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α^m CONTINUOUS MAPS IN TOPOLOGICAL SPACES

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Abstract: The main aim of the present paper is to introduce new classes of functions called α ^{m} continuous maps and α ^{m} irresolute maps. We obtain some characterizations of these classes and properties are studied.

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Key Words: α^m-closed set, α^m-continuous maps, α^m-irresolute maps

1. Introduction

Ganster and Reilly [1], Levine [2,3], Marcus [4], Mashour [5]et al have introduced LC-continuity, weak continuity, semi continuity, quasi continuity, α -continuity.Balachandran [6] have introduced and studied generalized semi - continuous maps,semi locally continuous maps,semi-generalized locally continuous maps and generalised locally continuous maps. Maki and Noiri studied the pasting lemma for α continuous mappings. Milby and R.Parimelazhagan introduced and studied the properties of α ^{m} -closed sets in topological spaces. Crossley and Hildebrand [7] introduced and investigated irresolute functions

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which are stronger than semi continuous maps but are indepedent of continuous maps.Since then several researchers have introduced several strong and weak forms of irresolute functions.Di Maio [8],Faro [9],Cammaroto and Noiri [10], Maheswari and Prasad[11], Sundaram [12] and Palanimani[13] have introduced and studied quasi irresolute and strongly irresolute maps,strongly α - irresolute maps,almost irresolute maps,α -irresolute maps and gc- irresolute maps are respectively.

2. Preliminaries

Before entering into our work, we recall the following definitions which are due to Levine.

Definition 1. [14] A subset A of a topological space (X, τ) is called a pre-open set if $A \subseteq$ int(cl(A)) and pre-closed set if $cl(int(A)) \subseteq A$.

Definition 2. [3] A subset A of a topological space (X, τ) is called a semi-open set if $A \subseteq cl(int(A))$ and semi closed set if $int(cl(A)) \subseteq A$.

Definition 3. [15] A subset A of a topological space (X, τ) is called an α-open set if $A \subseteq int(cl(int(A)))$ and an α -closed set if $cl(int(cl(A))) \subseteq A$.

Definition 4. [16] A subset A of a topological space (X, τ) is called a semi pre-open set (β-open set) if $A \subseteq cl(int(cl(A)))$ and semi-preclosed set (β closed set) if $int(cl(int(A))) \subseteq A$.

Definition 5. [17] A subset A of a topological space (X, τ) is called a g-closed if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is open in (X, τ).

Definition 6. [18] A subset A of a topological space (X, τ) is called a generalized α -closed (briefly $g\alpha$ -closed) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α-open in (X, τ).

Definition 7. [19] A subset A of a topological space (X, τ) is called weakly generalized closed set (briefly wg closed) if cl(int(A))$\subseteq U$whenever $A \subseteq U$ and U is open in X.

Definition 8. [20] A subset A of a topological space (X, τ) is called weakly generalized α -closed set (briefly $wg\alpha$-closed) if $\tau^{\alpha} - cl(int(A)) \subset U$ whenever $A \subset U$ and U is α - open in (X, τ).

Definition 9. [6] A function f: (X, τ) $\rightarrow$ (Y, σ) from a topological space X into a topological space Y is called g -continuous if f^{-1} (V) is g closed in X for every closed set V of Y.

Definition 10. [12] A function f: (X, τ) $\rightarrow$ (Y, σ) from a topological space X into a topological space Y is called semi -generalised continuous (briefly sg- continuous) if f^{-1} (V) is sg- closed in X for evey closedset V of X.

Definition 11. [21] A functionf: $X \rightarrow Y$ is said to be α- g continuous if f^{-1} (U) is α g - open in X for each open set U of Y.

Definition 12. [19] A function f: $X \rightarrow Y$ is said to be weakly generalised continuous(wg-continuous) if the inverse image of every open set in Y is wg-open in X.

Definition 13. [22] A function f: $X \rightarrow Y$ is w-continuous if f^{-1} (U) is w-openset in X for each openset U of Y.

Definition 14. [7] A function f: (X, τ) $\rightarrow$ (Y, σ) from a topological space X into a topological space Y is called irresolute if f^{-1} (V) is semi closed in X for every semi-closed set V of Y.

Definition 15. [23] A subset A of a topological space (X, τ) is called α^m-closed set if $int(cl(A)) \subseteq U$ whenever $A \subseteq U$ and U is α-open.

