International Journal of Pure and Applied Mathematics
Volume 106 No. 8 2016, 123-135
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
doi: 10.12732/ijpam.v106i8.16

ijpam.eu

THE ANNIHILATOR DOMINATION IN SOME STANDARD GRAPHS AND ARITHMETIC GRAPHS

Venkata Suryanarayana Rao Kavaturi[1], Srinivasan Vangipuram[2]
[1]Rajeev Gandhi Memorial College of Engineering and Technology
Nandyal, Andhra Pradesh, 518501, INDIA
[2]Sri Venkateswara University
Tirupathi, Andhra Pradesh, 517501, INDIA

Abstract: The paper concentrates on the theory of domination in graphs. The split domination in graphs was introduced by Kulli and Janakiram. In this paper, we define a new parameter on domination called the Annihilator dominating set and Annihilator dominating number and we have investigated some properties of the Annihilator domination number of some Standard graphs and Arithmetic graphs and obtained several interesting results.

Key Words: domination, annihilator domination set, annihilator domination number, standard graphs, arithmetic graphs

1. Introduction

Graph theory is one of the most thriving branches of modern mathematics. The last 40 years have withnessed wondeful growth of Graph theory due to its wide applications to descrete optimization problems, combinatorial problems and classical algebraic problems.It has a very wide range of applications to many fields like engineering, physical, social and biological sciences, linguistics etc., The theory of domination has been the nucleus of research activity in graph theory in recent times. This is largely due to a multiplicity of new parameters that can be developed from the basic definition of domination. The NP-completeness of the basic domination problems and its close relationship

Received: February 15, 2016
Published: June 3, 2016

to other NP-completeness problems has contributed to the enormous growth of research activity in domination theory.

In 1977 Cockayne and Hedetniemi made an interesting and extensive survey of the results know at that time about dominating sets in graphs. They have used the notation γ (G) for the domination number of a graph, which has become very popular since then.The survey paper of Cockayane and Hedetniemi has generated lot of interest in the study of domination in graphs. In a span of about twenty years after the survey, more than 1,200 research papers have been published on this topic, and the number of papers continued to be on the increase.Recent book on domination [2], has stimulated sufficient inspiration leading to the expansive growth of this field of study. Laskar and Walikar [5] developed various interesting results on domination related concepts in graph theory.

The split domination in graphs was introduced by Kulli & Janakiram [4]. Sampathkumar [6,7] obtained some interesting results on tensor product of graphs. Weichsel [12] obtained some interesting results on Kronecker product of graphs. Vasumathi & Vangipuram [8] and Vijayasaradhi & Vangipuram [9] and Suryanarayana Rao & Sreenivasan [10,11] obtained domination parameters of an arithmetic Graph and some product graphs and also they have obtained an elegant method for the construction of a arithmetic graph with the given domination parameter. Motivated by the study of domination and split domination, we define a new parameter on domination called the Annihilator dominating set and Annihilator dominating number and we have investigated some properties of the Annihilator domination number of some standard graphs and Arithmetic graphs. The terminology and notations used in this paper are the same as in Bondy and Murty [1] and Harary. F.[3]

2. Annihilator Domination – Some Standard Graphs

In this section, we define a new parameter of domination which is Annihilator domination set and Annihilator domination number and obtained some interesting results on Annihilator dominating set and Annihilator domination number of certain standard graphs.

Definition 2.1. A dominating set D of a graph G is said to be an annihilator dominating set, if its induced subgraph <V- D> is a graph containing only isolated vertices.

The annihilator domination number γ_a(G) of G is the minimum cardinality

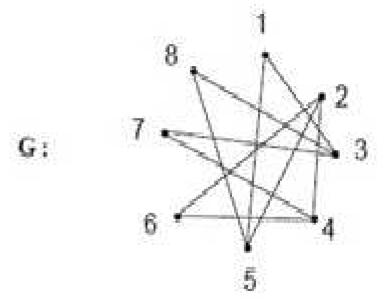

Figure 1: Graph of G. Annihilator dominating set $D = \{2, 3, 4, 5\}$.

