FINDING AN OPTIMAL SOLUTION OF THE INTERVAL INTEGER TRANSPORTATION PROBLEMS WITH ROUGH NATURE BY SPLIT AND SEPARATION METHOD

A. Akilbasha\textsuperscript{1}, G. Natarajan\textsuperscript{2}, P. Pandian\textsuperscript{3}

Department of Mathematics, SAS VIT University Vellore-14, Tamil Nadu, INDIA

Abstract: A new method namely, split and separation method based on zero point method [10] is proposed for finding an optimal solution for integer transportation problems where transportation cost, supply and demand are intervals. The proposed method is a rough variable method and also, it has been developed without using the midpoint and width of the interval in the objective function. The solution process is explained with a numerical example. The split and separation method can be served as an important tool for the decision makers when they are handling various types of logistic problems having rough variable parameters.

Key Words: rough variable interval integer transportation problem, upper approximation, lower approximation, optimal solution, zero point method

1. Introduction

Various efficient methods were developed for solving transportation problems with the assumption of precise source, destination parameter, and the penalty factors. In real life problems, these conditions may not be satisfied always. To deal with inexact coefficients in transportation problems, many researchers [2, 3, 4, 8, 9, 14, 15] have proposed fuzzy and interval programming techniques for solving them.
Das et al. [4] proposed a method, called fuzzy technique to solve interval transportation problem by considering the right bound and the midpoint of the interval. Sengupta and Pal [14] proposed a new fuzzy orientated method to solve interval transportation problems by considering the midpoint and width of the interval in the objective function. Akilbasha et al. [1] studied a new approach for solving bottleneck-cost transportation problems. Pandian and Natarajan [11] studied the fully interval integer transportation problem based on zero point method and also they have discussed in this paper that the fully triangular fuzzy numbers transportation problem, here they first considered the fully triangular fuzzy numbers transportation problem then convert it into interval integer transportation problem after that solved it by zero point method.

In this paper, we propose a new method namely, split and separation method to find an optimal solution for rough variable interval integer transportation problems where transportation cost, supply and demand are rough variable intervals. We develop the split and separation method without using the midpoint and width of the interval in the objective function of the fully rough interval transportation problem. The proposed method is based on zero point method [10]. The solution procedure is illustrated with a numerical example. The new method can be served as an important tool for the decision makers when they are handling various types of logistic problems having rough interval parameters.

2. Preliminaries

Let \( D \) denote the set of all closed bounded intervals on the real line \( \mathbb{R} \). That is, 
\[
D = \{[a, b] \mid a \leq b \text{ and } a, b \in \mathbb{R}\}.
\]

We need the following definitions of the basic arithmetic operators and partial ordering on closed bounded intervals which can be found in [8, 5].

**Definition 2.1.** Let \( A = [a, b] \) and \( B = [c, d] \) be in \( D \). Then:
\[
A \oplus B = [a + c, b + d];
\]
\[
A \ominus B = [a - d, b - c];
\]
\[
kA = [ka, kb] \text{ if } k \text{ is a positive real number};
\]
\[
kA = [kb, ka] \text{ if } k \text{ is a negative real number and};
\]
\[
A \otimes B = [p, q] \text{ where } p = \min\{ac, ad, bc, bd\} \text{ and } q = \max\{ac, ad, bc, bd\}.
\]
3. Fully Rough Variable Interval Integer Transportation Problems

Consider the following fully rough variable interval integer transportation problem (FRVIITP):

Minimize \( ([z_1, z_2], [z_3, z_4]) = \sum_{i=1}^{m} \sum_{j=1}^{n} ([c^1_{ij}, c^2_{ij}], [c^3_{ij}, c^4_{ij}]) \otimes ([x^1_{ij}, x^2_{ij}], [x^3_{ij}, x^4_{ij}]) \)

subject to

\[
\sum_{j=1}^{n} ([x^1_{ij}, x^2_{ij}, [x^3_{ij}, x^4_{ij}]) = ([a_i, p_i], [d_i, r_i]), i = 1, 2, \ldots, m \tag{1}
\]

\[
\sum_{i=1}^{m} ([x^1_{ij}, x^2_{ij}, [x^3_{ij}, x^4_{ij}]) = ([b_j, q_j], [e_j, s_j]), j = 1, 2, \ldots, n \tag{2}
\]

