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A PHARMACEUTICAL INVENTORY MODEL FOR HEALTHCARE INDUSTRIES WITH QUADRATIC DEMAND, LINEAR HOLDING COST AND SHORTAGES

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Abstract: In most of the pharmaceutical inventory models, the holding cost and demand rate have been considered as a constant function. However, in Healthcare Industries, these factors are depending on time. In view of this , we develop a pharmaceutical inventory model in which demand rate is quadratic function of time, holding cost is linear function of time, defective rate is constant, backlogging rate depends on the length of the next replenishment and shortages are allowed. The model is solved analytically by minimizing the total inventory cost. The sensitivity of the model has been checked with respect to the various major parameters of the system. The results reveal that the proposed inventory model is more applicable for Healthcare Industries.

Key Words: defective rate, pharmaceutical products, shortages, variable demand and variable holding cost

1. Introduction

As we enter the new millennium, healthcare organizations are facing new challenges and must continually improve their services to provide the highest quality at the best cost. Everyday hospitals deal with inventory complications, tracking materials, and patient validation. Inventory control is a complex and time-consuming process that every healthcare facility must deal with. Every

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pharmaceutical production process has unique, proprietary requirements that must be satisfied to protect patient outcomes while maximizing manufacturing quality and efficiency.

Deterioration is defined as decay, damage, spoilage evaporation and loss of utility of the product. Deterioration in pharmaceutical inventory is a realistic feature and need to consider it. Often we encounter the pharmaceutical products such as generic injectables, tablets, caplets, ophthalmics, ointments, creams, and liquids, drug etc., that have a defined period of life time. Pharmaceutical products are more commonly known as medicine or drugs. Due to defective items , pharmaceutical inventory system faces the problem of shortages and loss of good will or loss of profit. Shortage is a fraction of those customers whose demand is not satisfied in the current period reacts to this by not returning the next period.

Khanra in [1] discussed an Economic Order Quantity model which is developed for a deteriorating item having time dependent demand when delay in payment is permissible. The deterioration rate is assumed to be constant and the time varying demand rate is taken to be a quadratic function of time. Rakesh [2] presented an inventory model for deteriorating items in which shortages are allowed and it is assumed that the production rate is proportional to the demand rate.

Uthayakumar and Priyan in [3] developed an inventory model that integrates continuous review with production and distribution for a supply chain involving a pharmaceutical company and a hospital supply chain.
Mishra [4] developed a deterministic deteriorating inventory model in which demand rate and holding cost both is linear function of time, deterioration rate is constant, backlogging rate is variable and depend on the length of the next replenishment, shortages are allowed and partially backlogged. In this paper, we develop a pharmaceutical inventory model in which demand rate is quadratic function of time, holding cost is linear function of time, defective rate is constant, backlogging rate depends on the length of the next replenishment and shortages are allowed.

2. Notations and Assumptions

The mathematical model is based on the following notations and assumptions.

2.1. Notations

S	the setup Cost/ordering cost per order;
C	the purchase cost per unit;
θ	the defective rate;
$H(t)$	the inventory Holding Cost / Carrying Cost per unit per time unit;
π_b	the back ordered cost per unit short per time unit;
π_l	the cost of lost sales per unit;
t_1	the time at which the pharmaceutical inventory level reaches zero; $t_1 \geq 0$;
t_2	the length of period during which shortages are allowed; $t_2 \geq 0$;
T	$(= t_1 + t_2)$ the length of cycle time;
IM	the maximum pharmaceutical inventory level during [0, T];
IB	the maximum pharmaceutical inventory level during shortage period;
Q	(=IM+IB)the order quantity during a cycle of length T;
$I_1(t)$	the level of positive pharmaceutical inventory at time t;
$I_2(t)$	the level of negative pharmaceutical inventory at time t;
$TC(t_1, t_2)$	the total cost per time unit.

