

A NEW DEFUZZIFICATION MEASURE FOR SOLVING INTUITIONISTIC FUZZY ASSIGNMENT PROBLEM

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Abstract: This paper proposes a Hungarian approach of solving Intuitionistic Fuzzy Assignment problem using a new defuzzification measure. The assignment costs are taken as Triangular Intuitionistic Fuzzy Numbers (TIFN). The proposed method is illustrated with a numerical example.

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1. Introduction

Assignment problem is used world wide in solving real world problems. Suppose that ' n ' jobs have to be performed on ' m ' machines (one job to one machine). The objective is to assign the jobs to the machines at the minimum cost (or maximum profit) under the assumption that each machine can perform one job but with varying degree of efficiencies. Hence the assignment problem is a

particular case of number of tasks (Jobs or Origins or Sources) assigned to an equal number of facilities (Machines or Persons or Destination) of a Minimum cost (or Maximum Profit).

The concept of fuzzy set was introduced by Zadeh [18] in 1965 and it dealt with imprecision, vagueness in real life situations. In 1970, Belmann & Zadeh [5] proposed the concept of decision making problems involving uncertainty and imprecision. Amitkumar and Anila Gupta [1] investigated Assignment and travelling salesman problem with coefficients as LR Fuzzy parameters. Amit Kumar et al. [2] gave Fuzzy linear programming approach for solving Fuzzy Transportation problem with Transshipment. Amitkumar et al. [3], discussed the method for Solving Fully Fuzzy Assignment problems Using Triangular Fuzzy Numbers. P.K.De and Bharti Yadav [6] given a general approach for solving assignment problems involving with Fuzzy costs coefficients. M.S.Hungand W.O.Rom [8] found solving the assignment problem by relaxation. Kuhn H.W. [10] disclosed the Hungarian method for the assignment problem. Mukherjee S. and Basu.K [11] proposed application of fuzzy ranking method for solving assignment problem with fuzzy costs.

In 1986, K. Atanassov [4] introduced the idea of Intuitionistic Fuzzy Set(IFS). An Intuitionistic Fuzzy set (IFS) is characterized by membership degree and non-membership degree. The Intuitionistic Fuzzy set (IFS) theory has been studied and applied in different areas including decision making. P.Senthilkumar and R.Jahir Hussain [15] solved balanced Intuitionistic Fuzzy Assignment problem using ranking of triangular fuzzy numbers. Gaurav Kumar and Rakesh kumar Balaji [7] presented the solution of Interval Valued Intuitionistic Fuzzy Assignment problem using similarity measure and score function. B.Srinivas and G.Ganesan [17] solved Intuitionistic Fuzzy Assignment problem using Branch and Bound Method. K.Prabakaran and K.Ganesan [12] presented Fuzzy Hungarian Method for Solving Intuitionistic Fuzzy Assignment problems. Sagaya Roseline and Henry Amirtharaj [14] presented methods to find the solution for the Intuitionistic Fuzzy Assignment problem with ranking of Intuitionistic Fuzzy numbers. K.Pramila and G.Uthra [13] given optimal solution of an Intuitionistic Fuzzy Transportation problem. Shiny Jose and Sunny Kuriakose [16] discussed an algorithm for solving an assignment model in Intuitionistic fuzzy context. Jahir Hussain.R and Senthil Kumar.P [9] introduced an optimal more or less solution of mixed constraints Intuitionistic Fuzzy Transportation problems.

In this paper an Intuitionistic Fuzzy Assignment problem is considered. The Intuitionistic fuzzy numbers are converted into crisp values using a new defuzzification measure. The problem is then solved by the usual Hungarian

algorithm to get the optimal assignment. The rest of this paper is organized as follows. In Section 2, some basic definitions and a new measure for defuzzification of Intuitionistic fuzzy number are given. In Section 3, an Intuitionistic fuzzy Assignment problem is introduced. In Section 4, procedure and numerical example for the proposed method are given followed by conclusion in Section 5.

2. Definition

Definition 2.1. Let A be a classical set, $\mu_A(x)$ be a function from A to $[0,1]$. A fuzzy set A with the membership function $\mu_A(x)$ is defined as $A = \{(x, \mu_A(x)); x \in A \text{ and } \mu_A(x) \in [0,1]\}$.

