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SOME MORE PROPERTIES OF INTUITIONISTIC β-OPEN SETS

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Abstract: Intuitionistic β-open sets was studied recently by A.Singaravelan. In this paper some more properties of Intuitionistic β-open set (Iβ-open sets) and properties of intuitionistic β-closure (Iβ-cl) and intuitionistic β-interior (Iβ-int) are discussed.

AMS Subject Classification: 54A99
Key Words: Iβ-open sets, Iβ-closed sets, Iβ-closure, Iβ-interior

1. Introduction

In 1986, Atanassov [4] introduced the concept of Intuitionistic fuzzy sets as a generalization of fuzzy sets. Later in 1996, Coker [5] introduced the concept of Intuitionistic set and Intuitionistic points. This is a discrete form of intuitionistic fuzzy sets where all the sets are crisp set. In 2000, Coker [7] also introduced the concept of intuitionistic topological space and investigated basic properties of continuous functions and compactness. N. Levine [9] introduced semi open sets and semi continuity in topological space and M.E. Abd EI. Monsef et.al [1] introduced β-open sets and β-continuous mapping and discussed some basic properties. D.Andrijevic [3] introduced and discussed some more properties of semi pre open set in topological space. Gnanambal Ilango and Selvanayaki [8], introduced generalized pre regular closed sets in intuitionistic topological spaces and Singaravelan [11] introduced intuitionistic β-open set in intuitionistic topo-

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logical space.In this paper some more properties of intutionistic β-open sets and some more properties of intuitionistic β-interior and intuitionistic β-closure are discussed.

2. Preliminaries

Definition 1. [5] Let X is a non empty set. An intuitionistic set (IS for short) A is an object having the form $A =< X, A_1, A_2 >$, where A_1 and A_2 are subsets of X satisfying $A_1 \cap A_2 = \phi_\sim$. The set A_1 is called the set of members of A, while A_2 is called the set of non-members of A.

Definition 2. [5] Let X be a non empty set and let A, B are intuitionistic sets of the form $A =< X, A_1, A_2 >, B =< X, B_1, B_2 >$ respectively. Then:

(a) $A \subseteq B$ iff $A_1 \subseteq B_1$ and $B_2 \subseteq A_2$;

(b)$A = B$ iff $A \subseteq B$ and $B \subseteq A$;

(c)$A^c =< X, A_2, A_1 >$, (d) $[]A =< X, A_1, (A_1)^c >$;

(e) $A - B = A \cap B^C$;

(f) $\phi_\sim =< X, \phi_\sim, X_\sim >, X_\sim =< X, X_\sim, \phi_\sim >$;

(g) $A \cup B =< X, A_1 \cup B_1, A_2 \cap B_2 >$,;

(h) $A \cap B =< X, A_1 \cap B_1, A_2 \cup B_2 >$;

Furthermore, let $\{A_\alpha : \alpha \in J\}$ be an arbitrary family of intuitionistic sets in X, where $A_\alpha =< X, A_\alpha^{(1)}, A_\alpha^{(2)} >$ Then:

(i) $\cap A_\alpha =< X, \cap A_\alpha^{(1)}, \cup A_\alpha^{(2)} >$;

(j) $\cup A_\alpha =< X, \cup A_\alpha^{(1)}, \cap A_\alpha^{(2)} >$;

(k)$\langle\rangle\, A =< X, A_2^C, A_2 >$.

Definition 3. [6] An intuitionistic topology (for short IT) on a non empty set X is a family of IS's in X satisfying the following axioms:

(i) $\phi_\sim, X_\sim$;

(ii) $G_1 \cap G_2$ for any $G_1, G_2 \in \tau$.

(iii) $\cup G_\alpha$ for any arbitrary family $\{G_\alpha : \alpha \in J\} \subseteq \tau$ where (X, τ) is called an intuitionistic topological space (ITS(X)) and any intuitionistic set in is called an intuitionistic open set (IOS) in X. The complement A^c of an IOS A is called an intuitionistic closed set (ICS) in X.