2.2. Assumptions

- The demand rate is time dependent that is if a is fixed fraction of demand and b and c are fraction of demand which is vary with time then demand function is f(t) = a+bt+$ct^2, where$ $a > 0, b > 0$
- Holding cost is linear function of time H(t) = d+et, $d \geq 0, e \geq 0$ and the defective rate is constant.
- Shortages are allowed and partially backlogged.
- The lead time is zero and the replenishment rate is infinite.
- During stock out period, the backlogging rate is variable and is dependent on the length of the waiting time for next replenishment. So that the backlogging rate for negative pharmaceutical inventory is, $B(t) = \frac{1}{1+\delta(T-t)}$ δ is backlogging parameter and $(T - t)$ is waiting time $(t_1 \leq t \leq T)$.

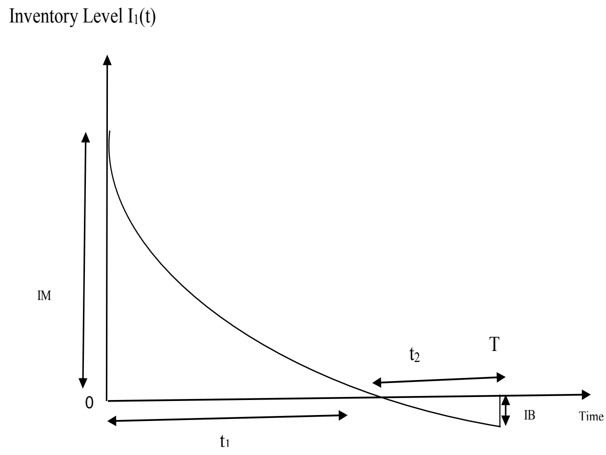

Figure 1: Graph of Inventory System

3. Mathematical Formulation and Solution of the Model

The rate of change of pharmaceutical inventory during positive stock period $[0, t_1]$ and shortage period $[t_1, T]$ is governed by the differential equations

$$\frac{d(I_1(t))}{dt} + \theta I_1(t) = -(a + bt + ct^2), \quad 0 \le t \le t_1, \tag{1}$$

$$\frac{d(I_2(t))}{dt} = \frac{-(a + bt + ct^2)}{1 + \delta(T - t)}, \quad t_1 \le t \le T, \tag{2}$$

with boundary condition $I_1(t) = I_2(t) = 0$ at $t = t_1$ and $I_1(t) = IM$ at $t = 0$.
Case 1. Pharmaceutical inventory Level without Shortage:

During the period $[0, t_1]$, the pharmaceutical inventory depletes due to the defective rate and demand. Hence, the inventory level at any time during $[0, t_1]$ is described by differential equation (1) with the boundary condition $I_1(t_1) = 0 \ at \ t = t_1$.

The solution of the equation (1) is

$$I_1(t) = -\frac{a}{\theta} - \frac{b}{\theta}(t - \frac{1}{\theta}) - \frac{c}{\theta}(t^2 - \frac{2t}{\theta} + \frac{2}{\theta^2}) + e^{\theta(t_1 - t)}[\frac{a}{\theta} + \frac{b}{\theta}(t_1 - \frac{1}{\theta}) + \frac{c}{\theta}(t_1^2 - \frac{2t_1}{\theta} + \frac{2}{\theta^2})], \quad 0 \le t \le t_1, \tag{3}$$

Case 2. Pharmaceutical inventory Level with Shortage: During the interval $[t_1, T]$ the pharmaceutical inventory level depends on demand and a fraction of

