International Journal of Pure and Applied Mathematics
Volume 106 No. 8 2016, 149-160
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
doi: 10.12732/ijpam.v106i8.18

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SOLVING INTUITIONISTIC FUZZY MULTI-OBJECTIVE LINEAR PROGRAMMING PROBLEMS USING RANKING FUNCTION

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Abstract: The concept of Fuzzy Numbers has been enhanced in many decision making problems of engineering optimization. Ranking of Fuzzy Numbers is one of the techniques that conceptualize Fuzzy Numbers to demonstrate the order of preference in decision making. This paper focuses on modified Maleki's and Yager's ranking functions on trapezoidal intuitionistic fuzzy numbers (TIFNS) to solve multi-objective intuitionistic fuzzy linear programming problem (MOIFLPP) in which both the coefficients of objective functions as well as the right-hand side are trapezoidal intuitionistic fuzzy numbers. Finally some illustrative examples for different cases with various states are given to check the effectiveness of the modified ranking functions.

Key Words: trapezoidal intuitionistic fuzzy number (TIFN), modified Maleki's ranking function, modified Yager's ranking function

1. Introduction

A lot of application problems can be modeled as mathematical problem which may be formulated with uncertainty. The concept of fuzzy multi-objective

Received: February 15, 2016
Published: July 23, 2016

linear programming was first proposed by Zimmerann [6]. Maleki [4] introduced fuzzy variables in linear programming problems and proposed a new method for solving these problems using ranking function.In this paper, we solving the intuitionistic fuzzy multi-objective linear programming problems, when the coefficients of two objective functions are intuitionistic fuzzy numbers, as well as right-hand side are intuitionistic fuzzy numbers too, and both the coefficients of objective function as well as right-hand side are intuitionistic fuzzy numbers by using Maleki linear ranking function when ($\alpha = \beta$) and $\alpha \neq \beta$ and using Yager linear ranking function when ($\alpha = \beta$) and $\alpha \neq \beta$. This paper is outlined as follows. In Section 2, we presented trapezoidal intuitionistic fuzzy numbers. In Section 3, we interested in ranking functions, but in Section 4, we study intuitionistic fuzzy multi-objective linear programming problems with fuzzy coefficients objective function and intuitionistic fuzzy right-hand side, then both of them. In Section 5, we illustrate all type of intuitionstic fuzzy multi-objective linear programming problems in numerical examples. Finally in Section 6 we make conclusion for this study.

2. Preliminaries

Definition 1. [1] Given a fixed set$X = x_1, x_2, \dots, x_n$, an intuitionistic fuzzy set (IFS) is defined as $\tilde{A}^I = (\langle x_i, \mu_{\tilde{A}^I}(x_i), \vartheta_{\tilde{A}^I}(x_i)\rangle / x_i \in X)$ which assigns to each element x_i, a membership degree $\mu_{\tilde{A}^I}(x_i)$ and a non-membership degree $\mu_{\tilde{A}^I}(x_i)$ under the condition $0 \leq \mu_{\tilde{A}^I}(x_i) + \nu_{\tilde{A}^I}(x_i) \leq 1$, for all $x_i \in X$

Definition 2 (Trapezoidal Intuitionistic Fuzzy Numbers)**.** There are various types of fuzzy numbers,in which the triangular and trapezoidal are the most important fuzzy memperships. In this research we use the trapezoidal intuitionistic fuzzy numbers. In fact the fuzzy number is defined by its corresponding membership function and non membership function as follows:

$$\mu_{\tilde{a}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} \end{cases}, \text{when } a_1 \leq x \leq a_2, \mu_{\tilde{a}}(x) = 1, \text{ when } a_2 \leq x \leq a_3$$

$$\mu_{\tilde{a}}(x) = \begin{cases} \dfrac{a_4 - x}{a_4 - a_3} \end{cases}, \text{when } a_3 \leq x \leq a_4, \mu_{\tilde{a}}(x) = 0, \text{ otherwise}$$

$$\nu_{\tilde{a}}(x) = \begin{cases} \dfrac{a_2 - x}{a_2 - a_1} \end{cases}, \text{when } a_1 \leq x \leq a_2, \nu_{\tilde{a}}(x) = 0, \text{ when } a_2 \leq x \leq a_3$$

$$\nu_{\tilde{a}}(x) = \begin{cases} \dfrac{x - a_3}{a_4 - a_3} \end{cases} \text{when } a_3 \leq x \leq a_4, \nu_{\tilde{a}}(x) = 1, \text{ otherwise}$$