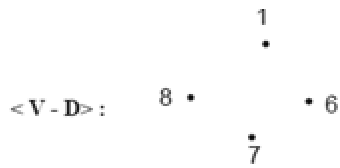

Figure 2: The induced sub graph < V-D >. The induced sub graph < **V-D** > of G is a graph with isolated vertices. Hence γ_a (G) =4.

of an annihilator dominating set.

We obtain several results on the annihilator dominating set and its relation with the other domination parameters.

From the definition, the following result is an immediate consequence.

Theorem 2.2. *In a graph G,* γ *(G)* $\leq \gamma_s$ *(G)* $\leq \gamma_a$ *(G).*

We have to obtain the annihilator domination number of some standard graphs.

Theorem 2.3. *If* $K_{m,n}$ *is a complete bipartite graph, with* $2 \leq m \leq n$*, then* $\gamma_a(K_{m,n}) = m$.

Proof. Let (X, Y) be the bi-partition of the graph $K_{m,n}$ with |X|= m and |Y|= n.

The removal of the m vertices in X will render the resulting graph <V-X> to be an independent set of n vertices only. It can be easily seen that the set X is an annihilator dominating set of minimum cardinality.

Therefore γ_a ($K_{m,n}$) = |X|= m.

Theorem 2.4. *If P_n is a path on n-vertices, then $\gamma_a(P_n)$ = [n / 2], where [X] represents the greatest integer less than or equal to X.*

Proof. The removal of the alternate vertices in the path P_n will result in the induced sub graph of the remaining vertices to be a set of isolated vertices.

Hence $\gamma_a(P_n)$ = [n / 2].

Theorem 2.5. *If $S_n$is a star on n-vertices, then γ_a (S_n) = 1.*

It is easily to deduce the following results.

Theorem 2.6. *If W_n is a wheel on n-vertices, Then*

$$W_n = \begin{cases} n/2 + 1, & \text{if } n \text{ is even,} \\ \frac{(n+1)}{2}, & \text{if } n \text{ is odd.} \end{cases}$$

Now we consider the following result on a cycle.

Theorem 2.7. *If C_n is a cycle on n-vertices, then γ_a (C_n) = $\lceil n/2 \rceil$, where $\lceil x \rceil$ represents the smallest integer $\geq$ x.*

Also, we mark the following result on a tree.

Theorem 2.8. *If T is a tree on n-vertices with p pendent vertices (p $\geq$ 3), then $\gamma_a(T) \leq n - p$.*

We will prove an interesting expression for annihilator domination number of a graph G, in terms of split domination number and domination number.

Theorem 2.9. *For any graph G, $\gamma_a(G) \geq \gamma_s(G) + \sum_{i=1}^{t}\gamma\ (G_i)$, where G_i's are the components of <V-D_s>, D_s being the split dominating set of G of minimum cardinality.*

Proof. Let G be a graph with vertex set V (G) = { v_1, v_2 . . . v_n}.

If D_s is the split dominating set of minimum cardinality of G, then the removal of D_s vertices will result in the induced sub graph <V – D_s> to be a disconnected graph with at least two components.

Let G_1, G_2 . . . G_t be the nontrivial components with atleast two vertices in the <V – D_s>.

Case 1. If each G_i is a component with exactly two vertices, Then it can be seen that, the split dominating set D_s together with the dominating sets of each of these components G_1, $G_2 \ldots G_t$ will be the annihilator dominating set of minimum cardinality.

Hence in this case

$$\gamma_a(G) = \gamma_s(G) + \sum_{i=1}^{t} \gamma(G_i).$$

Case 2. If some components "G_i" of $<V - D_s>$ have more than two vertices, then for the annihilator dominating set, we require the vertices of split dominating set of G, dominating sets of each components G_i and some more vertices whenever necessary in each component G_i.