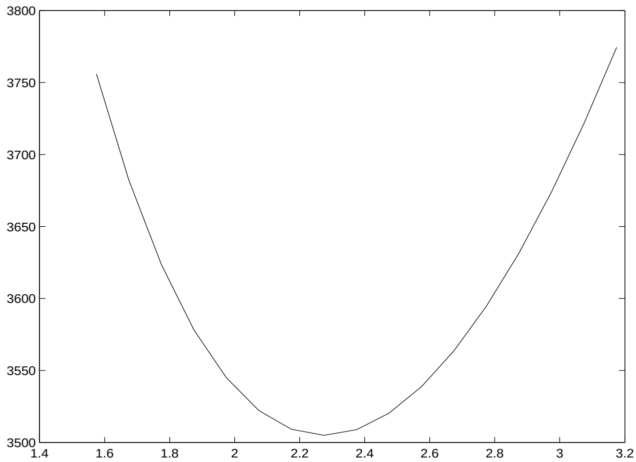

Figure 2: Total Cost vs t_1 at $t_2 = 0.3102$

demand is backlogged. The state of inventory during $[t_1, T]$ can be represented by the differential equation (2) with the boundary condition $I_1(t_1) = 0$ at $t = t_1$.

The lSolution of the equation (2) is

$$\begin{aligned}I_2(t) =& \frac{a}{\delta}\log[\frac{1+\delta(t_1+t_2-t)}{1+\delta t_2}] + \frac{b(t-t_1)}{\delta} \\ &+ \frac{b[1+\delta(t_1+t_2)]}{\delta^2}\log[\frac{1+\delta(t_1+t_2-t)}{1+\delta t_2}] \\ &+ \frac{c}{\delta^3}[\frac{(2+3\delta t_1+2\delta t_2+\delta t)(\delta t-\delta t_1)}{2} \\ &+ (1+\delta(t_1+t_2))^2\log(\frac{1+\delta(t_1+t_2-t)}{1+\delta t_2})], \quad t_1 \le t \le t_1+t_2, \qquad (4)\end{aligned}$$

Therefore the total cost per replenishment cycle consists of the following components:

1. Pharmaceutical inventory holding cost per cycle

$$IHC = \int_0^{t_1} H(t)I_1(t)dt = \int_0^{t_1} H(t)(d+et)dt,$$

$$\begin{aligned}IHC =& [-\frac{adt_1}{\theta} - \frac{bdt_1^2}{2\theta} - \frac{cdt_1^3}{3\theta} - \frac{ad}{\theta^2} + \frac{bd}{\theta^3} - \frac{2cd}{\theta^4} \\ &- \frac{aet_1^2}{2\theta} - \frac{bet_1^3}{3\theta} - \frac{bet_1^2}{2\theta^2} - \frac{cet_1^4}{4\theta} - \frac{cet_1^3}{3\theta^2} - \frac{aet_1}{\theta^2} - \frac{ae}{\theta^3}\end{aligned}$$

$$+\frac{be}{\theta^4}-\frac{2ce}{\theta^5}+\frac{ade^{\theta t_1}}{\theta^2}+\frac{bdt_1e^{t_1}}{\theta^2}-\frac{bde^{\theta t_1}}{\theta^3}$$
$$+\frac{cdt_1^2e^{\theta t_1}}{\theta^2}-\frac{2cdt_1e^{\theta t_1}}{\theta^3}+\frac{2cde^{\theta t_1}}{\theta^4}+\frac{aee^{\theta t_1}}{\theta^3}$$
$$+\frac{bet_1e^{\theta t_1}}{\theta^3}-\frac{bee^{\theta t_1}}{\theta^4}+\frac{cet_1^2e^{\theta t_1}}{\theta^3}-\frac{2cet_1e^{\theta t_1}}{\theta^4}+\frac{2cee^{\theta t_1}}{\theta^5}] \quad (5)$$

2. Backordered cost per cycle

$$BC=\Pi_b\int_{t_1}^{t_1+t_2}-I_2(t)dt,$$

$$BC=\Pi_b[\frac{at_2}{\delta}-\frac{a\log(1+\delta t_2)}{\delta^2}+\frac{bt_2}{\delta^2}+\frac{bt_1t_2}{\delta}+\frac{bt_2^2}{2\delta}-\frac{b\log(1+\delta t_2)}{\delta^3}$$
$$-\frac{bt_1\log(1+\delta t_2)}{\delta^2}-\frac{bt_2\log(1+\delta t_2)}{\delta^2}$$
$$+\frac{c}{\delta^3}(\frac{3\delta t_2^2}{2}+\delta^2t_1t_2^2+\frac{5\delta^2t_2^3}{6}$$
$$+t_2+\delta^2t_1^2t_2+2\delta t_1t_2-[1+\delta(t_1+t_2)]^2\log\frac{(1+\delta t_2)}{\delta})]. \quad (6)$$