Definition 4. [7] Let (X, τ) be an intuitionistic topological space (ITS(X)) and $A =< X, A_1, A_2 >$ be an IS in X. Then the interior and closure of A are defined by

Icl $(A) = \cap\{K : K \ is \ an \ ICS \ in \ X \ and \ A \subseteq K\}$;

Iint$(A) = \cup\{G : G \ is \ an \ IOS \ in \ X \ and \ G \subseteq A\}$.;

It can be shown that Icl(A) is an ICS and Iint(A) is an IOS in X and A is an ICS in X iff Icl$(A) = A$ and is an IOS in X iff Iint$(A) = A$.

Definition 5. [12] Let (X, τ) be an ITS(X). An intuitionistic set A of X is said to be (i) intuitionistic semiopen if $A \subseteq$ Icl(Iint(A)).;

(ii) intuitionistic preopen if $A \subseteq$ Iint(Icl(A)).;

(iii) intuitionistic regular open if A = Iint(Icl(A)) .;

(iv) intuitionistic α-open if $A \subseteq$ Iint(Icl(Iint(A))).;

(V) intuitionistic semi-preopen if $A \subseteq$ Icl(Iint(Icl(A))).;

The family of all intuitionistic, semi open pre open, intuitionistic regular open and intuitionistic α-open sets of (X, τ) are denoted by ISOS, IPOS, IROS and IαOS respectively.

Definition 6. [5] Let A, B, C and A_i be IS's in X $(i \in J)$. Then;

(a) $A \subseteq B$ and $B \subseteq C \Rightarrow A \subseteq C$;

(b) $A_i \subseteq B$ for each $i \in J \Rightarrow UA_i \subseteq B$;

(c) $B \subseteq A_i$ for each $i \in J \Rightarrow B \subseteq \cap A_i$;

(d) $(UA_i)^C = \cap A_i^C$, (e) $(\cap A_i)^C = \cup A_i^C$;

(f)$A \subseteq B \Leftrightarrow B^C \subseteq A^C$;

(g)$(A^C)^C = A$, (h)$(\phi_\sim)^C = X_\sim$ and (i)$(X_\sim)^C = \phi_\sim$.

Definition 7. [11] A subset A of an intuitionistic topological space X is intuitionistic β-open set, if there exists a intuitionistic preopen set U in X, such that U $\subseteq$ A $\subseteq$ Icl(U).The family of all intuitionistic β-open sets in X will be denoted by IβOS(X).

3. Some More Properties of Intuitionistic β-Open Sets

Theorem 8. *Let $B_i =< X, B_i^1, B_i^2 >$ for every $i \in J$ be family of intuitionistic β -open sets in a ITS X. Then $U_{i\in J}(B_i)$ is intuitionistic β-open in ITS(X).*

Proof. Let $B_i =< X, B_i^1, B_i^2 >$ be intuitionistic β-open sets for every $i \in J$.;

$\Rightarrow B_i^C =< X, B_i^2, B_i^1 >$ is $I\beta$-closed for every $i \in J$;

$\Rightarrow \cap B_i^C =< X, \cap B_i^2, \cup B_i^1 >$ is $I\beta$-closed for every $i \in J$;

$\Rightarrow \cap B_i^C = (\cup B_i)^C$ is $I\beta$-closed set for every $i \in J$;

by definition (6) $(A_i^C = (\cup A_i)^C)$;

$\Rightarrow (\cup B_i)^C = (< X, \cup B_i^1, \cap B_i^2 >)^C$.;

Hence $\cup B_i =< X, \cup B_i^1, \cap B_i^2 >$ is intuitionistic β-open set in the ITS(X). □

Corollary 9. *Let $A_i =< X, A_i^1, A_i^2 >$ for every $i \in J$ be family of intuitionistic β-closed sets in a ITS X. Then $\cap_{i\in J}(A_i)$is intuitionistic β-closed in ITS X.*