Hence in this case

$$\gamma_a(G) \geq \gamma_s(G) + \sum_{i=1}^{t} \gamma(G_i).$$

Therefore combining the two cases we conclude that

$$\gamma_a(G) \geq \gamma_s(G) + \sum_{i=1}^{t} \gamma(G_i).$$

3. Some Graphical Examples

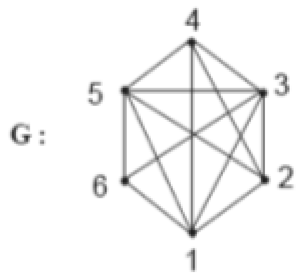

Figure 3: Graph of G. $V(G) = \{1,2,3,4,5,6\}$, $D_s(G) = \{1,3,5\}$, $V - D_s = \{2,4,6\}$.

Let G_1be the only non-trivial component of G joining the vertices 2,4. If D_a={1,2,3,5}, then

Figure 4: The induced Sub Graph of < V-D_a >.

Therefore D_a is an annihilator dominating set of G and

$$\gamma_a(G) = \gamma_s(G) + \gamma(G_1) = 3 + 1 = 4.$$

4. Annihilator Domination – Arithmetic Graphs

4.1. Arithmetic Graph

The arithmetic graph V_m is defined as a graph with its vertex set as the set of all divisors of m (excluding 1), where m is a natural number and $m = p_1^{a_1}\ p_2^{a_2}...p_r^{a_r}$, a canonical representation of m, where $p_i^{'}s$ are distinct primes and $a_i^{'}s{\geq}1$ and two distinct vertices a,b which are not of the same parity are adjacent in this graph if (a,b) = p_i,for 1= i = r.

The vertices a and b are said to be of the same parity if both a and b are the powers of the same prime, for instance a=p^2, b = p^5.

In this paper we obtain an expression for annihilator domination of the arithmetic graph.

Theorem 3.1. *If* $m = {p_1}^a{}_1.\ {p_2}^a{}_2$, *where* $p_{1,}p_2$ *are distinct primes, and* a_1, a_2 *are both* $\geq$ *1, then:*

(i) $\gamma_a\ [V_m] \leq 2a_1 + 1$, *if* $a_1 < a_2$.

(ii) $\gamma_a\ [V_m] \leq 2a_1$, *if* $a_1 = a_2$.

Proof. (i) If $a_1 < a_2$, we have the following proof.

The vertex set of V_m is

$$\{p_1, {p_1}^2, {p_1}^3 \ldots {p_1}^a{}_1, p_2, {p_2}^2, {p_2}^3 \ldots {p_2}^a{}_2,\\ p_1p_2, p_1{p_2}^2 \ldots p_1{p_2}^a{}_2, {p_1}^2p_2, {p_1}^2{p_2}^2 \ldots {p_1}^2{p_2}^a{}_2,\\ \ldots {p_1}^a{}_1p_2, {p_1}^a{}_1{p_2}^2, \ldots, {p_1}^a{}_1{p_2}^a{}_2\}.$$

We get $(a_1+1)(a_2+1) -1$ vertices: the set of vertices

$$D = \{p_1, {p_1}^2, \ldots, {p_1}^a{}_1, p_2, p_1p_2, {p_1}^2p_2, \ldots, {p_1}^a{}_1p_2\}$$

is an annihilator dominating set.

For, if v is any vertex in V – D, then v is of the form ${p_1}^i{}_{1.}\ {p_2}^i{}_2$, where $1 < i_2 < a_2$ and $0 \leq i_1 \leq a_1$.

We observe that if $i_1 > 1$ (then for all i_2), the vertex v is adjacent to p_1 in D and if $i_1 = 0$ (then for all values of i_2), the vertex v is adjacent with p_2 in D.

Thus D is a dominating set.

In V – D, any two vertices will be of the form ${p_1}^l{}_1{p_2}^l{}_2$, ${p_1}^m{}_1{p_2}^m{}_2$.

Case 1. If l_1, m_1 are both > 1, then since l_2, m_2 > 1.