3. Lost sales cost per cycle

$$LS=\Pi_l\int_{t_1}^{t_1+t_2}[(1-\frac{1}{1+\delta(t_1+t_2-t)})(a+bt+ct^2)]dt,$$

$$LS=\Pi_l[at_2+\frac{bt_2^2}{2}+bt_1t_2+\frac{ct_2^3}{3}+ct_1^2t_2+ct_1t_2^3-\frac{a\log(1+\delta t_2)}{\delta}+\frac{bt_2}{\delta}$$
$$-\frac{b[1+\delta(t_1+t_2)]\log(1+\delta t_2)}{\delta^2}$$
$$+\frac{c}{\delta^3}(\delta t_2+\frac{3\delta^2t_2^2}{2}+2\delta^2t_1t_2-[1+\delta(t_1+t_2)]^2\log(1+\delta t_2))]. \quad (7)$$

4. Purchase cost per cycle =(purchase cost per unit)*(Order quantity in one cycle) PC $= C\times Q$. When $t=0$ the level of inventory is maximum and it is denoted by $IM(=I_1(0))$ then from the equation (3)

$$IM=-\frac{a}{\theta}+\frac{b}{\theta^2}-\frac{2c}{\theta^3}+e^{\theta t_1}[\frac{a}{\theta}+\frac{b}{\theta}(t_1-\frac{1}{\theta})+\frac{c}{\theta}(t_1^2-\frac{2t_1}{\theta}+\frac{2}{\theta^2})]. \quad (8)$$

The maximum backordered inventory is obtained at $t = t_1 + t_2$ then from the Eq. (4), $IB = -I_2(t1 + t2)$,

$$IB = -\frac{a}{\delta}\log[\frac{1}{1+\delta t_2}] - \frac{bt_2}{\delta} - \frac{b[1+\delta(t_1+t_2)]}{\delta^2}\log\frac{1}{1+\delta t_2}$$
$$-\frac{c}{\delta^3}[\delta t_2 + 2\delta^2 t_1 t_2 + \frac{3\delta^2 t_2^2}{2} + [1+\delta(t_1+t_2)]^2\log\frac{1}{1+\delta t_2}]. \quad (9)$$

Thus, the order size during total time interval $[0, T]$, $Q = IM + IB$.Now from Eqs. (8) and (9).

$$Q = -\frac{a}{\theta} + \frac{b}{\theta^2} - \frac{2c}{\theta^3} + e^{\theta t_1}[\frac{a}{\theta} + \frac{b}{\theta}(t_1 - \frac{1}{\theta})$$
$$+\frac{c}{\theta}(t_1^2 - \frac{2t_1}{\theta} + \frac{2}{\theta^2})] - \frac{a}{\delta}\log[\frac{1}{1+\delta t_2}] - \frac{bt_2}{\delta}$$
$$\frac{b[1+\delta(t_1+t_2)]}{\delta^2}\log\frac{1}{1+\delta t_2} - \frac{c}{\delta^3}[\delta t_2 + 2\delta^2 t_1 t_2 + \frac{3\delta^2 t_2^2}{2}$$
$$+[1+\delta(t_1+t_2)]^2\log\frac{1}{1+\delta t_2}]. \quad (10)$$

