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ALUTHGE TRANSFORMATION OF N-CLASS A_k OPERATORS

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Abstract: In this paper a new class of an operator N-class A_k is defined and sum of its properties are discussed. And also we proved Aluthge transformation and $*$-Aluthge transformation of N-classA_k operators on Hilbert spaces.

AMS Subject Classification: 47A63, 47B37

Key Words: aluthge transformation, $*$-aluthge transformation, N-class A_k operators

1. Introduction

Let H be a non-zero Hilbert space and let $B(H)$ denotes the algebra of all bounded linear operators on H. We mean a closed linear manifold of H by subspace M of H and we mean a bounded linear transformation of H into itself by an operator T on H a subspace M invariant for T if $T(M) \subset M$, non-trivial if $\{0\} \neq M \neq H$. An operator T is called normal if $T^*T = TT^*$. An operator T is said to be quasinormal if $TT^*T = TT^*T$, its called hyponormal if $T^*T \geq TT^*$. Its called quasihyponormal if $T^{*2}T^2 \geq (T^*T)^2$. An operator $T \in B(H)$ is said to be paranormal if $\|Tx\|^2 \leq \left\|T^2x\right\|$ for all $x \in H$. Its said to be $*$-paranormal if $\|T^*x\|^2 \leq \left\|T^2x\right\|$ for all $x \in H$. An operator T is said to be class $A(k)$ if $|T|^2 \leq \left(T^* |T|^{2k} T\right)^{\frac{1}{k+1}}$. An operator T is called class A_k if $|T|^2 \leq \left(\left|T^{k+1}\right|\right)^{\frac{2}{k+1}}$ [4]. Therefore operator inclutions class $A(k) \subset$ class $A_k \subset$ N-class$A(k) \subset$ N-

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classA_k $\subset$ powres of N-classA_k. In[2] Aluthge defined an transformation $\widetilde{T}$ on T and proved p-hyponormal operators of Aluthge transformation.[10] has defined $*$-Aluthge transformation and discussed some properties of $*$-Aluthge transformation.

2. Aluthge Transformation of N-Class A_k Operators in Hilbert Space

Definition 1. An operator $T \in B(H)$ is N-classA_k if $|T|^2 \leq N \left|T^{k+1}\right|^{\frac{2}{k+1}}$ for a fixed $N > 0$.

Corollary 2. *If $T \in B(H)$ is N-classA_k if $|T|^2 \leq N \left|T^{k+1}\right|^{\frac{2}{k+1}}$ for a fixed $N > 0$ then the followings are hold,*
(i). If $N = 1$ then the operator is class A_k.
(ii). If $N = 1$ and $k = 1$ then the operator is class A.
(iii). If $k = 1$ then the operator is N-class A.

Theorem 3. *If T is a bounded linear operator on Hilbert space then we know that,*
(i). $T = U\,|T| = |T^*|\,U$ is polar decomposition of an operator T.
(ii). $T^* = U^*\,|T^*| = |T|\,U^*$ is polar decomposition of an operator T.

Theorem 4. *[3] If A is positive operator, Then the following inequalities hold for $x \in H$.*
1. $\langle A^r x, x\rangle \leq \langle A^r x, x\rangle^r \|x\|^{2(1-r)}$ for $0 < r \leq 1$
2. $\langle A^r x, x\rangle \geq \langle A^r x, x\rangle^r \|x\|^{2(1-r)}$ for $r > 1$

Theorem 5. *If T is N-class$A(k)$operator then T is N-classA_k operator.*

Theorem 6. *If T is N-class$A_k$operator if and only if*

$$\|Tx\|^2 \leq N \left\|T^{k+1}x\right\|^{\frac{2}{k+1}} \|x\|^{\frac{2k}{k+1}}.$$

Theorem 7. *If T is N-class$A_k$operator then T is hyponormal.*

Theorem 8. *If T is N-class$A_k$operator then T^* is N-class$A_k$operator .*

Corollary 9. *If T is N-class$A_k$operator then T^{-1} is N-class$A_k$operator .*

