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SOLUTION OF FRACTIONAL INTEGRO DIFFERENTIAL SYSTEM WITH FUZZY INITIAL CONDITION

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Abstract: In this work, we analyze the method of finding the solution of fractional integro differential equations of the form

$$^{C}D^{\alpha}y(t) = ay(t) + \int_0^t K(s-t)y(t)\mathrm{d}t,$$

with fuzzy initial condition, where $^{C}D^{\alpha}$ is a Caputo fractional derivative. A numerical illustration is provided to explain the proposed theory.

AMS Subject Classification: 92D25, 70K50, 35B36

Key Words: fractional integrodifferential equations, fuzzy differential equations

1. Introduction

Differential equations of fractional order have been the focus of many studies due to their frequent appearance in various applications such as fluid mechanics, viscoelasticity, biology, physics and engineering and etc. (see [1, 2]). A survey of applications of the fractional calculus in various fields of science are discussed in [2].

Fuzzy concepts were introduced by Zadeh and it is enriched by several authors. Fuzzy integrodifferential equations (FIDEs) are used to model the system with uncertainty in dynamical environments. When a physical system is modeled under the differential sense; It finally gives a fuzzy differential equation or

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a fuzzy integrodifferential equation and hence, the solution of integrodifferential equations have a major role in the fields of science and engineering. We cannot get a perfect model, while transforming a real world problem into a deterministic ordinary differential equation. For example, the initial values may contain uncertain parameters. For the initiation of this aspect of fuzzy theory, has been introduced. Consequently the study of the theory of FIDEs has recently been growing rapidly.

The existence and uniqueness results of the solutions for fuzzy integrodifferential equations can be found in [3, 4]. Approximation of the solution of the solution of linear fuzzy integrodifferential equation in discrete form was studied and error estimates were obtained. Numerical solution of the FFDE is studied by Mazandarani et al. using the Euler method [5]. Also, Agarwal et al. [6] investigated the fuzzy fractional integral equation under the compactness type condition.

The solutions of fuzzy fractional differential equations are constructed by using fuzzy Laplace transform is given in [7]. In this paper, the same method is used for determining the solution of fractional integro differential equations with fuzzy initial conditions, but with fractional derivative considered in the sense of the Caputo derivative.

In this paper

$$ {}^{C}D^{\alpha}y(t) = ay(t) + \int_0^t K(s-t)y(t)\mathrm{d}t, \tag{1} $$

with fuzzy initial condition is basically solved.

This work is organized as follows. In Section 2, we propose the Laplace Transform method to solve the fractional integrodifferential systems with fuzzy initial conditions. Then by using the method appeared in Section 2, fractional integrodifferential with fuzzy initial condition is solved. In Section 3 a numerical example is presented to illustrate the performance of the methods.

2. Fractional Linear Fuzzy Integrodifferential Equation

Basic defitions of fuzzy number and fuzzy Laplace transform are given in [8]. In this section, we will investigate solution of fuzzy integrodifferential equations with separable kernels. Let $y(t)$ be a fuzzy-valued function to be solved for, $K(t,s)$ is a known real-valued integral kernel. Let us consider the general linear fractional integrodifferential equation

$$ {}^{C}D^{\alpha}y(t) = ay(t) + \int_0^t K(s-t)y(t)\mathrm{d}t, \tag{2} $$

with the initial condition $y^{(k)}(0) = y_0; 0 < k \leq [\alpha] - 1$ and $y_0 = (\underline{y_0}, \overline{y_0})$ is a fuzzy number, $K \in C[J \times J \times \mathbb{R}^n, \mathbb{R}^n], J = [0, a]$, with $K(t, s, 0) = 0$ for all $t \in J$.

Fuzzy Laplace Transform Method: Consider the following fuzzy integrodifferential equation

$$^{C}D^{\alpha}y(t) = ay(t) + \int_0^t K(s-t)y(t)\mathrm{d}t),$$

with the initial condition $y'(0) = y_0$, where $y_0 = (\underline{y_0}, \overline{y_0})$
Taking Laplace transform on both sides

$$L(^{C}D^{\alpha}y(t)) = L(ay(t)) + L(\int_0^t K(s-t)y(t)\mathrm{d}t)),$$

by definition of Laplace transform and using the fuzzy Convolution theorem, we have

$$s^{\alpha} - s^{\alpha-1}y(0) = aL(y) + G(s)L(y)$$

where L(y) is the Laplace transform of y and G(s) is the Laplace Transform of K(t).

$$L(y) = \frac{s^{\alpha-1}y_0}{s^{\alpha} + a + G(s)}$$

Here y(0) is the fuzzy valued function this implies that,

$$l\{\underline{y}(t;r)\} = \frac{s^{\alpha-1}\underline{y_0}}{s^{\alpha} + a + G(s)}$$
$$l\{\overline{y}(t;r)\} = \frac{s^{\alpha-1}\overline{y_0}}{s^{\alpha} + a + G(s)} \quad 0 \leq r \leq 1.$$

By taking the inverse of fuzzy Laplace transformation on both sides of above relations we can easily obtain the value of $\underline{y}(t;r)$ and $\overline{y}(t;r)$ where $0 \leq r \leq 1$. Simillary fuzzy Laplace transform method can be applied to $1 < \alpha < 2$

3. Example

In this section, a example is solved by using method, which is explained in Section 3. Solutions are plotted by using MATLAB.