Hence Purchase Cost(PC) =C * Q:

$$PC = C \times (-\frac{a}{\theta} + \frac{b}{\theta^2} - \frac{2c}{\theta^3} + e^{\theta t_1}[\frac{a}{\theta} + \frac{b}{\theta}(t_1 - \frac{1}{\theta}) + \frac{c}{\theta}(t_1^2 - \frac{2t_1}{\theta} + \frac{2}{\theta^2})]$$
$$-\frac{a}{\delta}\log[\frac{1}{1+\delta t_2}] - \frac{bt_2}{\delta}$$
$$-\frac{b[1+\delta(t_1+t_2)]}{\delta^2}\log\frac{1}{1+\delta t_2} - \frac{c}{\delta^3}[\delta t_2 + 2\delta^2 t_1 t_2 + \frac{3\delta^2 t_2^2}{2}$$
$$+[1+\delta(t_1+t_2)]^2\log\frac{1}{1+\delta t_2}]). \quad (11)$$

5. Ordering cost $(OC) = S$. Therefore the total cost per time unit is given by

$$TC = \frac{1}{t_1+t_2}[OC + IHC + BC + LS + PC]. \quad (12)$$

Substituting the values of $OC, IHC, BC, LSandPC$, we obtain

$$TC(t_1, t_2) = \frac{1}{t_1+t_2}[S + H[-\frac{adt_1}{\theta} - \frac{bdt_1^2}{2\theta} - \frac{cdt_1^3}{3\theta} - \frac{ad}{\theta^2} + \frac{bd}{\theta^3}$$

$$
\begin{aligned}
&-\frac{2cd}{\theta^4}-\frac{aet_1^2}{2\theta}-\frac{bet_1^3}{3\theta}-\frac{bet_1^2}{2\theta^2}-\frac{cet_1^4}{4\theta}-\frac{cet_1^3}{3\theta^2}-\frac{aet_1}{\theta^2}\\
&-\frac{ae}{\theta^3}+\frac{be}{\theta^4}-\frac{2ce}{\theta^5}+\frac{ade^{\theta t_1}}{\theta^2}+\frac{bdt_1e^{t_1}}{\theta^2}-\frac{bde^{\theta t_1}}{\theta^3}\\
&+\frac{cdt_1^2e^{\theta t_1}}{\theta^2}-\frac{2cdt_1e^{\theta t_1}}{\theta^3}+\frac{2cde^{\theta t_1}}{\theta^4}+\frac{aee^{\theta t_1}}{\theta^3}+\frac{bet_1e^{\theta t_1}}{\theta^3}\\
&-\frac{bee^{\theta t_1}}{\theta^4}+\frac{cet_1^2e^{\theta t_1}}{\theta^3}-\frac{2cet_1e^{\theta t_1}}{\theta^4}+\frac{2cee^{\theta t_1}}{\theta^5}]\\
&+\Pi_b[\frac{at_2}{\delta}-\frac{a\log(1+\delta t_2)}{\delta^2}+\frac{bt_2}{\delta^2}+\frac{bt_1t_2}{\delta}+\frac{bt_2^2}{2\delta}\\
&-\frac{b\log(1+\delta t_2)}{\delta^3}-\frac{bt_1\log(1+\delta t_2)}{\delta^2}-\frac{bt_2\log(1+\delta t_2)}{\delta^2}+\frac{c}{\delta^3}(\frac{3\delta t_2^2}{2}\\
&+\delta^2t_1t_2^2+\frac{5\delta^2t_2^3}{6}+t_2+\delta^2t_1^2t_2\\
&+2\delta t_1t_2-[1+\delta(t_1+t_2)]^2\log\frac{(1+\delta t_2)}{\delta})]+\Pi_l[at_2+\frac{bt_2^2}{2}\\
&+bt_1t_2+\frac{ct_2^3}{3}+ct_1^2t_2+ct_1t_2^3-\frac{a\log(1+\delta t_2)}{\delta}\\
&+\frac{bt_2}{\delta}-\frac{b[1+\delta(t_1+t_2)]\log(1+\delta t_2)}{\delta^2}+\frac{c}{\delta^3}(\delta t_2\\
&+\frac{3\delta^2t_2^2}{2}+2\delta^2t_1t_2-[1+\delta(t_1+t_2)]^2\log\left(1+\delta t_2\right))]\\
&+C\times(-\frac{a}{\theta}+\frac{b}{\theta^2}-\frac{2c}{\theta^3}+e^{\theta t_1}[\frac{a}{\theta}+\frac{b}{\theta}(t_1-\frac{1}{\theta})\\
&+\frac{c}{\theta}(t_1^2-\frac{2t_1}{\theta}+\frac{2}{\theta^2})]-\frac{a}{\delta}\log[\frac{1}{1+\delta t_2}]-\frac{bt_2}{\delta}\\
&-\frac{b[1+\delta(t_1+t_2)]}{\delta^2}\log\frac{1}{1+\delta t_2}-\frac{c}{\delta^3}[\delta t_2+2\delta^2t_1t_2\\
&+\frac{3\delta^2t_2^2}{2}+[1+\delta(t_1+t_2)]^2\log\frac{1}{1+\delta t_2}])] \qquad (13)
\end{aligned}
$$