3. Ranking Function

The ranking function is the approach of ordering fuzzy numbers. Various types of ranking function have been introduced which are used to solve multi objective linear programming problems with fuzzy parameters. The ranking function of $F(R)$ is as follows: $\tilde{a} \geq \tilde{b}$ then $R(\tilde{a}) \geq R(\tilde{b})$, if $\tilde{a} > \tilde{b}$ then $R(\tilde{a}) > R(\tilde{b})$, if $\tilde{a} = \tilde{b}$ then $R(\tilde{a}) = R(\tilde{b})$

3.1. Maleki Ranking Function

$\tilde{a} = (a^l, a^u, \alpha, \beta)$,be a fuzzy numbers, then the ranking function is

$$R(\tilde{a}) = \int_0^1 (\inf \tilde{a}_\lambda + \sup \tilde{a}_\lambda)\, d\lambda \quad \text{when } \alpha = \beta$$

$$R(\tilde{a}) = a^l + a^u + \frac{1}{2}(\beta - \alpha) \quad \text{when } \alpha \neq \beta$$

3.2. Yager Ranking Function

$\tilde{a} = (a^l, a^u, \alpha, \beta)$,be a fuzzy numbers, then the ranking function is

$$R(\tilde{a}) = \frac{1}{2}\left[\int_0^1 a^l - \alpha\overline{L}^l(\lambda)\, d\lambda + \int_0^1 a^u - \beta\overline{R}^l(\lambda)\, d\lambda\right], \quad \text{when } \alpha = \beta$$

$$R(\tilde{a}) = \frac{1}{2}\left[a^l + a^u - \frac{4}{5}\alpha + \frac{2}{3}\beta\right], \quad \text{when } \alpha \neq \beta$$

4. Intuitionistic Fuzzy Multi-Objective Linear Programming Problem

The crisp multiobjective linear programming problem is defined as follows:

$$\max z_1 = \sum_{j-1}^{n} \tilde{c}_j x_j, \max z_2 = \sum_{k-1}^{n} \tilde{c}_k x_k,$$

subject to

$$\sum_{j-1}^{k} a_{ij} x_j \leq b_i.$$

In this paper we study three states of intuitionistic fuzzy multi objective linear programming problem. The first state are intuitionistic fuzzy numbers in the objective function coefficients ,the second state are intuitionistic fuzzy numbers in the right hand side coefficients, the third state are intuitionistic fuzzy numbers for both in the objective function coefficients and the right hand side coefficients.we have formulated the above three fuzzy multi objective linear programming problem states, as follows:

First State. In this section the objective function coefficients as trapezoidal intuitionistic fuzzy numbers which as

$$\max z_1 = \sum_{j=1}^{n} \tilde{c}_j x_j, \max z_2 = \sum_{k=1}^{n} \tilde{c}_k x_k \text{ subject to } \sum_{j=1}^{n} \tilde{a}_{ij} x_j \leq b_i$$

where c_j fuzzy coefficients of objective function, then we solve the fuzzy multi-objective linear programming by using two ranking function:

Maleki ranking function:

$$\max z = \sum_{j=1}^{n} \left[c_j^l + c_j^u + \frac{1}{2}(\beta - \alpha) \right] x_j, \text{subject to} \sum_{j=1}^{n} a_{ij} x_j \leq b_i.$$

Yager ranking function:

$$\max z = \sum_{j-1}^{n} \frac{1}{2} \left[c_j^l + c_j^u - \frac{4}{5}\alpha + \frac{2}{3}\beta \right] x_j, \text{ subject to } \sum_{j=1}^{n} a_{ij} x_j \leq b_i.$$