\[
x^1_{ij}, x^2_{ij}, x^3_{ij}, x^4_{ij} \geq 0, i = 1, 2, \ldots, m \quad \text{and} \quad j = 1, 2, \ldots, n \quad \text{are integers}
\]

\[
x^3_{ij} \leq x^1_{ij} < x^2_{ij} \leq x^4_{ij} \quad \text{for all } i \text{ and } j, \tag{3}
\]

where \( c^1_{ij}, c^2_{ij}, c^3_{ij}, \) and \( c^4_{ij} \) are positive real numbers for all \( i \) and \( j \). \( a_i, p_i, d_i \) and \( r_i \) are positive real numbers for all \( i \) and \( b_j, q_j, e_j \) and \( s_j \) are positive real numbers for all \( j \).

**Case (i):** Lower Approximation of the FRVIITP.

Minimize \( [z_1, z_2] = \sum_{i=1}^{m} \sum_{j=1}^{n} [c^1_{ij}, c^2_{ij}] \otimes [x^1_{ij}, x^2_{ij}] \)

subject to

\[
\sum_{j=1}^{n} [x^1_{ij}, x^2_{ij}] = [a_i, p_i], \quad i = 1, 2, \ldots, m, \tag{4}
\]

\[
\sum_{i=1}^{m} [x^1_{ij}, x^2_{ij}] = [b_j, q_j], \quad j = 1, 2, \ldots, n, \tag{5}
\]

\[
x^1_{ij}, x^2_{ij} \geq 0, i = 1, 2, \ldots, m \quad \text{and} \quad j = 1, 2, \ldots, n \quad \text{are integers}, \tag{6}
\]

where \( c^1_{ij} \) and \( c^2_{ij} \) are positive real numbers for all \( i \) and \( j \). \( a_i \) and \( p_i \) are positive real numbers for all \( i \) and \( b_j \) and \( q_j \) are positive real numbers for all \( j \).
Case (ii): Upper Approximation of the FRVIITP.

\[
\text{Minimize } [z_3, z_4] = \sum_{i=1}^{m} \sum_{j=1}^{n} [c_{ij}^3, c_{ij}^4] \otimes [x_{ij}^3, x_{ij}^4],
\]

subject to

\[
\sum_{j=1}^{n} [x_{ij}^3, x_{ij}^4] = [d_i, r_i], \quad i = 1, 2, \ldots, m, \quad (7)
\]

\[
\sum_{i=1}^{m} [x_{ij}^3, x_{ij}^4] = [e_j, s_j], \quad j = 1, 2, \ldots, n, \quad (8)
\]

\[
x_{ij}^3, x_{ij}^4 \geq 0, \quad i = 1, 2, \ldots, m \text{ and } j = 1, 2, \ldots, n \text{ are integers}, \quad (9)
\]

where \(c_{ij}^3\) and \(c_{ij}^4\) are positive real numbers for all \(i\) and \(j\). \(d_i\) and \(r_i\) are positive real numbers for all \(i\) and \(e_j\) and \(s_j\) are positive real numbers for all \(j\).

Definition 3.1. The set

\[
\{([x_{ij}^1, x_{ij}^2], [x_{ij}^3, x_{ij}^4]) \text{ for all } i = 1, 2, \ldots, m \text{ and } j = 1, 2, \ldots, n\}
\]

is said to be a feasible solution of the (FRVIITP) if they satisfy the equations (1), (2) and (3).

Definition 3.2. A feasible solution

\[
\{([x_{ij}^1, x_{ij}^2], [x_{ij}^3, x_{ij}^4]) \text{ for all } i = 1, 2, \ldots, m \text{ and } j = 1, 2, \ldots, n\}
\]

of the (FRVIITP) is said to be an optimal solution of (FRVIITP) if

\[
\sum_{i=1}^{m} \sum_{j=1}^{n} ([c_{ij}^1, c_{ij}^2], [c_{ij}^3, c_{ij}^4]) \otimes ([x_{ij}^1, x_{ij}^2], [x_{ij}^3, x_{ij}^4])
\]

\[
\leq \sum_{i=1}^{m} \sum_{j=1}^{n} ([u_{ij}^1, u_{ij}^2], [v_{ij}^1, v_{ij}^2]),
\]

for \(i=1,2,\ldots,m\) and \(j=1,2,\ldots,n\) and for all feasible

\[
\{[u_{ij}^1, u_{ij}^2], [v_{ij}^1, v_{ij}^2] \text{ for } i = 1, 2, \ldots, m \text{ and } j = 1, 2, \ldots, n\}.
\]
4. Rough Set

4.1. Rough Set

Rough set theory developed by Pawlak [12, 13] is a mathematical tool for dealing with uncertain and incomplete data without any prior knowledge about the data. We deal only with the available information provided by the data to generate conclusion.

