International Journal of Pure and Applied Mathematics
Volume 106 No. 8 2016, 21-25
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
doi: 10.12732/ijpam.v106i8.4
ijpam.eu

M-CONNECTEDNESS IN M-TOPOLOGY

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Abstract: Yager introduced the concept of multisets in the year 1986. Girish and Sunil Jacob John introduced the notion of a multiset topology in 2012. The purpose of this paper is to introduce the concept of multiset connectedness in a multiset topological space and characterize its properties.

1. Introduction

The theory of multiset(bag) as a natural extension of the set theory was introduced by Yager and Wayne D. Blizard in [2],[3]. Multiset topological spaces are studied by Girish & Sunil Jacob John in [8],[9].

Later Mahanta. J and Das. D developed his research on semi open and semi closed msets in multi topological spaces in [12]. The notion of γ-Operation, pre-open msets, α open mset, Semi open mset, β-open mset, b-open mset in M-topological spaces were discussed by El-Sheikh S. A, Omar R. A. K and Raafat in [4].

In this paper we will state certain theorems on M-connectedness in a multiset topological spaces.

Received: February 15, 2016
Published: March 3, 2016

2. Preliminaries

Definition 1. An mset M is a pair (X, C) where X is a non empty set and $C : X \to N$ is a function from X to the set N of all non negative integers. The function C is called the count function of M. Suppose $X = \{x_1, x_2, \ldots, x_n\}$, $C(x_i) = m_i$ for $1 \leq i \leq n$. Then the mset M is represented by

$$\left\{\frac{m_1}{x_1}, \frac{m_2}{x_2}, \ldots, \frac{m_n}{x_n}\right\}.$$

The elements having zero count need not be written in the above representation of a mset.

Definition 2. Let M and N be two msets drawn from a set X with count functions C_M and C_N respectively. Then we have

1. $M = N$ if $C_M(x) = C_N(x)\ \forall\, x \in X$.
2. $M \leq N$ if $C_M(x) \leq C_N(x)\ \forall\, x \in X$.
3. $P = M \cup N$ if $C_P(x) = \max\{C_M(x), C_N(x)\}\ \forall\, x \in X$.
4. $P = M \cap N$ if $C_P(x) = \min\{C_M(x), C_N(x)\}\ \forall\, x \in X$.
5. $P = M - N$ if $C_P(x) = \max\{C_M(x) - C_N(x), 0\}\ \forall\, x \in X$.

Notatation: Let X be a set from which msets are constructed.

$[X]^\infty =$ the set of all msets $\left\{\frac{k}{x} : x \in X, k \geq 0 \text{ is an integer}\right\}$

$[X]^W =$ the set of all msets $\left\{\frac{k}{x} : x \in X, \quad k \in \{0, 1, 2, \ldots, w\}\right\}$ where w is a fixed non negative integer.

Definition 3. If $M \in [X]^w$ then M', the complement of M is defined with count function $C_M(x) = w - C_M(x) \quad \forall\, x \in X$.

An mset $\varnothing$ with count function $C_\varnothing(x) = 0 \quad \forall\ x \in X$ is defined as the empty mset.

Throughout this paper M stands for a multiset drawn from the multiset space $[x]^w$ and $P^*(M)$ denotes the collection of all submsets of M.

Definition 4. Let $M \in [X]^w$ and $\tau \subseteq P^*(X)$. Then τ is called a multiset topology of M if

1. M and $\varnothing$ are in τ.
2. τ is closed under arbitrary union of msets.

3. τ is closed under finite intersection of msets.

Here, τ is called a multiset topology on M and the ordered pair (M, τ) is a multiset topological space with $M \in [X]^w$. Multiset topology is abbreviated as M-topology. The members of τ are called open msets in (M, τ).

Example.

1. $\tau = P^*(M)$ is called the discrete M-topology on M.

2. $\tau = \{\varnothing, M\}$ is called the indiscrete M-topology on M.