Theorem 10. *For any intuitionistic open set A in a ITS X and every $B \subseteq X$, $Icl(B) \cap A \subseteq Icl(A \cap B)$.*

Corollary 11. *For any intuitionistic closed set A in intuitionistic topological space X (for short ITS(X)) and every $B \subseteq X$, we have $Iint(B \cup A) \subseteq Iint(B) \cup A$.*

Theorem 12. *Let $A =< X, A_1, A_2 >$ be a subset of ITS X . Then the following conditions are equivalent.;*

(a) A is intutionistic semi open set.;

(b) $A \subseteq Icl(Iint(A))$ and (c) $Icl(A) = Icl(Iint(A))$.

Corollary 13. *Let $A =< X, A_1, A_2 >$ be a subset of ITS X . Then the following conditions are equivalent.;*

(a) A is intuitionistic semi closed set;

(b) $Iint(Icl(A)) \subseteq A$ and (c) $Iint(Icl(A)) = Iint(A)$.

Corollary 14. *Let $A =< X, A_1, A_2 >$ be subset of ITS X, then the following results hold.;*

(i) $Iscl(A) = A \cup Iint(Icl(A))$ *and (ii)* $Isint(A) = A \cap Icl(Iint(A))$;

(iii) $Ipcl(A) = A \cup Icl(Iint(A))$ *and (iv)* $Ipint(A) = A \cap Iint(Icl(A))$.

Theorem 15. *If A is intuitionistic α-closed and B is intuitionistic β-closed then A∪B is intuitionistic β-closed.*

Proof. Let $A =< X, A_1, A_2 >$ is a intuitionistic α-closed and $B =< X, B_1, B_2 >$ is intuitionisticβ-closed.;

$\Rightarrow Icl(Iint(Icl(A))) \subseteq A$ and $Iint(Icl(Iint(B))) \subseteq B$;

$\Rightarrow Icl(Iint(Icl(A))) \cup Iint(Icl(Iint(B))) \subseteq (AUB)$;

Let $D = Icl(Iint(Icl(A)))$ and $E = Icl(Iint(B))$;

$\Rightarrow DUIint(E) \subseteq (AUB)$;

$\Rightarrow Iint(DUE) \subseteq DUIint(E) \subseteq (AUB)$ (by corollary 10);

$\Rightarrow Iint(DUE) \subseteq (AUB)$;

$\Rightarrow Iint((Icl(Iint(Icl(A)))UIcl(Iint(B)) \subseteq (AUB)$;

$\Rightarrow Iint((Icl(Iint(A)))UIcl(Iint(B))) \subseteq (AUB)$;

$\Rightarrow Iint(Icl(Iint(AUB))) \subseteq (AUB)$;

Therefore $A \cup B$ is intuitionistic β-closed set. □

Corollary 16. *If A is intuitionistic α-open and B is intuitionistic β-open, then $A \cap B$ is intuitionistic β -open.*

Theorem 17. *If A is an intuitionistic closed and B is an intuitionistic β-closed, then $A \cup B$is intuitionistic β-closed set.*

Proof. Let $A =< X, A_1, A_2 >$ be an intuitionistic closed set, Then A= Icl(A) which implies Icl(A) = Icl(A) = Icl(Iint(A));

(by corollary 11) and let $B =< X, B1, B2 >$ is an intuitonistic;

β -closed, which implies $Iint(Icl(Iint(B))) \subseteq B$.;

$\Rightarrow AUIint(Icl(Iint(B))) \subseteq (AUB)$;

$\Rightarrow AUIint(Icl(Iint(B))) \subseteq (AUB)$;

Let $D = Icl(Iint(B))$, then $\Rightarrow AUIint(D) \subseteq (AUB)$;

$\Rightarrow Iint(DUA)AUIint(D) \subseteq (AUB)$(by corollary 10);

$\Rightarrow Iint(DUA) \subseteq (AUB) \Rightarrow Iint(Icl(Iint(B))UA) \subseteq (AUB)$;