We have $(p_1{}^{l}{}_1 p_2{}^{l}{}_2, p_1{}^{m}{}_1 p_2{}^{m}{}_2) = p_1{}^{n}{}_1 p_2{}^{n}{}_2$, where n_1, $n_2 > 1$.

Hence the vertices $p_1{}^{l}{}_1 p_2{}^{l}{}_2$, $p_1{}^{m}{}_1 p_2{}^{m}{}_2$ of V_m are not adjacent in this case.

Case 2. If $l_1 = 0$ and $m_1 > 0$, then since l_2, $m_2 > 1$.

We have $(p_1{}^{l}{}_1 p_2{}^{l}{}_2, p_1{}^{m}{}_1 p_2{}^{m}{}_2) = (p_2{}^{l}{}_2, p_1{}^{m}{}_1 p_2{}^{m}{}_2) = p_2{}^{c}{}_2$, where $c_2 > 1$.

Hence by definition of arithmetic graph 3.1.

The vertices $p_1{}^{l}{}_1 p_2{}^{l}{}_2$, $p_1{}^{m}{}_1 p_2{}^{m}{}_2$in V_m are not adjacent in this case.

Case 3. If $l_1 > 0$ and $m_1 = 0$, then since l_2, $m_2 > 1$.

We have $(p_1{}^{l}{}_1 p_2{}^{l}{}_2, p_1{}^{m}{}_1 p_2{}^{m}{}_2) = (p_1{}^{l}{}_1 p_2{}^{l}{}_2, p_2{}^{m}{}_2)= p_2{}^{b}{}_2$, where $b_2> 1$.

Hence by Definition 3.1, the vertices $p_1{}^{l}{}_1 p_2{}^{l}{}_2$, $p_1{}^{m}{}_1 p_2{}^{m}{}_2$are not adjacent in this case.

Case 4. If $l_1 = 0$ and $m_1 = 0$, then since l_2, $m_2 > 1$.

The two vertices $p_1{}^{l}{}_1 p_2{}^{l}{}_2$, $p_1{}^{m}{}_1 p_2{}^{m}{}_2$are reduced to $p_2{}^{l}{}_2$, $p_2{}^{m}{}_2$.

By definition 3.1, the vertices $p_1{}^{l}{}_1 p_2{}^{l}{}_2$, $p_1{}^{m}{}_1 p_2{}^{m}{}_2$in $V_m$are not adjacent.

Thus from all the above cases the vertices $p_1{}^{l}{}_1 p_2{}^{l}{}_2$, $p_1{}^{m}{}_1 p_2{}^{m}{}_2$are not adjacent in the induced sub graph < V- D>, so that D is an annihilator dominating set of V_m.

D is also a minimal annihilator dominating set.

Then D – {v} is not an annihilator dominating set.

If $v \in D =\{ p_1, p_1{}^2 \ldots p_1{}^{a}{}_1, p_2, p_1 p_2, p_1{}^2 p_2, \ldots, p_1{}^{a}{}_1 p_2\}$,

Then v is of the form $p_1{}^{i}{}_1 p_2{}^{i}{}_2$, where $1 \le i_1 \le a_1$, when $i_2 = 0$, $0 \le i_1 \le a_1$, when $i_2 = 1$.

Suppose $i_2 = 0$, then $1 \le i_1 \le a_1$, and the vertex $p_1{}^{i}{}_1 p_2{}^{i}{}_2$ becomes $p_1{}^{i}{}_1$. This vertex is adjacent to all the vertices of the form $p_1 p_2 b_2$, where $0 \le b_2 \le a_2$in the induced subgraph < V– (D–{v}) >.

If $i_2 = 0$, since $0 \le i_1 \le a_1$ when $i_2 = 1$, the vertex $v = p_1{}^{i}{}_1 p_2{}^{i}{}_2$ becomes $p_1 p_2{}^{i}{}_2$and this is adjacent to the vertices $p_2{}^2$, $p_2{}^3 \ldots\ldots p_2{}^{a}{}_2$ in <V–(D – {v})>.

Thus the set D – {v} is not an annihilator dominating set.