Definition 5. A submset N of an M-topological space M in $[X]^w$ is said to be closed if the mset $M - N$ is open.

Mathematicians studied, the concepts of M-basis, sub-M-basis, subspace of M-spaces, M-closure, M-interior, M-limit point, continuous multiset functions and extended some of the concepts in topology to M-topology.

Definition 6. The support set M^* of M is defined as $M^* = \{x \in X : C_M(x) > 0\}$. M^* is also called the root set of M.

Definition 7. A function f from an mset M to an mset N is defined to be a function f^* from M^* to N^* such that $f(\frac{k}{x}) = \frac{k^*}{f^*(x)}$ where $\frac{k^*}{f^*(x)} \in N$ and $\frac{k}{x} \in M$.

Example.

Let $X = \{a, b, c, d\}$ and $Y = \{x, y, z, w\}$. $M = \left\{\frac{5}{a}, \frac{4}{b}, \frac{4}{c}\right\}$ and $N = \left\{\frac{7}{x}, \frac{5}{y}, \frac{6}{z}, \frac{4}{w}\right\}$.

Then $M^* = \{a, b, c\}$, $N^* = \{x, y, z, w\}$.

Define

$$f^* : \begin{array}{l} a \to x \\ b \to y \\ c \to z \end{array}$$

Definition 8. $f : M \to N$ is said to be M-continuous if for each open submset V of N, $f^{-1}(V)$ is an open submset of M.

3. M-connectedness in M-topology

Let (M, τ) be an M-topological space. An M-separation of M is a pair M_1, M_2 of disjoint nonempty open sub msets of M whose union is M. An M-space (M, τ) is said to be M-connected if there does not exist an M-separation of M. A sub-mset N of an M-space M is M-connected if N is M-connected as a sub space of M.

Theorem 1. *An M-space (M, τ) is M-connected iff the only submsets of M that are both open and closed in M are the empty mset and M itself.*

Theorem 2. *If the multisets M_1 and M_2 form an M-separation of M and if N is an M-connected subspace of M then N lies entirely within M_1 or M_2.*

Theorem 3. *The union of a collection of M-connected sub-msets of M that have a point in common is M-connected.*

Theorem 4. *Let A be an M-connected subspace of M. If $A \subseteq B \subseteq \overline{A}$, then B is also M-connected.*

Theorem 5. *If $f : M \to N$ is M-continuous and if M is M-connected then $f(M) \subseteq N$ is M-connected.*

Example.

Let $M = \left\{\frac{2}{a}, \frac{3}{b}, \frac{4}{c}, \frac{5}{d}\right\}$ be an mset. Therefore an M-topology is defined as $\tau = \left\{M, \emptyset, \left\{\frac{2}{a}, \frac{3}{b}\right\}, \left\{\frac{4}{c}, \frac{5}{d}\right\}\right\}$.

Let $A = \left\{\frac{2}{a}, \frac{3}{b}\right\}$ and $B = \left\{\frac{4}{c}, \frac{5}{d}\right\}\}$ are open sub msets of M.

$\therefore A \cup B = \left\{\frac{2}{a}, \frac{3}{b}, \frac{4}{c}, \frac{5}{d}\right\} = M$ and $A \cap B = \emptyset$. Then there exist an M-seperation on M. Therefore M is an M-disconnected.

Example.

Let $M = \left\{\frac{2}{a}, \frac{3}{b}, \frac{4}{c}, \frac{5}{d}\right\}$ be an mset. Therefore an M-topology is defined as $\tau = \left\{M, \emptyset, \left\{\frac{1}{a}, \frac{3}{c}\right\}, \left\{\frac{2}{a}, \frac{2}{b}, \frac{2}{c}\right\}, \left\{\frac{2}{a}, \frac{2}{b}, \frac{3}{c}\right\}, \left\{\frac{1}{a}, \frac{2}{c}\right\}\right\}$.

Then there doesn't exist an M-seperation on M. Therefore M is an M-connected.

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