3. α^m Continuous Functions

In this section we introduce the concept of α^m continuous functions.

Definition 16. A map f: $X \rightarrow Y$ from a topological space X into a topological space Y is called α^m continuous if the inverse image of every closed set in Y is α^m closed set in X.

Theorem 17. *If a map f: $X \rightarrow Y$ from a topological space into a topological space Y is continuous then it is α^m continuous but not conversely.*

Proof. Let f: $X \rightarrow Y$ be continuous.Let F be any closed set in Y.Then the inverse image of f^{-1} (F) is closed in X.Since every closed set is α^m closed set(previous paper)[23], f $^{-1}(F)$ is α^m closed in X.Therefore f is α^m continuous.The converse need not be true. □

Theorem 18. *Let f: (X, τ) $\rightarrow$ (Y, σ) be a map from a topological space(X,τ) into a topological space (Y,σ)*

(I) The following statements are equivalent.

(a) f is α^m continuous

(b) The inverse image of each openset in Y is α^m open in X.

(II) If f:(X, τ) $\to$ (Y,σ) is α^m continuous then f(cl(A))$\subset$ $\overline{f(A)}$ for every subset A of X;(here cl*(A) as defined by Dunham[24].Further be defined a topological τ^*gclosure by τ^* =$\{G$: cl*(G^c)=$G^c\}$)*

(III)The following statements are equivalent.

(a)For each point x $\in$ X and each open set V containing f(x).Thereexist a α^m open set U containing X suchthat f(U)$\subset$ V.

(b)For every subset A of X, f(cl(A))$\subset$ $\overline{f(A)}$ holds.*

(c)The map f: (X, τ^*) $\to$ (Y, σ) from a topological space.
(X, τ^*) defined by Dunham[24] into topological space (Y, σ) is continuous.

Proof. (I) Assume that f: $X \to Y$ is α^m continuous. Let G be open in Y.Then G^c is closed in Y.Since f is α^m continuous, f^{-1} (G^c) is α^m closed in X. But f^{-1} (G^c)=X- f^{-1} (G).Thus X- f^{-1} (G) is α^m closed in X and so f^{-1} (G) is α^m open in X. Therefore (a) $\Rightarrow$(b).

Conversely assume that the inverse image of each open set in Y is α^m open in X.Let F be any closedset in Y.ThenF^c is open in Y.By assumption,f^{-1} (F^c) is α^m open in X.But f^{-1} (F^c) = X- f^{-1} (F).Thus X- f^{-1} (F) is α^m open in X and so f^{-1} (F) is α^m closed in X. Therefore f is α^m continuous.Hence (b) $\Rightarrow$(a).Thus (a) and (b) are equivalent.

(II)Assume that f is α^m continuous.Let A be any subset of X.Then $\overline{f(A)}$ is a closed set in Y.Since f is α^m continuous f $^{-1}(\overline{f(A)})$ is α^m closed in X and it contains A.But cl*(A) is the intersection of all α^m closed sets containing A.Therefore cl*(A)$\subset$ f $^{-1}(\overline{f(A)})$ and so f(cl*(A))$\subset$ $\overline{f(A)}$.

(III) (a) $\Rightarrow$(b) Let y$\in$ f(cl*(A)) and let V be any open neighbourhood of y.Then thereexist a point x $\in$ X and a α^m open set U suchthat f(x)=y, x $\in$ U, x$\in$ cl*(A) and f(U) $\subset$ V. Since x $\in$ cl*(A), U $\cap$ A $\neq \phi$ holds and hence $f(A)$$\cap$ V $\neq \phi$.Therefore we have y=f(x) $\in(\overline{f(A)})$.