Second State. In this state, we make the right hand side coefficients as intuitionistic trapezoidal fuzzy numbers which as

$$\max z_1 = \sum_{j=1}^{n} c_j x_j, \max z_2 = \sum_{k=1}^{n} c_k x_k \text{ subject to } \sum_{j=1}^{n} a_{ij} x_j \leq \tilde{b}_i.$$

Multi objective linear programming problem when using maleki ranking function:

$$\sum_{j=1}^{n} a_{ij} x_j \leq \left[b_j^l + b_j^u + \frac{1}{2}(\beta - \alpha) \right]$$

Multi objective linear programming problem when using Yager ranking function:

$$\max z_1 = \sum_{j=1}^{n} c_j x_j,\ \max z_2 = \sum_{k=1}^{n} c_k x_k,$$

subject to

$$\sum_{j=1}^{n} a_{ij} \leq \frac{1}{2}\left[b_j^l + b_j^u - \frac{4}{5}\alpha + \frac{2}{3}\beta\right].$$

Third State. In this state, we make both the objective function coefficients and right hand side as trapezoidal intuitionistic fuzzy numbers which as

$$\max z_1 = \sum_{j=1}^{n} \tilde{c}_j x_j, \max z_2 = \sum_{k=1}^{n} \tilde{c}_k x_k \text{ subject to } \sum_{j=1}^{n} \tilde{a}_{ij} x_j \leq b_i$$

Multi objective linear programming problem when using Maleki ranking function

$$\sum_{j=1}^{n} a_{ij} \leq \frac{1}{2}\left[b_j^l + b_j^u - \frac{4}{5}\alpha + \frac{2}{3}\beta\right] x_i,$$

subject to

$$\sum_{j=1}^{n} a_{ij} \leq \frac{1}{2}\left[b_j^l + b_j^u - \frac{4}{5}\alpha + \frac{2}{3}\beta\right].$$

Multi objective linear programming problem when using Yager ranking function

$$\max z = \sum_{j=1}^{n} \frac{1}{2}\left[c_j^l + c_j^u - \frac{4}{5}\alpha + \frac{2}{3}\beta\right] x_j,$$

subject to

$$\sum_{j=1}^{n} a_{ij} \leq \frac{1}{2}\left[b_j^l + b_j^u - \frac{4}{5}\alpha + \frac{2}{3}\beta\right].$$

5. Numerical Examples

In this section we discussed all states by the following examples which is suggested by the researchers in case of $\alpha = \beta$ or $\alpha \neq \beta$.

Example 1. The following example is suggested by the researchers in case of $\alpha = \beta$.

(a)

$$\begin{aligned}\max \tilde{Z}_1 &= (4,2,3,3)(1,5,2,2)\tilde{x_1} + (1,3,2,2)(3,1,2,2)\tilde{x_2} \\ &+ (1,1,3,3)(1,1,2,2)\tilde{x}_3, \\ \max \tilde{Z}_2 &= (1,2,2,2)(2,1,3,3)\tilde{x}_1 + (3,2,2,2)(4,1,2,2)\tilde{x}_2 \\ &+ (2,2,3,3)(3,1,2,2)\tilde{x_3}\end{aligned}$$

subject to

$$2\tilde{x}_1 - \tilde{x}_2 + 2\tilde{x}_3 \leq 4,\ \tilde{x}_1 + 4\tilde{x}_3 \leq 4,\ \tilde{x}_1 + 3\tilde{x}_2 + 2\tilde{x_3} \leq 7,\ \tilde{x}_1, \tilde{x}_2, \tilde{x}_3 \geq 0$$

By using Maleki ranking function we get

$$\max z_1 = 6x_1 + 4x_2 + 2x_3,\ \max z_2 = 3x_1 + 5x_2 + 4x_3,$$

subject to

$$2x_1 - x_2 + 2x_3 \leq 4, x_1 + 4x_3 \leq 4, x_1 + 3x_2 + 2x_3 \leq 7$$

We solve the above crisp linear programming by using through TORA software program we get the following solution, $x_1 = 2.71, x_2 = 1.43,\ x_3 = 0, z = 22$.