Hence D is a minimal annihilator dominating set.

Thus $\gamma_a [V_m] \le |D|= 2a_1 + 1$.

(ii) If $a_1 = a_2$, then $m = p_1{}^{a}{}_1 p_2{}^{a}{}_1$ and we have the following proof.

Let $D = \{ p_1, p_1{}^2 \ldots\ldots p_1{}^{a}{}_1, p_2, p_2{}^2, \ldots, p_1{}^{a}{}_1\}$.

This an annihilator dominating set of V_m.

For, any vertex in V–D is of the from $p_1{}^{i}{}_1 p_2{}^{i}{}_2$, where $1 \le \{i_1, i_2\} \le a_1$.

This vertex is adjacent with p_1, p_2 in D. Thus D is a dominating set.

Moreover, if u. v are any two vertices in V – D, then u, v will be of the form

$u = p_1{}^{i}{}_1 p_2{}^{i}{}_2$, $v = p_1{}^{j}{}_1 p_2{}^{j}{}_2$ where $1 \leq \{i_1, i_2, j_1, j_2\} \leq a_1$.

But u and v are not adjacent in the induced sub graph $< V - D >$,

since $(p_1{}^{i}{}_1 p_2{}^{i}{}_2, p_1{}^{j}{}_1 p_2{}^{j}{}_2) = p_1{}^{b}{}_1 p_2{}^{b}{}_2$, where $b_1, b_2 \geq 1$.

Thus D is an annihilator dominating set.

Further it is an annihilator dominating set of minimal cardinality.

For, If we remove any vertex v in D then v is of the form $p_1{}^{i}{}_1$ or$p_2{}^{i}{}_2$,where $1 \leq \{i_1, i_2\} \leq a_1$.

If v is of the form $p_1{}^{i}{}_1$ for $1 \leq i_1 \leq a_1$, then v is adjacent with

$p_1\ p_2, p_1 p_2{}^2 \ldots p_1\ p_2{}^{a}{}_1$ in $<$ V-{D - {v}} $>$.

On the other hand if v is of the form $p_2{}^{i}{}_2$, for $1 \leq i_2 \leq a_2$,

then v is adjacent with $p_1 p_2, p_1{}^2\ p_2 \ldots\ldots p_1{}^{a}{}_1 p_2$ in $<$ V-{D-{v}}$>$.

Then D–{ v} is not an annihilator dominating set.

So D is a minimal annihilator dominating set.

Hence $\gamma_a[V_m] \leq |D| = 2a_1$.

5. Construction of a Graph whose Annihilator Domination Number Does Not Exceed a Given Number *n*

If there is a real life situation where there is a specified number of a collection of pesticides which can destroy a specified varieties of pests, then the network of pests in which these pesticides can be used so as to destroy the pests and isolate the remaining varieties of the pests in order to control the damage can be done by using the following construction.

This is equivalent to the construction of a graph which has a given number as the cardinality of an annihilator dominating set of the graph.

For this construction we proceed as follows:

Case I. If n is even; Choose $m = p_1{}^{n/2}.\ p_2{}^{n/2}$, where p_1, p_2 are two distinct primes.

By the above Theorem 5.1, the graph V_m has a minimal annihilator dominating set $D = \{ p_1, p_1{}^2 \ldots\ldots p_1{}^{n/2}, p_2, p_2{}^2, \ldots, p_2{}^{n/2}\}$ which is of cardinality n.

Case II. If n is odd; choose $m = p_1{}^{(n-1)/2}\ p_2{}^{a}{}_2$,where $a_2 > \frac{(n-1)}{2}$and p_1, p_2 are distinct primes.

Then, by the previous theorem 5.1, the graph V_m has a minimal annihilator dominating set,

$$D = \{p_1, {p_1}^2 \ldots\ldots {p_1}^{(n-1)/2}, p_2, p_1p_2, {p_1}^2p_2, ..., {p_1}^{(n-1)/2}p_2\}$$

which is of cardinality n.