(b) $\Rightarrow$ (a) Let x $\in$ X and V be any openset containing f(x). Let A=f^{-1} (V^c),then x $\notin$A. Now cl*(A) $\subset$ f^{-1} (f(cl*(A))) $\subset$ f^{-1} (V^c)=A. (ie)cl*(A) $\subset$ A. But A $\subset$ cl*(A).Therefore A=cl*(A). Then, since x $\notin$ cl*(A),thereexist a α^m open set U containing x suchthat U $\cap$A =ϕ and hence f(U) $\subset$ f(A^c) $\subset$V. □

Theorem 19. *Let f: $X \to Y$ be α^m continuous map from a topological space X into a topological space Y and let H be a closed subset of Y.Then the restriction of f/H :H $\to$ Y is α^m continuous where H is endowed with the relative topology.*

Proof. Let F be any closed subset in Y.Since f is α^m continuous, f^{-1} (F) is α^m closed in X.Levine has proved that intersection of a closed set and also we proved that intersection of α^m closed sets is α^m closed. Thus if f $^{-1}$ (F) $\cap$ H = H $_1$.then H $_1$ is a α^m closed in X. since(f/H)$^{-1}$(F)= H_1, it is sufficient to proved that H_1 is α^m closed in H.Let G be any open set of H suchthat $G_1 \supset H_1$. Let G_1 =G $\cap$ H where G is open in X. Now $H_1 \subset$ G $\cap$ H $\subset$ G.Since H_1 is α^m closed in X,$\overline{H_1} \subset$ G.Now cl$_H$ (H_1)=$\overline{H_1} \cap$ H $\subset$G $\cap$ H=G_1,where cl$_H$(A) is the closure of a subset A (CH) in a subspace H of X.Therefore f/H is α^m continuous.

□

Theorem 20. *Let α=A $\cup$ B be a topological space with topology τ and Y be topological space with topology σ. Let f:(A, τ /A) $\rightarrow$ (Y,σ) and g:(B, τ/B) $\rightarrow$ (Y,σ) be α^m continuous maps suchthat f(x)=g(x) for every x $\in$ A $\cap$ B.Suppose that A and B are α^m closed sets in X.Then the combination α: (X, τ) $\rightarrow$ (Y,σ) α^m continuous .*

Proof. Let F be any closed set in Y. Clearly α^{-1}(F) = f^{-1} (F) $\cup$ g^{-1} (F) = C $\cup$ D where C= f^{-1} (F) and D = g^{-1} (F).But C is α^m closed in A and A is α^m closed in X and so C is α^m closed set in X;since we have proved that if B $\subset$ A $\subset$ X, B is α^m closed in A and A is α^m closed in X then B is α^m closed in X.Similarly D is α^m closed in X. Also C $\cup$ D is α^m closed in X.Therefore α^{-1}(F) is α^m closed in X. Hence α is α^m continuous.

□

4. α^m-Irresolute

Definition 21. A map f: $X \rightarrow Y$ from a topological space X into a topological space Y is called α^m - irresolute if the inverse image of every α^m closed set in Y is α^m closed set in X.

Theorem 22. *Let f: $X \rightarrow Y$,g: $Y \rightarrow Z$ be two maps.Then their composition gof:$X \rightarrow Z$ is α^m - continuous if f is α^m - irresolute and g is α^m - continuous.*

Proof. Let V be an open set in Z.Then (gof)$^{-1}$ (V)=f^{-1}(g^{-1}(V))= f^{-1}(U) where U= g^{-1}(V) is α^m open in Y as g is α^m -continuous.Since f is α^m - irresolute, f^{-1}(U) is α^m open in X.Thus gof is α^m continuous. □

Theorem 23. *Let f: $X \rightarrow Y$,g: $Y \rightarrow Z$ be two α^m -irresolute maps.Then their composition gof:$X \rightarrow Z$ is α^m- irresolute map.*

Proof. Let V be an α^m- open set in Z.Consider $(\text{gof})^{-1}$ (v)= f^{-1}(v) where U= g^{-1}(v) is α^m open in Y as g is α^m -irresolute.Since f is α^m -irresolute, f^{-1}(U) is α^m open in X.Thus gof is α^m irresolute map. □

Theorem 24. *Let f: $X \to Y$, g: $Y \to Z$ be two maps such that gof:$X \to Z$ is α^m closed map.Then(i) if f is continuous and surjective then g is α^m closed and (ii) if g is irresolute and injective then f is α^m closed.*

Proof. (i)Let H be closed set in Y.Since f^{-1}(H) is closed in X. (gof)(f^{-1}(H)) is α^m closed set in Z and hence g(H) is α^m closed in Z.Thus g is α^m closed.
(ii)Let F be closed set of X.Then (gof)(F) is α^m closed in Z and g^{-1}(gof)(F) is α^m closed in Y.Since g is injective f(F)= g^{-1}(gof)(F) is α^m closed in Y.Then f is α^m closed. □

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