The another objective function is

$$\max z_2 = 3x_1 + 5x_2 + 4x_3,$$

subject to

$$\begin{aligned}&2x_1 - x_2 + 2x_3 \leq 4, x_1 + 4x_3 \leq 4, \\ &x_1 + 3x_2 + 2x_3 \leq 7, 2.71x_1 + 1.43x_2 \leq 22\end{aligned}$$

By using simplex method, the solution is $x_1 = 2.71, x_2 = 1.43, x_3 = 0, z = 15.29$.

If each stage of preemptive optimization yields a single objective optimum, the final solution is an efficient point of the full multi-objective model.

By using Yager ranking function we get

$$\max z_1 = 2.8x_1 + 1.9x_2 + 0.8x_3,$$

subject to

$$2x_1 - x_2 + 2x_3 \leq 4, x_1 + 4x_3 \leq 4, x_1 + 3x_2 + 2x_3 \leq 7$$

by using simplex method the solution is $x_1 = 2.71, x_2 = 1.43, x_3 = 0, z = 10.31$.

The another objective function is

$$\max z_2 = 1.37x_1 + 2.87x_2 + 1.8x_3,$$

subject to

$$2x_1 - x_2 + 2x_3 \leq 4, x_1 + 4x_3 \leq 4,$$
$$x_1 + 3x_2 + 2x_3 \leq 7, 2.71x_1 + 1.43x_2 \leq 10.31$$

On solving, the solutions is $x_1 = 2.71, x_2 = 1.43, x_3 = 0, z = 7.82$.

(b)

$$\max \tilde{Z}_1 = 4\tilde{x}_1 + 3\tilde{x}_2 + 5\tilde{x}_3, \ \max = 3\tilde{x}_1 + 5\tilde{x}_2 + 9\tilde{x}_3,$$

subject to

$$2x_1 - x_2 + 2x_3 \leq (2,6,3,3)(3,5,2,2), x_1 + 4x_3 \leq (2,7,2,2), (4,5,3,3),$$
$$x_1 + 3x_2 + 2x_3 \leq (5,9,3,3)(6,8,2,2)$$

By using Maleki ranking function we get

$$2x_1 - x_2 + 2x_3 \leq 8, x_1 + 4x_3 \leq 9, x_1 + 3x_2 + 2x_3 \leq 14$$
$$\max z_1 = 4x_1 + 3x_2 + 5x_3,$$

subject to

$$2x_1 - x_2 + 2x_3 \leq 8, x_1 + 4x_3 \leq 9, x_1 + 3x_2 + 2x_3 \leq 14$$

By using simplex method the solution is $x_1 = 5.43, x_2 = 2.86, x_3 = 0, z = 30.29$.

The another objective function is

$$\max z_2 = 3x_1 + 5x_2 + 9x_3,$$

subject to

$$2x_1 - x_2 + 2x_3 \leq 8, x_1 + 4x_3 \leq 9,$$
$$x_1 + 3x_2 + 2x_3 \leq 14, 5.43x_1 + 2.86x_2 \leq 30.29$$

The solution is $x_1 = 0, x_2 = 3.17, x_3 = 2.25, z = 36.08$.
If each stage of preemptive optimization yields a single objective optimum, the final solution is an efficient point of the full multi-objective model.

By using Yager ranking function we get

$$\max z_1 = 4x_1 + 3x_2 + 5x_3,$$

subject to,

$$2x_1 - x_2 + 2x_3 \leq 3.8, x_1 + 4x_3 \leq 3.3, x_1 + 3x_2 + 2x_3 \leq 6.8$$

by using simplex method, the solution is, $x_1 = 2.60, x_2 = 1.40, x_3 = 0, z = 14.60$.

$$\max z_2 = 3x_1 + 5x_2 + 9x_3,$$

subject to

$2x_1 - x_2 + 2x_3 \leq 3.8, x_1 + 4x_3 \leq 3.3,$
$x_1 + 3x_2 + 2x_3 \leq 6.8, 2.60x_1 + 1.40x_2 \leq 14.60$

by using simplex method the solution is, $x_1 = 0, x_2 = 1.72, x_3 = 0.83, z = 16.01$.