The necessary condition for the total cost per time unit,to be minimize is

$$\frac{\partial TC}{\partial t_1}=0 \text{ and } \frac{\partial TC}{\partial t_2}=0.$$

Provided

$$(\frac{\partial^2 TC}{\partial t_1^2})(\frac{\partial^2 TC}{\partial t_2^2})-(\frac{\partial^2 TC}{\partial t_1\partial t_2})>0 \text{ and } (\frac{\partial^2 TC}{\partial^2 t})>0.$$

Since the nature of the cost function is highly non linear thus the convexity of the function shown graphically in the next section.

4. Numerical Example and Sensitivity Analysis

Consider a pharmaceutical inventory system with the following parameter in proper unit $A = 5000$, $H = 0.5$,$d = 0.5$, $e = 0.01$, $C = 5$,$\Pi_b = 12$, $\Pi_l = 15$, $\delta = 1.5$, $a = 25$, $b = 40$,$c = 50$, $\theta = 0.005$. The computer output of the program by using matlab is $t_1 = 2.2748$, $t_2 = 0.3102$ and $TC = 3505$. i.e. the value of t_1 at which the inventory level become zero is 2.2748 unit and shortage period is 0.3102 unit. The effect of changes in the parameter of the inventory model is as follows From Tab. 1 .We observed that the parameter a, b, and θ is more sensitive comparatively to other parameters of the model. If we plot the total cost function (13) with some values of fixed t_2 at 0.3102 and t_1 varies from 1.5748 to 3.1748 then we get strictly convex graph of total cost function (TC) given by the Figs. 2.

5. Conclusion

In this paper, we developed a model for defective item with time dependent demand, holding cost and partial backlogging and give analytical solution of the model that minimize the total inventory cost. The model is very practical for the healthcare industries in which the demand rate and holding cost is depending upon the time. The sensitivity of the model has checked with respect to the various parameter of the system and it is observed that the solution of the model is quite stable. This model can further be extended by taking more realistic assumptions such as fuzzy demand rate, variable defective rate and permissible delay in payment etc.

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$changingparameter$	$Variation$	t_1	t_2	TC
a	25	2.2748	0.3102	3505
	25.25	2.2749	0.3101	3506.4
	25.50	2.2752	0.3098	3507.8
	25.75	2.2754	0.3096	3510.7
	26	2.2756	0.3094	3510.73
b	40	2.2748	0.3102	3505
	41	2.2732	0.3083	3512.8
	42	2.2717	0.3063	3520.6
	43	2.2702	0.3043	3528.3
c	50	2.2748	0.3102	3505
	51	2.2569	0.3126	3527.2
	52	2.2393	0.31502	3549.3
	53	2.222	0.3174	3571.1
θ	0.0050	2.2748	0.3102	3505
	0.0055	2.2735	0.3107	3505.6
	0.0060	2.2722	0.3111	3506.1
	0.0065	2.2710	0.3115	3506.7
d	0.5	2.2748	0.3102	3505
	0.6	2.2514	0.3182	3515.7
	0.7	2.2291	0.3259	3526
	0.8	2.2075	0.3335	3536

Table 1: Effect of changes in the parameter of the inventory model

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