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A REGION GROWING SEGMENTATION ALGORITHM USING METRIC TOPOLOGICAL NEIGHBOURHOODS

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Abstract: One of the important techniques in image processing is image segmentation. It plays a vital role in the field of medicine for segmentation of tumor from the medical images. One of the useful methods of segmentation is region growing. In this article, a novel region growing segmentation algorithm is proposed based on the neighbourhoods of different new metric topology to segment tumor from brain magnetic resonance images (MRI). The quality of the segmented images is measured by the traditional performance evaluation measures Accuracy, Sensitivity, Specificity, PSNR and the new evaluation measure entropy.

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Key Words: metrics, metric topology, neighbourhoods, region growing segmentation

1. Introduction

Medical image segmentation is an important technique used to visualize the affected region in medical images for diagnosis and treatment of tumor. It is a process of extracting the required region from the image which is a set of connected pixels satisfying some predefined condition. Image segmentation algorithms are based on two categories i.e., detecting discontinuities based on

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the abrupt changes in gray level of the pixel and detecting similar pixels having some homogeneous criterion. Region growing segmentation method is based on the second category.

A novel technique for detecting and calculating the area of tumor from brain MRI by thresholding segmentation was developed by Alyaa H. Ali et al [2] . V. Murali and S. Boopathy [12] presented a comparative study of edge detection techniques like Prewitt, Canny and Sobel implemented on brain tumor images and measured the consistency using the root mean square error between the input image and the output image and proved that Gaussian filter, Canny edge detector method and thresholding are the best techniques for edge detection. Prathibha Sharma et al [14] provided an efficient algorithm for detection of edges of brain tumor. Here, the location of the tumor is identified by digital imaging techniques and basic morphological operations.

S. Allin Christe, K. Malathy and A. Kandaswamy [1] implemented fuzzy - c means segmentation and k - means segmentation. Also a new improved hybrid segmentation technique called fuzzy k - means of image segmentation based on statistical feature clustering was proposed and the clustered segmented tissue images were compared with ground truth images and proved that accuracy is improved rather than pixel based segmentation.

Aman Chandra Kaushik and Vandana Sharma [3] proposed a content based active contour, which uses both intensity and texture information. Here, region growing method is used for segmenting the region of interest, and edge detection for boundary segmentation of tumor.

Manisha Sharma and Vandana Chouhan [10] surveyed the different evaluation parameters used to perform a quantitative comparison between image segmentation like variation of information, rand index, global consistency error, boundary displacement error, segmentation accuracy, entropy etc and showed that the measures boundary displacement error, global consistency error, variation of information and rand index are used for cluster based methods. Global consistency is used for region based methods and precision and recall are used for boundary based methods. J. Pastore et al [13] described an automatic segmentation method for segmentation of tumors from computerized axial tomography images by means of alternating sequential filters of mathematical morpohology and continuous topology concepts. An automatic segmentation method relevant to medical image segmentation using topological concepts was developed by Gnanambal Ilango and R. Marudhachalam [6].

B. Shanthi Gowri and Gnanambal Ilango [15] proposed a new metric topology based region growing algorithm and measured the quality of segmentation by the evaluation measure accuracy. Gnanambal Ilango et al [16] developed a seeded region growing algorithm using metric topology and introduced a new performance evaluation measure entropy. The performance of segmentation is measured using the evaluation measures accuracy, PSNR and entropy.

In this article, a new metric topological neighbourhoods based region growing algorithm is proposed. The proposed algorithm is implemented on MRI of brain affected by tumor. The segmented images are compared with the ground truth images. The quality of segmentation is measured using the evaluation measures Accuracy, PSNR, Sensitivity, Specificity and entropy.

2. Materials and Methods

2.1. Seeded Region Growing Method

Seeded region growing method is one of the region based methods of segmentation which depends on intensity and edges. The procedure for this method is as follows:

i) Select a seed point from the region to be segmented.

ii) Define a grouping criterion based on gray level and a stopping rule.

iii) Grow the region by merging the neighbours of the seed point satisfying the grouping criterion with the seed point.

iv) Stop the region growth if there is no more pixel satisfy the grouping criterion.