(c)

$$\max \tilde{Z}_1 = (4,2,3,3)(1,5,2,2)\tilde{x}_1 + (1,3,2,2)(3,1,2,2)\tilde{x}_2 + (1,1,3,3)(1,1,2,2)\tilde{x}_3,$$
$$\max \tilde{Z}_2 = (1,2,2,2)(2,1,3,3)\tilde{x}_1 + (3,2,2,2)(4,1,2,2)\tilde{x}_2 + (2,2,3,3)(3,1,2,2)\tilde{x}_3$$

subject to

$2x_1 - x_2 + 2x_3 \leq (2,6,3,3)(3,5,2,2), x_1 + 4x_3 \leq (2,7,2,2),(4,5,3,3),$
$x_1 + 3x_2 + 2x_3 \leq (5,9,3,3)(6,8,2,2), \quad \tilde{x}_1, \tilde{x}_2, \tilde{x}_3 \geq 0$

By using Maleki ranking function

$$\max z_1 = 6x_1 + 4x_2 + 2x_3, \max z_2 = 3x_1 + 5x_2 + 4x_3$$

subject to

$2x_1 - x_2 + 2x_3 \leq 8, x_1 + 4x_3 \leq 9, x_1 + 3x_2 + 2x_3 \leq 14$

by using simplex method the solution is $x_1 = 5.43, x_2 = 2.86, x_3 = 0, z = 44$.
The another objective function is

$$\max z_2 = 3x_1 + 5x_2 + 4x_3$$

subject to

$2x_1 - x_2 + 2x_3 \leq 8, x_1 + 4x_3 \leq 9,$
$x_1 + 3x_2 + 2x_3 \leq 14, 5.43x_1 + 2.86x_2 \leq 44$

by using simplex method $x_1 = 5.43, x_2 = 2.86, x_3 = 0, z = 30.57$.
By using Yager ranking function we get

$$\max z_1 = 2.8x_1 + 1.9x_2 + 0.8x_3$$

subject to

$2x_1 - x_2 + 2x_3 \leq 3.8, x_1 + 4x_3 \leq 3.3, x_1 + 3x_2 + 2x_3 \leq 6.8$

by using simplex method $x_1 = 2.60, x_2 = 1.40, x_3 = 0, z = 9.94$.
The another objective function is

$$\max z_2 = 1.37x_1 + 2.87x_2 + 1.8x_3,$$

subject to

$2x_1 - x_2 + 2x_3 \leq 3.8, x_1 + 4x_3 \leq 3.3,$

$x_1 + 3x_2 + 2x_3 \leq 6.82, 60x_1 + 1.40x_2 \leq 9.94$

the solution is $x_1 = 2.60, x_2 = 1.40, x_3 = 0, z = 7.58$.

Example 2. The following example is suggested by the researchers in case of $\alpha \neq \beta$.

(a)

$$\begin{aligned}\max \tilde{Z}_1 &= (4,2,3,1)(5,1,2,3)\tilde{x}_1 + (1,3,2,1)(2,2,3,1)\tilde{x}_2 \\ &+ (1,1,2,3)(1,1,3,2)\tilde{x}_3 \\ \max &= (1,2,1,2)(2,1,3,1)\tilde{x}_1 + (3,2,1,3)(4,1,2,1)\tilde{x}_2 \\ &+ (2,2,1,3)(3,1,3,2)\tilde{x}_3\end{aligned}$$

subject to

$2\tilde{x}_1 - \tilde{x}_2 + 2\tilde{x}_3 \leq 4, \tilde{x}_1 + 4\tilde{x}_3 \leq 4, \tilde{x}_1 + 3\tilde{x}_2 + 2\tilde{x}_3 \leq 7,\ \tilde{x}_1, \tilde{x}_2, \tilde{x}_3 \geq 0$

Solution : By using Maleki ranking function we get

$$\max z_1 = 5x_1 + 3.5x_2 + 2.5x_3, \max z_2 = 3.5x_1 + 6x_2 + 5x_3$$

subject to

$2x_1 - x_2 + 2x_3 \leq 4, x_1 + 4x_3 \leq 4, x_1 + 3x_2 + 2x_3 \leq 7$

The solution is $x_1 = 2.71, x_2 = 1.43, x_3 = 0, z = 18.57$.