Example 3.1. Consider the linear fuzzy integrodifferential system of the form

$$^{C}D^{\alpha}y(t) + \int_0^t y(t)\mathrm{d}t = 0, \tag{3}$$

where $0 < \alpha < 1$ and with the initial condition $y(0) = (r-1, 1-r)$.
This can be written in the standard form $^{C}D^{\alpha}y(t) = ax(t) + \int_0^t K(s-t)y(t)\mathrm{d}t$, where $A = 0$ and $K(s,t) = 1$. Taking Laplace Transform on both sides we get

$$s^{\alpha}L(y) - s^{\alpha-1}y(0) + \frac{1}{s}L(y) = 0$$

From this we have

$$L(y) = \frac{s^{\alpha}}{s^{\alpha+1}+1}y(0)$$

In compact form we have

$$\begin{aligned} l\{\underline{y}(t;r)\} =&(r-1)\frac{s^{\alpha}}{s^{\alpha+1}+1} \\ l\{\overline{y}(t;r)\} =&(1-r)\frac{s^{\alpha}}{s^{\alpha+1}+1} \quad 0 \le r \le 1. \end{aligned}$$

by taking inverse Laplace transform we get

$$\begin{aligned} &\underline{y}(t;r) = (r-1)E_{\alpha+1}(-t^{\alpha+1}) \\ &\overline{y}(t;r) = (1-r)E_{\alpha+1}(-t^{\alpha+1}) \quad 0 \le r \le 1. \end{aligned}$$

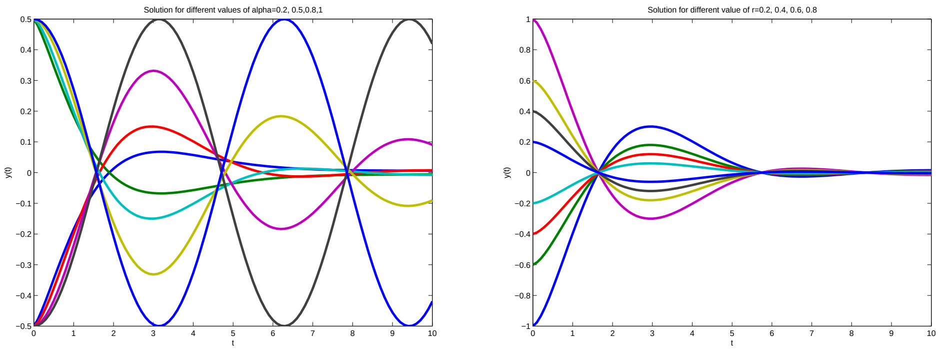
The solutions are plotted for different values of $\alpha = 0.2, 0.5, 0.8, 1$ and for a different values of $r = 0.2, 0.4, 0.6, 0.8$ This can be seen in the following figures. When $\alpha = 1/2$ we get the solutions as

$$\begin{aligned} \underline{y}(t;r) &= (r-1)E_{3/2}(-t^{3/2}) \\ \overline{y}(t;r) &= (1-r)E_{3/2}(-t^{3/2}) \quad 0 \le r \le 1. \end{aligned}$$

while in the case of integer order i.e when $\alpha = 1$ we get the solutions of the form

$$\begin{aligned} \underline{y}(t;r) &= (r-1)cos(t) \\ \overline{y}(t;r) &= (1-r)cos(t) \quad 0 \le r \le 1. \end{aligned}$$

The comparison between integer order and a fractional order is given in the following figure 2.



(a) $\alpha = 0.2, 0.5, 0.8, 1$, $r = 0.5$ (b) $r = 0.2, 0.4, 0.6, 0.8$, $\alpha = 0.5$

Figure 1: The solutions for different values of α and r

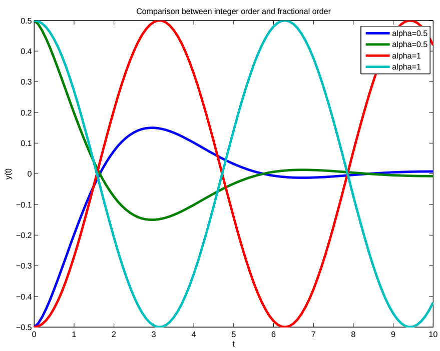


Figure 2: Comparison between integer order and fractional order

4. Conclusion

In this paper, solution of fuzzy integrodifferential equations with fuzzy initial condition is presented by using the fuzzy Laplace transform method. To show the applicability of the method, a example is presented and they are compared with the integer order system. Our future works include the solution of nonlinear fractional neutral and integrodifferential equations with fuzzy initial conditions and fuzzy coefficients.

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