his life, for that

1.	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	17.	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$
2.	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	18.	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$
3.	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	19.	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$
4.	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	20.	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$
5.	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	21.	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$
6.	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	22.	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$
7.	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	23.	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$
8.	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	24.	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$
9.	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	25.	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$
10.	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	26.	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$
11.	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	27.	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$
12.	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	28.	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$
13.	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	29.	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$
14.	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	30.	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$
15.	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	31.	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$
16.	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	32.	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$

case we introduce the new mathematical concept like Sostak fuzzy topology. According to statement 1 of Definition 4, empty set and full set will be present, that shows the varying level of a person from balanced form to others.

2.1. Modeling through Sostak Fuzzy Topology

Suppose question number (i) is a very relevant question and he answered it correctly. So we say that he may belong to that particular category is $\frac{1}{32}$. And if all questions are like that, that is relevant to him, we can treat it as a full set, or completely open and having membership value 1. If all questions are not relevant to him we can treat it as an empty set or not open that is complement of completely open, as that it is also present and its membership value is again 1. So we can see that it satisfies statement 1 of Definition 4 of Sostak Fuzzy Topology. Other axioms are solved using an example.

Example 5. Let $X = \{a, b\}$. Define the fuzzy sets A,B in X as:

$$A(x) = \begin{cases} 1 \text{ if } x = a \\ 0 \text{ if } x = b \end{cases}$$

$$B(x) = \begin{cases} 0 \text{ if } x = a \\ 1 \text{ if } x = b \end{cases}$$

We define a mapping $\tau : I^X \to [0,1]$ by: $\tau(X) = \tau(\phi) = 1, \tau(A) = 0.15, \tau(B)$ =0.80, and $\tau(C) = 0.10$, if $\tau = \{\phi, X, A, B\}$. Note, for example, that $A \vee B = X$, $\tau(A \vee B) = 1 \geq 0.15 = \tau(A) \wedge \tau(B)$. It is clearly that (X, τ) is a Sostak Fuzzy Topology on X, which is not a Chang's Fuzzy Topology.

So here we tried for a mathematical modeling of Ayurvedic doshas. While we were trying it was found that mathematical modeling of Ayurvedic doshas is not so apt when we use Chang's definition of Fuzzy Topology, and when we use Sostak's Fuzzy Topology it is perfectly suit. We would like to go through extensive application using Sostak Fuzzy Topology on another work.

3. Validity of the Online Test on Ayurvedic Doshas

We discussed earlier that our main aim of this paper is to check the level of validity of those test which are available online to test Ayurvedic doshas. For that we classify the question into three parts as earlier. (a) Very relevant question (b) Moderately relevant question (c) not a relevant question. Category (c) type questions are those questions, if a doctor is asking directly to a patient he can asses the patient, that is to which category he belongs, But if the patient is directly doing it through online questions may not be relevant to his mind and the answer he quotes may be wrong.
Let $U = \{Q_1, Q_2, Q_3, \ldots, Q_{32}\}$ be set of 32 questions, which may be characterized by a set of parameters $E = \{$very relevant question, Moderately relevant question, Not relevant question$\}$. In this study we also consider mood of each person. So our parametrization will extend to supporting parameters such as less attentive, partially attentive, Fully attentive.

Assuming that the fuzzy soft set (F, A) describe the questions which are very relevant, fuzzy soft set (G, B) describe the questions which are Moderately relevant, fuzzy soft set (H, C) describe the questions which are not relevant.For convenience we take only some questions from the above 32 and assumes that while going through online test a person feels these questions as a mixture of theses three category questions. If (F, A) and (G, B) be two fuzzy soft sets then (F, A) AND (G, B) is a fuzzy soft set denoted by $(F, A) \wedge (G, B)$.

3.1. Algorithm

1. Input the fuzzy soft sets (F,A), (G,B) and (H,C).
2. Compute the corresponding resultant fuzzy soft set (I,D) from the fuzzy soft sets (F,A), (G,B) and (H,C).
3. Place the obtained value in tabular form.
4. Construct the comparison table of fuzzy soft set(I,D).
5. Compute the score of Q_i, for all i. that is the row sum.
6. Least numerical values shows that the questions will go wrongly interpreted.