Definition 1. [11] A metric on a set X is a function $d : X \times X \to R$ having the following properties:

i) $d(x, y) \geq 0,\ \forall\ x, y \in X$; equality holds iff $x = y$.

ii) $d(x, y) = d(y, x),\ \forall\ x, y \in X$.

iii) $d(x, z) \leq d(x, y) + d(y, z)\ \forall\ x, y, z \in X$.

Given a metric d on X, the number $d(x, y)$ is called as the distance between x and y in the metric d. For a given $\varepsilon > 0$, consider the set

$$B_d(x, \varepsilon) = \{y\ /d(x, y) < \varepsilon\} = B(x, \varepsilon)$$

of all points y whose distance from x is less than ε. Here $B_d(x, \varepsilon)$ is called the ε – ball centered at x. If d is a metric on the set X, then the collection of all ε -

balls $B_d(x,\varepsilon)$ for $x \in X$ and $\varepsilon > 0$, is a basis for a topology on X, called metric topology induced by d. A set U is open in the metric topology induced by d iff for each $y \in U$, there is a $\delta > 0$ such that $B_d(y,\delta) \subset U$. If X is a topological space, X is said to be metrizable if there exists a metric d on the set X that induces the topology of X. A metric space is a metrizable space X together with a specific metric d that gives the topology of X.

Definition 2. [8] An image may be defined as a two-dimensional function $f(x,y)$, where x and y are spatial (plane) co-ordinates, and the amplitude of f at any pair of coordinates (x,y) is called the intensity or gray level of the image at that point. If x, y and the intensity values of f are all finite, discrete quantities, we call the image a digital image. A digital image is composed of a finite number of elements (x,y) each of which has a particular location and value. These elements are called picture elements or pixels.

Definition 3. [7] Let $S^2 = S \times S = \{(x,y); x = 0,1,2,...M-1 \; and \quad y = 0,1,2,...N-1\}$ be the set of spatial coordinates of a digital image.

For any metric space (S^2,d), any $p \in S^2$ and any $\varepsilon > 0$, consider the set $N_{d,\varepsilon}(p) = \{q; d(p,q) < \varepsilon + 1\}$. $N_{d,\varepsilon}(p)$ is called the (open) ε - neighbourhood of p in S^2.

Definition 4. [9] Let $p = (x_1,y_1), q = (x_2,y_2) \in S^2$. Consider the functions i) $d_4 : S \times S \to R$ defined by $d_4(p,q) = |x_1 - x_2| + |y_1 - y_2|$. Then (S^2, d_4) is a metric space. The metric d_4 is called the City-Block metric or Manhattan metric.

ii) $d_8 : S \times S \to R$ defined by $d_8(p,q) = max\{|x_1 - x_2|, |y_1 - y_2|\}$. Then (S^2, d_8) is a metric space. The metric d_8 is called the Chessboard metric.

Definition 5. [7] For any point p, the 4 - neighbourhood of p denoted by $N_{d_4,1}(p)$ is defined as $N_{d_4,1}(p) = \{q \in S^2; d_4(p,q) < 2\}$. $N_{d_4,1}(p)$ is also denoted by $N_4(p)$ and the 8 - neighbourhood of p denoted by $N_{d_8,1}(p)$ is defined as $N_{d_8,1}(p) = \{q \in S^2; d_8(p,q) < 2\}$. $N_{d_8,1}(p)$ is also denoted by $N_8(p)$.

Definition 6. [7] For any point p, the 4 - neighbours of p are $N_{d_4,1}(p) - \{p\}$ and the 8 - neighbours of p are $N_{d_8,1}(p) - \{p\}$.

Definition 7. [17] Let $M \subseteq S \times S$ where $M = \{(x, y \pm k); where \; k = 0,1,2,3,...\}$. Let $p = (x_1,y_1)$, $q = (x_2,y_2) \in M$. Consider the function $d_V : M \to R$ defined by $d_V(p,q) = |x_1 - x_2| + |y_1 - y_2|$. (M, d_V) is a metric space. For any point p, the V_ε neighbourhood of p is defined as $N_{d_V,\varepsilon}(p) = \{q; d_V(p,q) < \varepsilon + 1\}$. Hence the V_1 neighbourhood of p is defined as $N_{d_V,1}(p) = \{q; d_V(p,q) < 2\}$ and the V_2 neighbourhood of p is $N_{d_V,2}(p) = \{q; d_V(p,q) < 3\}$.