The another objective function is

$$\max z_2 = 3.5x_1 + 6x_2 + 5x_3$$

subject to

$2x_1 - x_2 + 2x_3 \leq 4, x_1 + 4x_3 \leq 4,$

$x_1 + 3x_2 + 2x_3 \leq 7, 2.71x_1 + 1.43x_2 \leq 18.57$

The solution is $x_1 = 2.71, x_2 = 1.43, x_3 = 0, z = 18.07$.

By using Yager ranking function we get

$$\max z_1 = 2.13x_1 + 1.53x_2 + 1.2x_3,$$

subject to

$2x_1 - x_2 + 2x_3 \leq 4, x_1 + 4x_3 \leq 4, x_1 + 3x_2 + 2x_3 \leq 7$

The solution is $x_1 = 2.71, x_2 = 1.43, x_3 = 0, z = 7.97$.

The another objective function is

$$\max z_2 = 1.76x_1 + 3.1x_2 + 2.6x_3$$

subject to

$2x_1 - x_2 + 2x_3 \leq 4, x_1 + 4x_3 \leq 4,$

$x_1 + 3x_2 + 2x_3 \leq 7, 2.71x_1 + 1.43x_2 \leq 7.97$

The solution is $x_1 = 2.25, x_2 = 1.31, x_3 = 0.4, z = 9.08$.

(b)

$$\max \tilde{Z}_1 = 4\tilde{x}_1 + 3\tilde{x}_2 + 5\tilde{x}_3, \text{ max} = 3\tilde{x}_1 + 5\tilde{x}_2 + 9\tilde{x}_3$$

subject to

$2x_1 - x_2 + 2x_3 \leq (2,6,1,3)(3,5,2,3), x_1 + 4x_3 \leq (2,7,2,3)(1,8,3,2),$

$x_1 + 3x_2 + 2x_3 \leq (5,9,1,2)(6,8,2,3)$

By using Maleki ranking function we get

$$\max z_1 = 4x_1 + 3x_2 + 5x_3$$

Subject to

$2x_1 - x_2 + 2x_3 \leq 9, x_1 + 4x_3 \leq 9.5, x_1 + 3x_2 + 2x_3 \leq 14.5$

The solution is $x_1 = 5.93, x_2 = 2.86, x_3 = 0, z = 32.29$.

The another objective function is

$$\max z_2 = 3x_1 + 5x_2 + 9x_3$$

subject to

$2x_1 - x_2 + 2x_3 \leq 9, x_1 + 4x_3 \leq 9.5,$

$x_1 + 3x_2 + 2x_3 \leq 14.5, 5.93x_1 + 2.86x_2 \leq 32.29$

The solution is $x_1 = 0, x_2 = 3.25, x_3 = 2.38, z = 37.63$.
If each stage of preemptive optimization yields a single objective optimum, the final solution is an efficient point of the full multi-objective model.

By using Yager ranking function we get

$$\max z_1 = 4x_1 + 3x_2 + 5x_3,$$

subject to

$2x_1 - x_2 + 2x_3 \leq 4.6, x_1 + 4x_3 \leq 4.7, x_1 + 3x_2 + 2x_3$

by using simplex method the solution is $x_1 = 3.01, x_2 = 1.42, x_3 = 0, z = 16.29$.

$$\max z_2 = 3x_1 + 5x_2 + 9x_3$$

subject to

$2x_1 - x_2 + 2x_3 \leq 4.6, x_1 + 4x_3 \leq 4.7,$

$x_1 + 3x_2 + 2x_3 \leq 7.26, 3.01x_1 + 1.42x_2 \leq 16.29$

The solutions are $x_1 = 0, x_2 = 1.64, x_3 = 1.18, z = 18.76$.