Definition 2.2. Let X be a given set. An intuitionistic fuzzy set A in X is given by, $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$, where: $\mu_A, \nu_A: X \rightarrow [0,1]$, where $\mu_A(x)$ is the degree of membership of the element x in A and $\nu_A(x)$ is the degree of non membership of x in A and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$

Definition 2.3. A fuzzy number $A(a_1, a_2, a_3)$ is defined to be a triangular fuzzy number if its membership functions $\mu_A: R \rightarrow [0,1]$ is equal to

$$\mu_A(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & \text{if } x \in [a_1, a_2] \\ \frac{a_3-x}{a_3-a_2} & \text{if } x \in [a_2, a_3] \\ 0 & \text{otherwise} \end{cases}$$

where $a_1 \leq a_2 \leq a_3$.

Definition 2.4 A^* is Triangular Intuitionistic Fuzzy Number (TIFN) with parameters

$$a_1^* \leq a_1 \leq a_2 \leq a_3 \leq a_3^*$$

and denoted by $A^* = (a_1, a_2, a_3)(a_1^*, a_2, a_3^*)$ having the membership and non-membership function as follows

$$\mu_A(x) = \begin{cases} 0 & \text{if } x < a_1 \\ \frac{x-a_1}{a_2-a_1} & \text{if } a_1 \leq x \leq a_2 \\ 1 & \text{if } x = a_2 \\ \frac{x-a_1}{a_2-a_1} & \text{if } a_2 \leq x \leq a_3 \\ 0 & \text{if } x > a_3 \end{cases}$$

$$\nu_A(x) = \begin{cases} 0 & \text{if } x < a_1^* \\ \frac{x-a_1}{a_2-a_1} & \text{if } a_1^* \leq x \leq a_2 \\ 1 & \text{if } x = a_2 \\ \frac{x-a_1}{a_2-a_1} & \text{if } a_2 \leq x \leq a_3^* \\ 0 & \text{if } x > a_3^* \end{cases}$$

Particular Case:

Let $A^* = (a_1, a_2, a_3)(a_1^*, a_2, a_3^*)$ be a Triangular Intuitionistic Fuzzy number. Then the following cases arise.

Case(i): If $a_1^* = a_1, a_3^* = a_3$ then A^* represent Triangular Fuzzy number(TFN)

Case(ii): If $a_1^* = a_1 = a_2 = a_3 = a_3^* = m$, then A^* represent a real number m .

Operations on Intuitionisitic fuzzy number:

Let $A^* = (a_1, a_2, a_3)(a_1^*, a_2, a_3^*)$ and $B^* = (b_1, b_2, b_3)(b_1^*, b_2, b_3^*)$ be two Triangular Intuitionistic Fuzzy Numbers the arithmetic operations on A^* and B^* are given below:

Addition:

$$(a_1, a_2, a_3)(a_1^*, a_2, a_3^*) + (b_1, b_2, b_3)(b_1^*, b_2, b_3^*) = (a_1 + b_1, a_2 + b_2, a_3 + b_3; a_1^* + b_1^*, a_2 + b_2, a_3^* + b_3^*).$$

Subtraction:

$$(a_1, a_2, a_3)(a_1^*, a_2, a_3^*) - (b_1, b_2, b_3)(b_1^*, b_2, b_3^*) = (a_1 - b_3, a_2 - b_2, a_3 - b_1, a_1^* - b_3^*, a_2 - b_2, a_3^* - b_1^*).$$

Multiplication:

$$(a_1, a_2, a_3)(a_1^*, a_2, a_3^*) * (b_1, b_2, b_3)(b_1^*, b_2, b_3^*) = (a_1 b_1, a_2 b_2, a_3 b_3)(a_1^* b_1^*, a_2 b_2, a_3^* b_3^*).$$

Scalar Multiplication:

$$(a_1, a_2, a_3)(a_1^*, a_2, a_3^*) = (ka_1, ka_2, ka_3)(ka_1^*, ka_2, ka_3^*) \text{ if } k > 0 \\ = (ka_3, ka_2, ka_1)(ka_3^*, ka_2, ka_1^*) \text{ if } k < 0.$$

Defuzzification:

We define an average function $D(A^*) = \frac{(a_1 + 4a_2 + a_3) + (a_1^* + 4a_2 + a_3^*)}{12}$ to defuzzify a given Triangular Intuitionistic Fuzzy Number.

