International Journal of Pure and Applied Mathematics
Volume 106 No. 8 2016, 1-6
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
doi: 10.12732/ijpam.v106i8.1
ijpam.eu

CORDIAL LABELING OF MONGOLIAN TENT M_n

Sharmila Mary Arul[1], K. Subashini[2]
[1,2]Department of Mathematics
Jeppiaar Engineering College
Chennai, INDIA

Abstract: The *Cordial labeling* of graph G is an injection $f : V(G) \to \{0,1\}$ such that each edge uv in G is assigned the label $|f(u) - f(v)|$ with the property that $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$, where $v_f(i)$ for $i = 0,1$ denote the number of vertices with label i. The graph which admits cordial labeling is called the *Cordial graph*. In this paper, we prove that the Mongolian Tent is cordial.

AMS Subject Classification: 05C78
Key Words: cordial labeling, Mongolian Tent

1. Introduction

Graph labeling methods trace their origin to the graceful labeling introduced by Rosa[7] in 1967. For the past five decades variations in labeling methods have evolved. One such labeling method is the cordial labeling introduced by Cahit[4] in 1987. The *cordial labeling* of graph G is an injection $f : V(G) \to \{0,1\}$ such that each edge uv in G is assigned the label $|f(u) - f(v)|$ with the property that $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$, where $v_f(i)$ for $i = 0,1$ denotes the number of vertices with label i. The graph which admits cordial labeling is called the *Cordial graph*. Various graphs are shown to be cordial. Andar et al. [1,2,3] have proved that the helms, closed helms, flowers, gears and sunflower graphs and multiple shells are cordial. Again in [1,2,3] the one point union

Received: February 15, 2016
Published: March 3, 2016

of helms, flowers, gears, sunflower graphs are shown to be cordial. Cahit [5] has proved that every tree is cordial. In [5] Cahit has shown that all fans are cordial, the wheel W_n when $n \neq 3(mod4)$ is cordial, the complete graph K_n is cordial if and only if $n \leq 3$, the bipartite graph $K_{m,n}$ is cordial for all m and n, the friendship graph $C_3^{(t)}$ is cordial if and only if $t \neq 2(mod4)$. An extensive survey of cordial labeling methods is available in [6] by Gallian.

In this paper, we prove that the Mongolian Tent is cordial.

2. Main Results

In this section, first we recall the definition for cordial labeling and Mongolian tent. Later, we prove that the Mongolian tent satisfies the condition of cordial labeling.

Definition 1. The *cordial labeling* of graph G is an injection $f : V(G) \rightarrow \{0, 1\}$ such that each edge uv in G is assigned the label $|f(u) - f(v)|$ with the property that $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$, where $v_f(i)$ for $i = 0, 1$ denote the number of vertices with label i. The graph which admits cordial labeling is called the *cordial graph*.

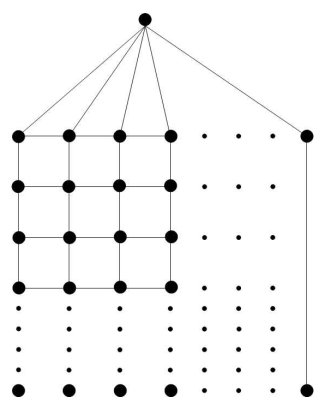

Figure 1: Mongolian tent

Definition 2. A *Mongolian tent* M_n is defined as the graph obtained from $P_n \times P_n$ by adding a new vertex above the grid and joining every vertex of the top row to the new vertex(See Figure 1).