(a)

$$\max \tilde{Z}_1 = (4,2,3,1)(5,1,2,3)\tilde{x}_1 + (1,3,2,1)(2,2,3,1)\tilde{x}_2$$

$$+ (1,1,2,3)(1,1,3,2)\tilde{x}_3$$

$$\max \tilde{Z}_2 = (1,2,1,2)(2,1,3,1)\tilde{x}_1 + (3,2,1,3)(4,1,2,1)\tilde{x}_2$$
$$+ (2,2,1,3)(3,1,3,2)$$

subject to

$$2x_1 - x_2 + 2x_3 \leq (2,6,1,3)(3,5,2,3), x_1 + 4x_3 \leq (2,7,2,3)(1,8,3,2),$$
$$x_1 + 3x_2 + 2x_3 \leq (5,9,1,2)(6,8,2,3), \qquad \tilde{x}_1, \tilde{x}_2, \tilde{x_3} \geq 0$$

By using Maleki ranking function

$$\max z_1 = 5x_1 + 3.5x_2 + 2.5x_3, \max z_2 = 3.5x_1 + 6x_2 + 5x_3,$$

subject to

$$2x_1 - x_2 + 2x_3 \leq 9, x_1 + 4x_3 \leq 9.5, x_1 + 3x_2 + 2x_3 \leq 1$$

The solution is $x_1 = 5.93, x_2 = 2.86, x_3 = 0, z = 34.01$.

The another objective function is

$$\max z_2 = 3.5x_1 + 6x_2 + 5x_3,$$

subject to

$$2x_1 - x_2 + 2x_3 \leq 9, x_1 + 4x_3 \leq 9.5,$$
$$x_1 + 3x_2 + 2x_3 \leq 14.5, 5.43x_1 + 2.86x_2 \leq 34.01$$

The solution is $x_1 = 4.53, x_2 = 2.51, x_3 = 1.23, z = 37.02$.

By using Yager ranking function we get

$$\max z_1 = 2.13x_1 + 1.53x_2 + 1.2x_3$$

subject to

$$2x_1 - x_2 + 2x_3 \leq 4.6, x_1 + 4x_3 \leq 4.7, x_1 + 3x_2 + 2x_3 \leq 7.26$$

The solution is $x_1 = 3.01, x_2 = 1.42, x_3 = 0, z = 8.58$.

The another objective function is

$$\max z_2 = 1.76x_1 + 3.1x_2 + 2.6x_3$$

subject to

$$2x_1 - x_2 + 2x_3 \leq 4.6, x_1 + 4x_3 \leq 4.7,$$
$$x_1 + 3x_2 + 2x_3 \leq 7.26, 3.01x_1 + 1.42x_2 \leq 8.5$$

The solution is $x_1 = 2.26, x_2 = 1.26, x_3 = 0.61, z = 9.47$.

6. Conclusion

The coefficients of the objective function and the right-hand side with intuitionstic fuzzy numbers are ranked with two special ranking functions for Maleki and Yager linear ranking function. For all twelve states which is studied in the paper, with $\alpha = \beta$ and $\alpha \neq \beta$.

When the objective function coefficients are intuitionstic fuzzy numbers the preferable solution is $x_1 = 2.71, x_2 = 1.43, x_3 = 0, z = 15.29$.

When the right hand side coefficients are intuitionstic fuzzy numbers the preferable solution is $x_1 = 0, x_2 = 3.25, x_3 = 2.38, z = 37.63$.

When the objective function coefficients and right hand side coefficients are intuitionstic fuzzy numbers the preferable solution is $x_1 = 4.53, x_2 = 2.51, x_3 = 1.23, z = 37.02$.

References

[1] Atanassov.K.T (1986), Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, 20, 87-96.

[2] Atanassov.K.T (1989), More on intuitionistic fuzzy sets, *Fuzzy sets and systems*, 33, 37-46.

[3] Bellman,R.E. and Zadeh, L.A., "Decision making in a fuzzy environment", *Management Science*, 17, 141-164,1970.

[4] Maleki, H.R. 2002, Ranking functions and their applications to fuzzy linear programming, *Far East Journal Mathematics Sciences*, 4(2002),pp: 283-301.

[5] Yager,R,R.1981, A procedure for ordering fuzzy subsets of the unit interval, *Information Sciences*, vol.24, no.2, pp:143-161.

[6] Zimmerman H. J., Fuzzy Programming and Linear Programming with Several Objective Functions, *Fuzzy Sets and Systems*, Vol. 1, 1978, 45-55.