3. Intuitionistic Fuzzy Assignment Problem

Consider the situation of assigning n machines to n jobs and each machine is capable of doing each job at different costs. Let C_{ij}^* be an Intuitionistic fuzzy cost of assigning the i^{th} machine to the j^{th} job. Let x_{ij} be the decision variable denoting the assignment of the i^{th} machine to the j^{th} job. The objective is to minimize the total cost.

The mathematical model of an Intuitionistic Fuzzy Assignment Problem is given by

$$\text{Minimize } z^* = \sum_{i=1}^n \sum_{j=1}^n C_{ij}^* x_{ij}$$

Subject to

$$\sum_{i=1}^n x_{ij} = 1, \quad \text{for } j=1, 2, \dots, n$$

$$\sum_{j=1}^n x_{ij} = 1, \quad \text{for } i=1, 2, \dots, n$$

$$x_{ij} \in [0, 1]$$

$$\text{where } x_{ij} = \begin{cases} 1, & \text{if the } i^{th} \text{ machine is assigned to } j^{th} \text{ job} \\ 0, & \text{if the } i^{th} \text{ machine is assigned to } j^{th} \text{ job} \end{cases}$$

$$C_{ij}^* = (C_{ij}^1, C_{ij}^2, C_{ij}^3)(C_{ij}^{1*}, C_{ij}^2, C_{ij}^{3*})$$

4. Procedure

1. Check whether the fuzzy assignment problem is balanced.
2. If balanced go to step 3 (no.of rows = no.of columns)
3. If not balanced go to step 2 (no.of rows $\neq$ no.of columns)
4. If the given Fuzzy assignment problem is not balanced then add dummy row (or) dummy column to change the problem into balanced. (Introduce a dummy row/dummy column with the cost value as zero).
5. Defuzzify the Intuitionistic Triangular Fuzzy Number.

6. Solve the assignment by the usual Hungarian method to get the optimal solution.

Example: An Intuitionistic Fuzzy Assignment Problem with rows representing 3 machines M_1, M_2, M_3 and columns representing the 3 Jobs J_1, J_2, J_3 is considered. The cost matrix C_{ij}^* whose elements are Intuitionistic Triangular Fuzzy Number is given below. The problem is to find the minimum cost.

$$\begin{pmatrix} (4, 7, 14) (3, 7, 24) & (12, 14, 26) (11, 14, 28) & (8, 15, 25) (7, 15, 30) \\ (6, 18, 25) (5, 18, 28) & (13, 19, 28) (12, 19, 36) & (20, 25, 33) (19, 25, 38) \\ (20, 25, 38) (19, 25, 42) & (2, 11, 14) (1, 11, 19) & (6, 12, 15) (5, 12, 20) \end{pmatrix}$$

After defuzzification by $D(A^*)$ the reduced problem is

$$\begin{pmatrix} 8.417 & 15.75 & 15.833 \\ 17.333 & 20.083 & 25.833 \\ 26.583 & 10.333 & 11.833 \end{pmatrix}$$

By Hungarian method, the assignment is

$$\begin{pmatrix} (0) & 4.583 & 3.166 \\ \emptyset & (0) & 4.25 \\ 19 & \emptyset & (0) \end{pmatrix}$$

Finally, the Optimum assignment schedule is

$$M_1 \rightarrow J_1, M_2 \rightarrow J_2, M_3 \rightarrow J_3$$

The Optimum cost = 40.33

5. Conclusion

In this paper, an Intuitionistic Fuzzy Assignment Problem whose cost being taken as Intuitionistic Triangular Fuzzy Number is considered. The optimal solution is obtained by defuzzifying the costs into crisp values by a new measure and solving by usual Hungarian method. The Proposed defuzzification measure is easy to compute and cost effective. Future work is being projected to tackle intuitionistic fuzzy assignment problems with linguistic expressions.

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