Thus the tools of number theory enable us to develop a simple method of constructing a graph with a given cardinality of the annihilator dominating set with amazing ease.

Illustration

Given n = 5 (odd); choose any two primes P_1, P_2 and let $m = {P_1}^2{P_2}^3$ (with a_1=2,a_2=3 and $a_1 < a_2$).

The vertices of V_m are the divisors of m (except 1):

$$P_1, P_2, {P_1}^2, {P_2}^2, {P_2}^3, P_1P_2, P_1{P_2}^2, P_1{P_2}^3, {P_1}^2P_2, {P_1}^2{P_2}^2, {P_1}^2{P_2}^3.$$

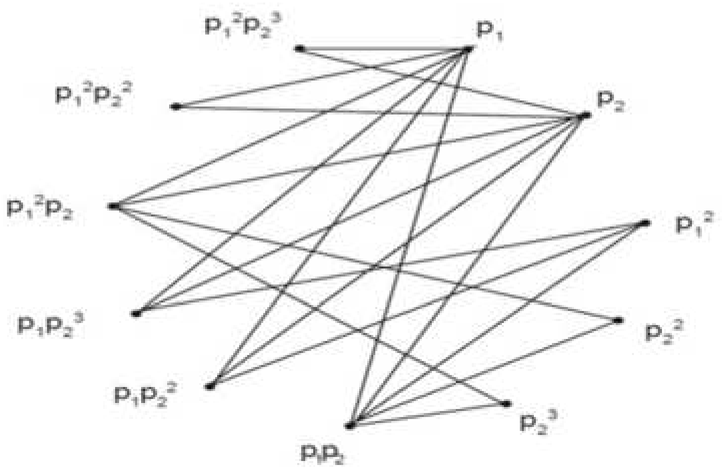

Figure 5: The graph of V_m with m= ${P_1}^2{P_2}^3$.

The annihilator dominating set

$$D = \{P_1, {P_1}^2, P_2, P_1P_2, {P_1}^2P_2\},$$

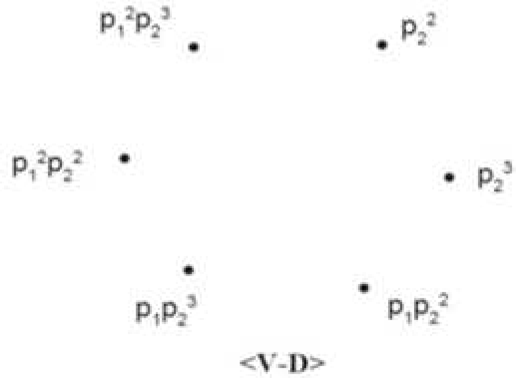

Figure 6: The induced sub graph $< V - D >$.

$jDj = 5$.

Hence γ_a $[V_m] \leq |D| = 2a_1 + 1 = 2x2 + 1 = 5$.

Given n = 4 (even); choose any two primes P_1, P_2 and let

$$m = {P_1}^2{P_2}^2$$

(with a_1=2,a_2=2 and $a_1 = a_2$)

The vertices of V_m are the divisors of m (except 1):

$$P_1, P_2, {P_1}^2, {P_2}^2, P_1P_2, P_1{P_2}^2, {P_1}^2P_2, {P_1}^2{P_2}^2.$$

The annihilator dominating set

$$D = \{P_{1,}{P_1}^2, P_2, {P_2}^2\},$$

$jDj = 4$.

Hence γ_a $[V_m] \leq |D| = 2a_1 = 2x2 = 4$.

6. Conclusion

This concept has very wide applications in real life situations. We list below two important applications among the several applications of annihilator dominating set.

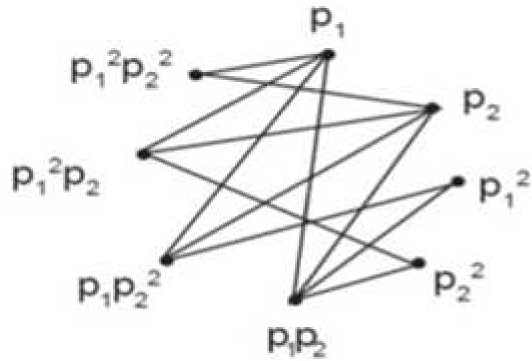

Figure 7: The graph of V_m with m= $P_1{}^2P_2{}^3$.