Let $U$ be a finite non empty set called the universal and let $R$ be a binary relation defined on $U$. Let $R$ be a equivalence relation and $R(x)$ be the equivalence class of the relation which contain $X$. $R$ shall be referred as indiscernibility relation.

For any $X \subseteq U$, the lower and upper approximation of $X$ is defined by $R(X) = \{x \in U : R(x) \subseteq X\}$, $\overline{R}(X) = \{x \in U : R(x) \cap X \neq \phi\}$.

The lower approximation $R(X)$ is exact set contained in $X$ so that the object in $R(X)$ are members of $x$ with certainty on the basis of knowledge in $R$, where the objects in the upper approximation $\overline{R}(X)$ can be classified as possible members of $X$. The difference between the upper and lower approximation of $X$ will be called as $R$-boundary of $X$ and is defined by $BNR(X) = \overline{R}(X) - R(X)$.

The set $X$ is $R$-exact if $BNR(X) = \phi$, otherwise the set is $R$-rough set.

4.2. Rough Variable

The concept of rough variable is introduced by Liu [6] as uncertain variable. The following definitions are based on Liu [6, 7].

Definition 4.3. Let $\Lambda$ be a non empty set, $A$ be an $\sigma$ - algebra of subsets of $\Lambda$, $\Delta$ be an element in $A$, and $\pi$ be a non negative, real- valued, additive set function on $A$. Then $(\Lambda, \Delta, A, \pi)$ is called a rough space.

Definition 4.4. A rough variable $\xi$ on the rough space $(\Lambda, \Delta, A, \pi)$ is a measurable function from $\Lambda$ to the set of real numbers $\mathbb{R}$ such that for every Borel set $B$ of $\mathbb{R}$, we have $\{\lambda \in \Lambda | \xi(\lambda) \in B\} \in A$. Then the lower and upper approximation of the rough variable $\xi$ are defined as follows $\underline{\xi} = \{\xi(\lambda) | \lambda \in \Delta\}$, $\overline{\xi} = \{\xi(\lambda) | \lambda \in \Lambda\}$.

5. Split and Separation Method

The split and separation method proceeds as follows:
Step 1: split the given rough variable interval integer transportation problem into two interval integer transportation problems that is lower and upper approximation problem of the rough variable interval integer transportation problem.

Step 2: case (i): Upper Approximation Problem Construct the UBITP of the upper approximation problem then solve it by zero point method. Let \( \{ \hat{x}_{ij}^4, \text{ for all } i \& j \} \) be an optimal solution of the UBITP of the upper approximation problem.

Step 3: Construct the LBITP of the upper approximation problem then solve it with the upper bound constraints \( x_{ij}^3 \leq \hat{x}_{ij}^4 \), for all i & j by using the zero point method. Let \( \{ \hat{x}_{ij}^3, \text{ for all } i \& j \} \) be an optimal solution of the LBITP of the upper approximation problem with \( \hat{x}_{ij}^3 \leq \hat{x}_{ij}^4 \), for all i & j.

Step 4: case (ii): Lower Approximation Problem Construct the UBITP of the lower approximation problem then solve it by zero point method. Let \( \{ \hat{x}_{ij}^2, \text{ for all } i \& j \} \) be an optimal solution of the UBITP of the lower approximation problem.

Step 5: Construct the LBITP of the lower approximation problem then solve it with the upper bound constraints \( x_{ij}^1 \leq \hat{x}_{ij}^2 \), for all i & j by using the zero point method. Let \( \{ \hat{x}_{ij}^1, \text{ for all } i \& j \} \) be an optimal solution of the LBITP of the lower approximation problem with \( \hat{x}_{ij}^1 \leq \hat{x}_{ij}^2 \), for all i & j.

Step 6: The optimal solution of the given FRVIITP is

\[
\{(\hat{x}_{ij}^1, \hat{x}_{ij}^2), [\hat{x}_{ij}^3, \hat{x}_{ij}^4]\}, \text{for all } i = 1, 2, ...m \text{ and } j = 1, 2, ...n.\]

The proposed algorithm is illustrated by the following example.