Theorem 10. *Let T is N-class$A_k$operator then $T = U\,|T|$ be the polar decomposition of T is N-class$A_k$operator.*

Proof. From by the definition of N-classA_k operator for every $x \in H$.

$$|T|^2 \leq N \left|T^{k+1}\right|^{\frac{2}{k+1}}$$

$$(U^* |T^*| U |T|) \leq N \left\{ \left(U^{*k+1} \left|T^{*k+1}\right| U^{k+1} \left|T^{k+1}\right| \right) \right\}^{\frac{1}{k+1}}$$

$$|T|^2 \leq N \left|T^{k+1}\right|^{\frac{2}{k+1}}$$

Therefore $T = U|T|$ be the polar decomposition of T is N-class$A_k$operator. □

Theorem 11. *Let T is N-class$A_k$operator and S is an unitary operator such that $TS = ST$ then $C = TS$ is also N-class$A_k$operator.*

Theorem 12. *Let A and B be positive operators. Then for each $p \geq 0$ and $r \geq 0$ the following assertion hold.*
1. *If $\left(\beta^{\frac{r}{2}} A^p \beta^{\frac{r}{2}}\right)^{\frac{r}{p+r}} \geq \beta^r$ then $\left(\beta^{\frac{p}{2}} A^r \beta^{\frac{p}{2}}\right)^{\frac{p}{p+r}} \leq A^p$*
2. *If $\left(\beta^{\frac{p}{2}} A^r \beta^{\frac{p}{2}}\right)^{\frac{p}{p+r}} \leq A^p$, $N(A) \subset N(\beta)$ then $\left(\beta^{\frac{r}{2}} A^p \beta^{\frac{r}{2}}\right)^{\frac{r}{p+r}} \geq \beta^r$.*

Theorem 13. *Let $T = U|T|$ be the polar decomposition of T is N-class$A_k$operator for $0 < p \leq 1$ then,$\widetilde{T}_{s,t} = |T|^s U |T|^t$ is $2(p + min(s,t))$ N-classA_k operator for $s, t > 0$ such that $max(s,t) \geq p$.*

Proof. From by the definition of N-classA_k operator for every $x \in H$.

$$\left(\widetilde{T}^*_{s,t}\widetilde{T}_{s,t}\right)^{\frac{p+min(s,t)}{s+t}} \leq N \left[\left\{ \left(\widetilde{T}^{*k+1}_{s,t} \widetilde{T}^{k+1}_{s,t} \right) \right\}^{\frac{1}{k+1}} \right]^{\frac{p+min(s,t)}{s+t}}$$

$$U^* \left(\beta^{\frac{t}{2}} A^{\frac{s}{2}} \beta^{\frac{t}{2}}\right)^{\frac{p+min(s,t)}{s+t}} U \leq N U^* \left[\left\{ \left(\beta^{\frac{t}{2}} A^{\frac{s}{2}} \beta^{\frac{t}{2}}\right)^{k+1} \right\}^{\frac{1}{k+1}} \right]^{\frac{p+min(s,t)}{s+t}} U$$

$$U^* (|T^*|)^{2(p+min(s,t))} U \leq N U^* \left[\left\{ (|T^*|)^{k+1} \right\}^{\frac{1}{k+1}} \right]^{2(p+min(s,t))} U$$

$$(|T|)^{2(p+min(s,t))} \leq N \left[\left\{ (|T|)^{k+1} \right\}^{\frac{1}{k+1}} \right]^{2(p+min(s,t))}$$

□

3. $*$-Aluthge Transformation of N-Class A_k Operators in Hilbert Space

Definition 14. Let $T = U|T|$ be the polar decomposition of an operator T, then $*$-Althuge transformation T is $\widetilde{T}^{(*)} = |T^*|^{\frac{1}{2}} U |T^*|^{\frac{1}{2}}$.

Definition 15. Let $T = U|T|$ be the polar decomposition of an operator T. Then adjoint of $*$ - Aluthge transformation T is $\left(\widetilde{T}^{(*)}\right)^* = |T^*|^{\frac{1}{2}} U^* |T^*|^{\frac{1}{2}}$.