The V_1 neighbours of p are $N_{d_V,1}(p) - \{p\}$ and the V_2 neighbours of p are $N_{d_V,2}(p) - \{p\}$.

Definition 8. [17] Let $M \subseteq S \times S$ where $M = \{(x \pm k, y); where\ k = 0, 1, 2, 3, ...\}$. Let $p = (x_1, y_1)$, $q = (x_2, y_2) \in M$. Consider the function $d_H : M \to R$ defined by $d_H(p, q) = |x_1 - x_2| + |y_1 - y_2|$. (M, d_H) is a metric space. For any point p, the ε -H neighbourhood of p is defined as $N_{d_H,\varepsilon}(p) = \{q; d_H(p, q) < \varepsilon + 1\}$. Hence the 1-$H$ neighbourhood of p is defined as $N_{d_H,1}(p) = \{q; d_H(p, q) < 2\}$ and the 2-H neighbourhood of p is defined as $N_{d_H,2}(p) = \{q; d_H(p, q) < 3\}$. The 1-$H$ neighbours of p are $N_{d_H,1}(p) - \{p\}$ and the 2-H neighbours of p are $N_{d_H,2}(p) - \{p\}$.

Definition 9. [7] Let $M \subseteq S \times S$ where $M = \{\{(x+k, y+k)\} \cup \{(x-k, y-k)\}; where\ k = 0, 1, 2, 3, ...\}$. Let $p = (x_1, y_1)$, $q = (x_2, y_2) \in M$. Consider the function $d_{RT} : M \to R$ defined by $d_{RT}(p, q) = \frac{1}{2}[|x_1 - x_2| + |y_1 - y_2|]$. (M, d_{RT}) is a metric space. For any point p, the RT_ε neighbourhood of p is defined as $N_{d_{RT},\varepsilon}(p) = \{q; d_{RT}(p, q) < \varepsilon + 1\}$. Hence the RT_1 neighbourhood of p is defined as $N_{d_{RT},1}(p) = \{q; d_{RT}(p, q) < 2\}$ and the RT_2 neighbourhood of p is defined as $N_{d_{RT},2}(p) = \{q; d_{RT}(p, q) < 3\}$. $N_{d_{RT},1}(p)$ is also denoted by $R_3(p)$. The RT_1 neighbours of p are $N_{d_{RT},1}(p) - \{p\}$ and the RT_2 neighbours of p are $N_{d_{RT},2}(p) - \{p\}$.

Definition 10. (Grouping criterion)[15] Let X be an image in levels of gray and $\Im$ a topology associated with X and let $Y \subset \Im$. Let 'd ' be a metric on X. Define $\phi : Y \times X \to R$ such that $\phi((A, x)) = m$ where m is the maximum of the gray level difference between x and the elements of A. Let $A \in Y$. Given a fixed ε and δ, an element $x \in X$ is said to belong to A if $d(x, y) < \varepsilon + 1$ for some $y \in A$ and $\phi((A, x)) < \delta$.

3. Proposed Segmentation Algorithm

3.1. RTHV Algorithm

Let X be an image in levels of gray.

Step I From the histogram of the image, select the gray level of the region of interest to be segmented.

Step II Let $A_1 = \{x\}$ where x is the seed point of the region of interest.

Step III Choose $\varepsilon = 1$ and δ=2. If $\exists$ a point $y \in X$ where $y \notin A_1$ such that $d_{RT}(x, y) < \varepsilon + 1$ and $\phi((A_1, y)) < \delta$. (0)then include y in A_1 and rename it as A_2. Repeat Step III again and again for the elements of A_2 till $\exists$ no point $y \in X$ satisfying the condition (1). That is, all the RT neighbours of elements of A_1 are obtained.

Step IV If $\exists$ a point $y \in X$ where $y \notin A_2$ such that $d_H(z, y) < \varepsilon + 1$ where $z \in A_2$ and $\phi((A_2, y)) < \delta$, then include y in A_2 and rename it as A_3. Repeat Step IV again and again for the elements of A_3 till $\exists$ no point $y \in X$ satisfying the condition (2). That is, all the horizontal neighbours of elements of A_2 are obtained.