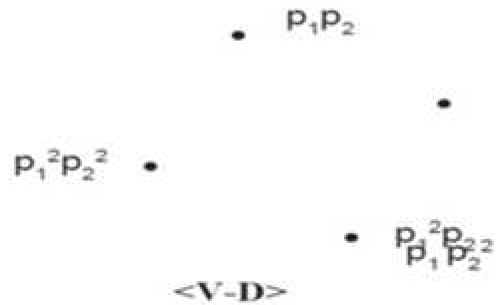

Figure 8: The induced sub graph $< V - D >$.

In the field of agriculture, control of pests plays a major role. Several pests interact among themselves, resulting in the large scale production of pests causing wide spread damage to the agriculture products. The one reasonable solution to this problem will be to isolate the pests, so that the interaction among the members of pest complex is prevented and it is easy for the agriculturists to eliminate the isolated types of the pests. We identified the network of the several types of pests as a graph with a vertex in the graph being specific types of the pest and the other pests with which this type will interact denoting the adjacency of the graph. We have to identify an annihilator dominating set D in this graph so that the induced sub graph <V-D> is a set of isolated vertices.

In requirements of the Department of Defense, it is observed that the strength of a unit in operation depends on the strength of the interaction of the several camps for the supply of essential armaments, weapons and essential commodities. The problem for the enemy side will be to eliminate certain posts or camps resulting in the collapse of the channels of interaction among the camps. In Graph Theoretic model, the network of the camps of the unit represents a Graph with its vertex set as the set of camps and the channels with which each camp interacts is the adjacency of a vertex with the vertices in the graph. The problem is to identify a set of vertices D in the graph, so that the removal of the vertex set D should help in the resulting graph of the induced sub graph <V-D> to be a graph with a set of isolated vertices only.

References

[1] Bondy and Murty, *Graph Theory with Applications*, Macmillan (1976).

[2] T.W. Haynes, S.T. Hedetniemi, P.J. Slater, *Fundamentals of Domination in Graphs*, Marcel Dekkar, Inc-New York (1998).

[3] F. Harary, *Graph Theory*, Addison-Wesley, Massachusetts (1969).

[4] V.R. Kulli, B. Janakiram, The split domination number of a graph, *Graph Theory Notes of New York*, New York Acadamy of Sciences, **XXXII**, 16-19 (1997).

[5] R.C. Laskar, H.B. Walikar, On domination related concepts in graph theory, *Lecture Notes in Match.*, **885** (1981), 308-320.

[6] E. Sampathkumar, On some new domination parameters of a graph – A Survey, In: *Proceedings of a Symposium on Graph Theory and Combinatorics*, Kochi, Kerala, India, 17-19 May 1991, 7-13.

[7] E. Sampathkumar, On tensor product graphs, *J. Austrial. Math. Soc., Series A*, **20** (1975), 268-273.

[8] N. Vasumathi, S. Vangipuram, Existence of a graph with a given domination parameter, In: *Proceedings of the Fourth Ramanujan Symposium on Algebra and its Applications*, University of Madras, Madras, 187-195 (1995).

[9] Vijaya Saradhi, Vangipuram, Irregular graphs, *Graph Theory Notes of New York*, **41** (2001), 33-36.

[10] K.V. Suryanarayana Rao, V. Sreeenivasan, The split domination in arithmetic graphs, *International Journal of Computer Applications*, **29**, No. 3 (2011).

[11] K.V. Suryanarayana Rao, V. Sreeenivasan, The split domination in product graphs, *I. J. Information Engineering and Electronic Business*, **4** (2013), 51-57.

[12] P.M. Weichsel, The Kronecker product of graphs, *Proc. Am. Math. Soc.*, **13** (1962), 47-52.