$\Rightarrow Iint(Icl(Iint(B))UIcl(A) = Icl(Iint(A))) \subseteq (AUB);$

$\Rightarrow Iint(Icl(Iint(B)UIcl(Iint(A))) \subseteq (AUB);$

$\Rightarrow Iint(Icl(Iint(AUB))) \subseteq (A \cup B);$

Therefore $A \cup B$ is an intuitionistic β-closed set. □

Corollary 18. *If A is an intuitionistic open and B is an intuitionistic β-open, then $A \cap B$ is Iβ-open set.*

4. Properties of Intuitionistic β-Closure and Intuitionistic β-Interior

Definition 19. Let (X, τ) be an intuitionistic topological space and let $A =< X, A_1, A_2 >$ be an intuitionistic set of X. Then $I\beta - cl(A) = \cap\{F : F \text{ is intuitionistic } \beta - \text{closed in } X \text{ and } A \subseteq F\}$.

Definition 20. Let (X, τ) be an intuitionistic topological space and let $A =< X, A_1, A_2 >$ be an intuitionistic set of X. Then $I\beta - int(A) = \cup\{F : F \text{ is intuitionistic } \beta - \text{open in } X \text{ and } F \subseteq A\}$.

Theorem 21. *If $A =< X, A_1, A_2 >$ is a subset of ITS X, then $Isint(Iscl(A)) = Iscl(A) \cap Icl(Iint(Icl(A)))$.*

Proof. By corollary [13], Isint (Iscl(A)) = Iscl(A)∩ Icl(Iint(Iscl(A)));

= Iscl(A)∩Icl(Iint(A ∪ Iint(Icl(A))));

=Iscl(A) ∩ Icl(Iint(A) ∪ Iint(Icl(A)));

= Iscl(A) ∩Icl(Iint(A ∪ Icl(A)));

= Iscl(A) ∩Icl(Iint(Icl(A)));

Iscl(A) ∩ Icl(Iint(Icl(A)) ⊆ Isint (Iscl(A))⇒ (1);

$Isint(Iscl(A)) = Iscl(A)\ capIcl(Iint(Icl(A)));$

$\Rightarrow Isint(Iscl(A)) \subseteq$ Iscl(A) ∩ Icl(Iint(Icl(A)))⇒ (2);

From (1) and (2) we have;

Isint(Iscl(A)) = Iscl(A) ∩ Icl(Iint(Icl(A))). □

Corollary 22. *If $A =< X, A_1, A_2 >$ is a subset of ITS X, then Iscl(Isint(A)) = Isint(A) ∪ Iint(Icl(Iint(A))).*

Corollary 23. *Let $A =< X, A_1, A_2 >$ be an intuitionistic set of ITS(X), then Iβ-int (Iβ-cl(A)) = Iβ-cl(Iβ-int(A)).*

Theorem 24. *If $A =< X, A_1, A_2 >$ is a subset of intuitionistic topological space X, then:*

Iscl(Isint(A)) ⊆ Iβ-int(Iβ-cl(A)) ⊆ Isint(Iscl(A)).

Proof. Let $A =< X, A_1, A_2 >$ be a subset of intuitionistic topological space X, then:

Iscl(Isint(A)) = Isint(A)∪ Iint(Icl(Iint(A))) [by corollary 21];

= (A ∩ Icl(Iint(A))) ∪ (Iint(Icl(Iint(A)))) [by corollary 13];

= (A ∪ Iint(Icl(Iint(A)))) ∩ Icl(Iint(A));

(A ∪ Iint(Icl(Iint(A)))) ∩ Icl(Iint(Icl(A))) = Iβ-int(Iβ-cl(A));

Iscl(Isint(A)) ⊆ Iβ-int(Iβ-cl(A))⊆ (A U Iint(Icl(A))) ∩ Icl(Iint(Icl(A))) = Isint(Iscl(A));

Thus Iscl(Isint(A))⊆ Iβ-int(Iβ-cl(A)) ⊆ Isint(Iscl(A)). □

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