Step V If $\exists$ a point $y \in X$ where $y \notin A_3$ such that $d_V(z, y) < \varepsilon + 1$ where $z \in A_3$ and $\phi((A_3, y)) < \delta$, then include y in A_3 and rename it as A_4. Repeat Step V again and again for the elements of A_4 till $\exists$ no point $y \in X$ satisfying the condition (3). That is, all the vertical neighbours of elements of A_3 are obtained.

The set A_4 is the segmented region of interest.

4. Evaluation Measures

4.1. Accuracy [15]

Let X be the set of pixels in the image. Define the ground truth $T \subset X$ as the set of pixels that were labeled as tumor by the expert. Similarly define the segmented tumor $S \subset X$ as the set of pixels that were labeled as tumor by the algorithm. $\bar{T}$ and $\bar{S}$ be the set of pixels that were labeled as non-tumor by the expert and algorithm respectively. The true positive (TP) set is defined as $TP = T \cap S$. i.e., the set of pixels common to T and S. i.e., the set of pixels that were labeled as tumor by the expert and algorithm. The true negative (TN) set is defined as $TN = \bar{T} \cap \bar{S}$. i.e., the set of pixels common to $\bar{T}$ and $\bar{S}$. i.e.,the set of pixels that were labeled as non-tumor by the expert and the algorithm. The false negative (FN) set is defined as $FN = T \cap \bar{S}$. i.e., the set of pixels common to T and $\bar{S}$. i.e., the set of pixels that were labeled as tumor by the expert and non-tumor by the algorithm. The false positive (FP) set is defined as $FP = \bar{T} \cap S$. i.e., the set of pixels common to $\bar{T}$ and S. i.e., the set of pixels that were labeled as non-tumor by the expert and tumor by the algorithm.

The segmentation evaluation measure ‘Accuracy’ is defined as

$$\text{Accuracy} = \frac{n(TP) + n(TN)}{n(TP) + n(TN) + n(FP) + n(FN)}.$$

4.2. PSNR[4]

The Mean Square Error (MSE) is defined as
MSE $=\frac{1}{mn}\sum_{i=0}^{m-1}\sum_{j=0}^{n-1}[I(i,j)-K(i,j)]^2$ where

I(i,j) = Input image and K(i,j) = Output image. The Peak Signal to Noise ratio (PSNR) is defined as the ratio of peak signal power to noise power in logarithmic scale.

PSNR $= 10log_{10}[\frac{Max_I^2}{MSE}]$

where Max$_I$ =Maximum possible pixel value of the image. The typical values of PSNR in lossy image is between 20 and 50 db.

4.3. Sensitivity and Specificity[5]

Sensitivity (TPR- True Positive Rate) and Specificity (True Negative Rate are the two statistical measures of the performance of a binary classification test in image segmentation. In statistics it is also known as classification function. Sensitivity measures the percentage of actual positive values which are correctly identified whereas Specificity measures the percentage of negative values which are correctly identified. Mathematically, this can be expressed as:
Sensitivity $= \frac{n(TP)}{n(TP)+n(FN)}$
Specificity $= \frac{n(TN)}{n(TN)+n(FP)}$

4.4. Entropy[16]

A new parameter for the validation in terms of amount of information retained after segmentation is the entropy. Entropy (E) which is the amount of information per pixel in the source is formulated as E $= \sum_i p_i log_2 p_i$ where p_i = Number of pixels expressed in bits.

5. Experimental Work

The proposed region growing segmentation algorithm is applied to MRI of brain affected by tumor. Here, from the histogram of the image, the gray level of the region of interest is selected. The seed point is chosen as the point having maximum number of metric topological ε - neighbourhood. For the

Evaluation Measures	IMAGE 1	IMAGE 2	IMAGE 3	IMAGE 4
Accuracy	0.9853	0.9890	0.9921	0.9947
% of correct Segmentation	98.53%	98.90%	99.21%	99.47%
Sensitivity	0.9930	0.9918	0.9987	0.9988
Specificity	0.9850	0.9889	0.9918	0.9947
entropy of original image	7.1440	6.9093	7.4553	5.8936
entropy of segmented image	7.1432	6.8887	7.4641	5.9118
PSNR	25.4523	24.2312	25.1653	25.9041

Table 1: Evaluation Measures

homogeneity criterion, a grouping criterion is defined based on metric and gray level difference between pixels. The connected region of interest is grown by including the ε -neighbours of the seed point satisfying the grouping criterion. The proposed algorithm is implemented using MATLAB. The performance of the proposed algorithm is evaluated using the segmentation evaluation measures Accuracy, Sensitivity, Specificity, PSNR and entropy.