Step by step procedure is as follows.

		V. R			M.R			N.R	
	L.A	P.A	F.A	L.A	P.A	F.A	L.A	P.A	F.A
Q_1	0.73	0.83	0.96	0.72	0.82	0.89	0.61	0.71	0.79
Q_2	0.41	0.21	0.10	0.43	0.23	0.16	0.49	0.39	0.33
Q_3	0.44	0.25	0.12	0.46	0.27	0.20	0.45	0.35	0.30
Q_4	0.63	0.76	0.89	0.62	0.72	0.81	0.60	0.71	0.71
Q_5	0.72	0.82	0.91	0.70	0.77	0.83	0.59	0.63	0.73
Q_6	0.73	0.84	0.94	0.71	0.73	0.87	0.61	0.67	0.77

where V.R means very relevant, M.R means moderately relevant, N.R means not relevant, and L.A means less attentive to the question, P.A means Partially attentive to the question, F.A means fully attentive to the question.

Consider (F, A) and (G, B) fuzzy soft sets if we perform (F, A) AND (G, B)then we have $3 \times 3 = 9$ parameters of the form e_{ij}, where $e_{ij} = V.R \wedge M.R$. If we require fuzzy soft set for the parameters P $=\{e_{11}, e_{13}, e_{23}, e_{31}, e_{32}, e_{33}\}$ So the resultant fuzzy soft set is as follows.

.	e_{11}	e_{13}	e_{23}	e_{31}	e_{32}	e_{33}
Q_1	0.72	0.73	0.83	0.72	0.82	0.89
Q_2	0.41	0.16	0.16	0.10	0.10	0.10
Q_3	0.44	0.20	0.20	0.12	0.12	0.12
Q_4	0.62	0.63	0.76	0.62	0.72	0.81
Q_5	0.70	0.72	0.82	0.70	0.77	0.83
Q_6	0.71	0.73	0.84	0.71	0.73	0.87

Another resultant fuzzy soft set is obtained by taking the operation with N.R also. It is as follows

	$e_{11} \wedge n_1$	$e_{13} \wedge n_2$	$e_{23} \wedge n_3$	$e_{31} \wedge n_1$	$e_{32} \wedge n_2$	$e_{33} \wedge n_3$
Q_1	0.61	0.71	0.79	0.61	0.71	0.79
Q_2	0.441	0.16	0.16	0.10	0.10	0.10
Q_3	0.44	0.20	0.20	0.10	0.10	0.10
Q_4	0.60	0.63	0.71	0.60	0.71	0.71
Q_5	0.59	0.63	0.73	0.59	0.63	0.73
Q_6	0.61	0.67	0.77	0.61	0.67	0.77

Now the comparison table is as follows, it is calculated like the entry c_{ij} indicates a numerical measure, which is an integer number and q_i dominates q_j in c_{ij} number of parameters out of given parameters.

.	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6
Q_1	6	6	6	6	6	6
Q_2	0	6	3	0	0	0
Q_3	0	6	6	0	0	0
Q_4	1	6	6	6	5	1
Q_5	0	6	6	3	6	0
Q_6	2	6	6	5	6	6

Next we compute the row sum

.	Row sum
Q_1	36
Q_2	9
Q_3	12
Q_4	25
Q_5	21
Q_6	31

From the table it is clear that Q_2 and Q_3 are more vague questions, that the examinees may not get the exact mood while doing these kinds of problems. Out of these 32 questions 2 or 3 questions are of such type, which may lead to an incorrect interpretation. So without consulting an Ayurvedic doctor we cant say our kind of doshas in a perfect way, as when the doctor asks these questions in a face to face contact he can say with in seconds that which kind of doshas we dominates.

4. Conclusion

Here we attended to connect the purely medical concepts to a mathematical concept, there by giving a new way to medical field. Each and every kind of deceases deals with fuzzy concepts. The abstractness of fuzzy topology can be narrow it down to a layman by taking the exact fuzzy topology like Sostak Fuzzy Topology.

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