Theorem 16. *[5] If T is a bounded linear operator on a Hilbert space, Then we know that*

(i). $\widetilde{T} = |T|^{\frac{1}{2}} U |T|^{\frac{1}{2}}$ is the Aluthge transformation then adjoint of aluthge transformation $\widetilde{T}^*$ is given by $\widetilde{T}^{(*)} = |T|^{\frac{1}{2}} U^* |T|^{\frac{1}{2}}$.

(ii). $\widetilde{T}^{(*)} = \left(\widetilde{T}^*\right)^* = |T^*|^{\frac{1}{2}} U |T^*|^{\frac{1}{2}}$ is the $*$ - aluthge transformation then adjoint of $*$ - aluthge transformation $\left(\widetilde{T}^{(*)}\right)^* = |T^*|^{\frac{1}{2}} U^* |T^*|^{\frac{1}{2}}$.

Theorem 17. *Let $T = U|T|$ is N - class A_k operator and U is isometry then $\widetilde{T}$ is N - class A_k operators.*

Proof. Form the definition of N - class A_k operator.

$$(U^* |T^*| U |T|) \leq N \left\{ (U^* |T^*| U |T|)^{k+1} \right\}^{\frac{1}{k+1}}$$

$$\left(|T|^{\frac{1}{2}} U^* |T|^{\frac{1}{2}} |T|^{\frac{1}{2}} U |T|^{\frac{1}{2}}\right) \leq N \left\{ \left(|T|^{\frac{1}{2}} U^* |T|^{\frac{1}{2}} |T|^{\frac{1}{2}} U |T|^{\frac{1}{2}}\right)^{k+1} \right\}^{\frac{1}{k+1}}$$

$$\left|\widetilde{T}\right|^2 \leq N \left(\left|\widetilde{T}\right|^{k+1} \right)^{\frac{2}{k+1}}$$

Therefore $\widetilde{T}$ is N - class A_k operator. □

Theorem 18. *Let $T \in B(H)$ and $\widetilde{T}$ is N - class A_k operator then $\widetilde{T}^*$ is N - class A_k operator.*

Theorem 19. *Let $T \in B(H)$ and $\widetilde{T}^*$ is N - class A_k operator then $\widetilde{T}$ is N - class A_k operator.*

Theorem 20. *Let $T \in B(H)$ and $\widetilde{T}^{(*)}$ is N - class A_k operator then $\left(\widetilde{T}^{(*)}\right)^*$ is N - class A_k operator.*

Proof. Form the definition of N - class A_k operator.

$$\left(\left(\widetilde{T}^{(*)}\right)^{*}\widetilde{T}^{(*)}\right) \leq N\left\{\left(\left(\widetilde{T}^{(*)}\right)^{*}\widetilde{T}^{(*)}\right)^{k+1}\right\}^{\frac{1}{k+1}}$$

$$\left(|T^*|^{\frac{1}{2}}U^*|T^*|^{\frac{1}{2}}|T^*|^{\frac{1}{2}}U|T^*|^{\frac{1}{2}}\right) \leq N\left\{\left(|T^*|^{\frac{1}{2}}U^*|T^*|^{\frac{1}{2}}|T^*|^{\frac{1}{2}}U|T^*|^{\frac{1}{2}}\right)^{k+1}\right\}^{\frac{1}{k+1}}$$

$$\left|\left(\widetilde{T}^{(*)}\right)^{*}\right|^{2} \leq N\left\{\left|\left(\widetilde{T}^{(*)}\right)^{*}\right|^{k+1}\right\}^{\frac{2}{k+1}}.$$

Therefore $\left(\widetilde{T}^{(*)}\right)^{*}$ is N - class A_k operator. □

Theorem 21. *Let $T = U|T|$ is N - class A_k operator and U is isometry operator if and only if $\left(\widetilde{T}^{(*)}\right)^{*}$ is N - class A_k.*

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