6. Results and Discussion

The MRIs of brain affected by tumor are segmented using the proposed RTHV algorithm. Figure 1 shows the ground truth images. Figure 2 shows the original magnetic resonance images of brain affected by tumor, segmented and segmented images with boundary by the RTHV algorithm. Table 1 shows the Accuracy values, Percentage of correct segmentation, sensitivity, specificity, entropy values of the original and segmented images and PSNR of the proposed RTHV algorithm applied to magnetic resonance images of brain affected by tumor.

7. Conclusion

In this work a new region growing segmentation algorithm based on metric topological ε - neighbourhoods and grouping criterion is introduced. To demonstrate the performance of the proposed region growing segmentation algorithm based on metric topological ε - neighbourhoods, the experiments have been conducted on MRIs of brain affected by tumor. The performance of the proposed segmentation algorithm is measured using the segmentation

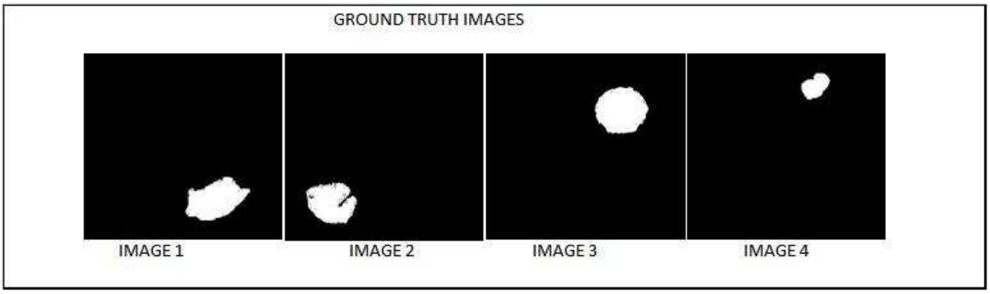

Figure 1: Ground Truth Images

evaluation measure accuracy, sensitivity, specificity, PSNR and entropy. The experimental results indicate that the quality of the correct segmentation is 98.53% and above. This work is a new metric topological approach for the segmentation of medical images.

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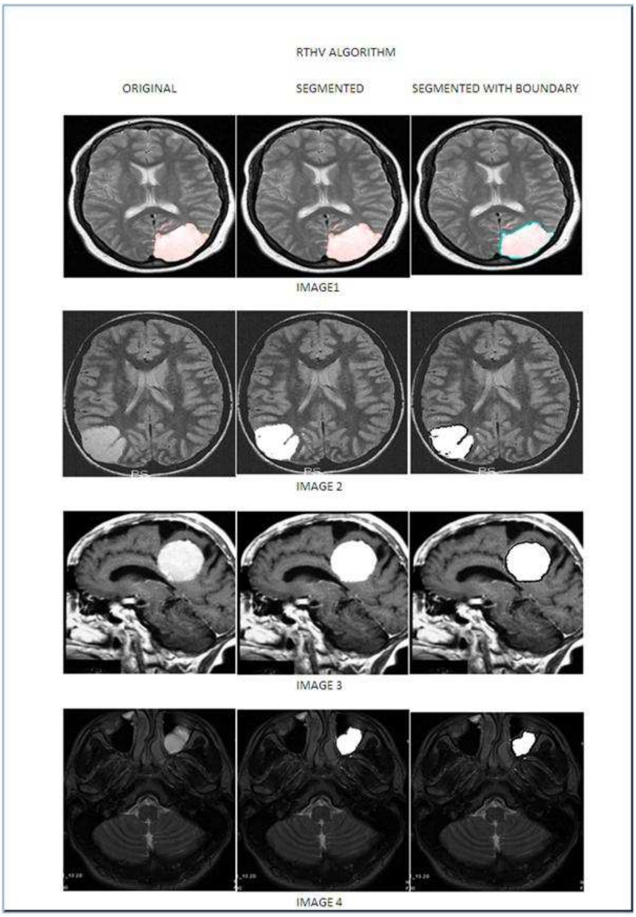

Figure 2: Segmentation by RTHV Algorithm

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