Theorem 3. *The Mongolian tent M_n is cordial.*

Proof. Let $\{v_{i,j}/(1 \leq i \leq n, 1 \leq j \leq n)\}$ be the vertices of the grid $P_n \times P_n$ and v be the vertex above the grid.
Let M_n be the Mongolian tent (See Figure 1) with
$V=\{v_{i,j}/(1 \leq i \leq n, 1 \leq j \leq n)\} \cup \{v\}$ and $E = E_1 \cup E_2 \cup E_3$ where

$E_1 = \{(v_{i,j}, v_{i,j+1})/(1 \leq i \leq n, 1 \leq j \leq n-1)\}$
$E_2 = \{(v_{i,j}, v_{i+1,j})/(1 \leq i \leq n-1, 1 \leq j \leq n)\}$
$E_3 = \{(v_{1,j}, v)/(1 \leq j \leq n)\}$.

Let $p = n^2 + 1$ be the number of vertices in M_n and
let $q = n(2n-1)$ be the number of edges in M_n.

Case 1: when n is even (See Figure 2)

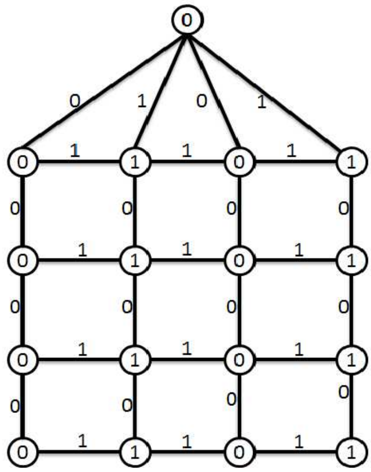

Figure 2: Cordial labeling of Mongolian tent M_4

Define $f(v) = 0$.

$$f(v_{i,j}) = \begin{cases} 1, & 1 \leq i \leq n, \quad j = 2k, \quad 1 \leq k \leq \lfloor n/2 \rfloor \\ 0, & 1 \leq i \leq n, \quad j = 2k-1, \quad 1 \leq k \leq \lfloor n/2 \rfloor \end{cases}$$

In view of the above defined pattern we have,

$$v_f(1) = \lfloor p/2 \rfloor \text{ and } v_f(0) = \lfloor p/2 \rfloor + 1$$

Thus $|v_f(0) - v_f(1)| \leq 1$ where
$v_f(1)$ denote the number of vertices with label 1 and
$v_f(0)$ denote the number of vertices with label 0.
Similarly we have

$$e_f(1) = q/2 \text{ and } e_f(0) = q/2.$$

Hence $|e_f(0) - e_f(1)| = |(q/2) - (q/2)| = 0 \leq 1$.
Thus $|e_f(0) - e_f(1)| \leq 1$ where
$e_f(1)$ denote the number of edges with label 1 and
$e_f(0)$ denote the number of edges with label 0.

Case 2: when n is odd
Define $f(v) = 0$.

$$f(v_{i,j}) = \begin{cases} 1, & 1 \leq i \leq n, \quad j = 2k, \quad 1 \leq k \leq \lfloor n/2 \rfloor \\ 0, & 1 \leq i \leq n, \quad j = 2k+1, \quad 1 \leq k \leq \lfloor n/2 \rfloor \end{cases}$$

Case 2.1: when $n \equiv 1(mod4)$ (See Figure 3)

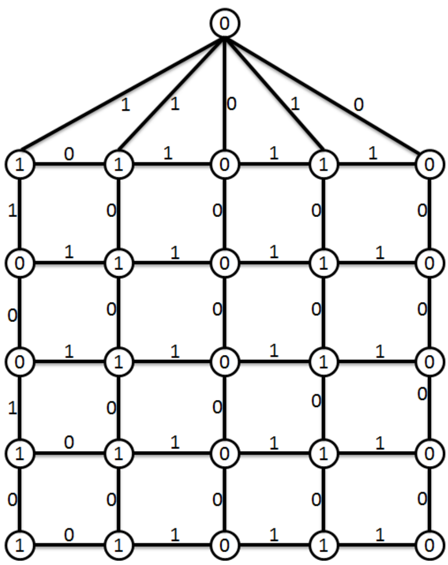

Figure 3: Cordial labeling of Mongolian tent M_5

Define $f(v_{1,1}) = 1$.