**Example 1.** Consider the following FRVIITP

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>([6, 8], [5, 9])</td>
<td>([11, 13], [10, 14])</td>
<td>([9, 10], [7, 11])</td>
<td>([9, 13], [6, 17])</td>
</tr>
<tr>
<td></td>
<td>([3, 4], [2, 6])</td>
<td>([2, 3], [1, 8])</td>
<td>([5, 7], [4, 9])</td>
<td>([9, 10], [7, 12])</td>
</tr>
<tr>
<td></td>
<td>([4, 5], [2, 6])</td>
<td>([2, 3], [1, 4])</td>
<td>([9, 10], [8, 12])</td>
<td>([7, 14], [4, 16])</td>
</tr>
<tr>
<td>Demand</td>
<td>([13, 19], [10, 23])</td>
<td>([4, 8], [2, 10])</td>
<td>([8, 10], [5, 12])</td>
<td></td>
</tr>
</tbody>
</table>

Now, consider the UBITP of the upper approximation problem of FRVIITP then using the zero point method, we get the optimal solution to the UBITP.
of the upper approximation of the given problem is \( \dot{x}_{11}^4 = 5, \dot{x}_{13}^4 = 12, \dot{x}_{21}^4 = 12, \dot{x}_{31}^4 = 6, \dot{x}_{32}^4 = 10. \)

Very similar to the above problem, after using the zero point method we get the optimal solutions of:

(1) the LBITP of the upper approximation problem (with the upper bounded constraints \( x_{ij}^3 \leq \dot{x}_{ij}^4 \), for all \( i = 1,2,...m \) and \( j = 1,2,...n \) and are integers). (2) the UBITP of the lower approximation problem;

(3) the LBITP of the lower approximation problem (with the upper bounded constraints \( x_{ij}^1 \leq \dot{x}_{ij}^2 \), for all \( i = 1,2,...m \) and \( j = 1,2,...n \) and are integers) of FRVIITP is respectively given below:

(1) \( \dot{x}_{11}^3 = 1, \dot{x}_{13}^3 = 5, \dot{x}_{21}^3 = 7, \dot{x}_{31}^3 = 2, \dot{x}_{32}^3 = 2; \)
(2) \( \dot{x}_{11}^2 = 3, \dot{x}_{13}^2 = 10, \dot{x}_{21}^2 = 10, \dot{x}_{31}^2 = 6, \dot{x}_{32}^2 = 8; \)
(3) \( \dot{x}_{11}^1 = 1, \dot{x}_{13}^1 = 8, \dot{x}_{21}^1 = 9, \dot{x}_{31}^1 = 3, \dot{x}_{32}^1 = 4. \)

Thus, an optimal solution to the given FRVIITP is

\[
([\dot{x}_{11}^1, \dot{x}_{11}^2], [\dot{x}_{11}^3, \dot{x}_{11}^4]) = ([1, 3], [1, 5]);
([\dot{x}_{13}^1, \dot{x}_{13}^2], [\dot{x}_{13}^3, \dot{x}_{13}^4]) = ([8, 10], [5, 12]);
([\dot{x}_{21}^1, \dot{x}_{21}^2], [\dot{x}_{21}^3, \dot{x}_{21}^4]) = ([9, 10], [7, 12]);
([\dot{x}_{31}^1, \dot{x}_{31}^2], [\dot{x}_{31}^3, \dot{x}_{31}^4]) = ([3, 6], [2, 6]);
([\dot{x}_{32}^1, \dot{x}_{32}^2], [\dot{x}_{32}^3, \dot{x}_{32}^4]) = ([4, 8], [2, 10]);
\]

and also, the minimum transportation cost is \([125, 218], [60, 325]\).

6. Conclusion

In this work, the cost of transportation from the source to destination is considered to be rough costs are assigned. The availability as well as the demand is also considered to be rough interval parameters. The split and separation method based on the zero point method provides an optimal value of the objective function for the fully rough interval integer transportation problem. This method is a systematic procedure, both easy to understand and to apply. Interval integer transportation problem with uncertain variable such as fuzzy is discussed by many researchers, but an interval integer transportation problem with rough interval parameters are not discussed before. This proposed method gives more options and very helpful to the decision makers who are handling cost, availability and demand are in rough interval parameters.
References