$$f(v_{i,1}) = \begin{cases} 1, & i \equiv 0, 1(mod4) \\ 0, & i \equiv 2, 3(mod4) \end{cases}$$

Case 2.2: when $n \equiv 3(mod4)$ (See Figure 4)

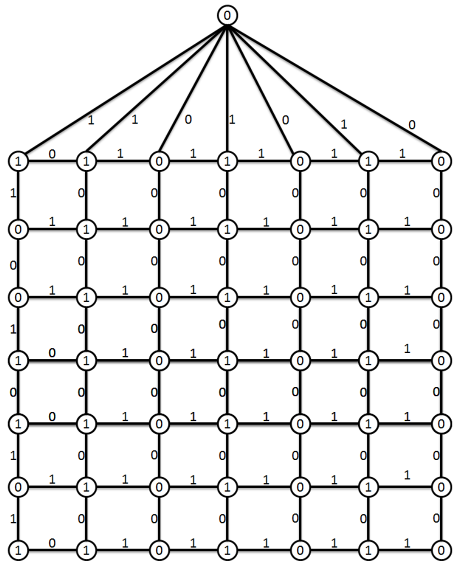

Figure 4: Cordial labeling of Mongolian tent M_7

Define $f(v_{1,1}) = f(v_{n,1}) = 1$.
Define $f(v_{n-1,1}) = 0$.
For $2 \leq i \leq n-2$ define $f(v_{i,1})$ as follows

$$f(v_{i,1}) = \begin{cases} 1, & i \equiv 0, 1(mod4) \\ 0, & i \equiv 2, 3(mod4) \end{cases}$$

In view of the above defined pattern we have,

$$v_f(1) = p/2 \text{ and } v_f(0) = p/2.$$

Hence $|v_f(0) - v_f(1)| = |(p/2) - (p/2)| = 0 \leq 1$.
Thus $|v_f(0) - v_f(1)| \leq 1$.
Similarly from case 2.1,
$e_f(1) = \lfloor q/2 \rfloor$ and $e_f(0) = \lfloor q/2 \rfloor + 1$
Hence$|e_f(0) - e_f(1)| = |(\lfloor q/2 \rfloor + 1) - (\lfloor q/2 \rfloor)| \leq 1$.
Thus $|e_f(0) - e_f(1)| \leq 1$.
From case 2.2,
$e_f(1) = \lfloor q/2 \rfloor + 1$ and $e_f(0) = \lfloor q/2 \rfloor$.
Hence$|e_f(0) - e_f(1)| = |(\lfloor q/2 \rfloor) - (\lfloor q/2 \rfloor + 1)| \leq 1$.
Thus $|e_f(0) - e_f(1)| \leq 1$. □

3. Conclusion

In this paper we have obtained the Cordial labeling for the Mongolian tent. Finding the cordial labeling for other unicyclic graphs are under investigation and are quite challenging.

References

[1] Andar M., Boxwala S. and Limaye N., "Cordial labelings of some wheel related graphs", J. Combin. Math. Combin. Comput.Vol. 41(2002), 203-208.

[2] Andar M., Boxwala S. and Limaye N., "A note on cordial labelings of multiple shells", Trends Math.(2002), 77-80.

[3] Andar M., Boxwala S. and Limaye N., "New families of cordial graphs", J.Combin. Math. Combin. Comput., Vol. 53(2005), 117-154.

[4] Cahit I., "Cordial Graphs: A weaker version of graceful and Harmonic Graphs", Arts Combinatoria, Vol. 23(1987), 201-207.

[5] Cahit I., "On cordial and 3-equitable labeling of graphs", Util. Math.,Vol. 37(1990), 189-198.

[6] Gallian J.A., "A dynamic survey of graph labeling", the Electronics Journal of Combinatorics, Vol. 16(2013), 53-63.

[7] Rosa A., "On certain valuation of the vertices of a graph",Theory of graphs(International Symposium Rome, Gordan and Breach, N.Y and Dunod Paris, July(1966), 349-355.