

**APPLICATION OF FDTD METHOD IN SIMULATION
OF FLAT-TOP BEAM IN OPTICAL FIBER**

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Abstract: In this work, an investigation of the propagation characteristics of a flat-top laser beam in Kerr nonlinear media using Finite-Difference-Time-Domain Method (FDTD) has been presented. First an appropriate nonlinear Schrödinger equation (NLSE) has been chosen to find the beam properties in optical fiber, then the discretization method is used to find central difference in space for FDTD simulation, which is written in C language. This simulation work has revealed the unstable beam behavior in the system.

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1. Introduction

In mathematics and physics, the term soliton has been used for a self-reinforcing

solitary wave that maintains its shape during propagation. The solitons arise due to cancellation of nonlinear and dispersive/diffractive effects of the propagating medium. In mathematics, the solitons are the solutions of nonlinear partial differential equations which describe the physical systems. In the field of optical fiber communication, solitons are categorized as temporal, spatial and spatiotemporal soliton.

2. Mathematical Model

In this work, spatial soliton [1] is discussed, which arises due to perfect balance of diffraction in fiber with fiber nonlinearity during beam propagation through optical fiber. The beam dynamics of spatial soliton is modeled by well known nonlinear Schrödinger equation (NLSE) [2], which is shown below

$$i \frac{\partial \Psi}{\partial z} = \nabla^2 \Psi + \kappa |\Psi|^2 \Psi \quad (1)$$

where Ψ is the normalized slowly varying amplitude of the propagating electric field envelope and κ is coefficient of nonlinearity. The first term on right hand side of Eq. (1) is due to diffraction whereas the second term is due to nonlinearity of the medium. The perfect balance between first and second term results in soliton. In this work, the Eq. (1) has been solved numerically using well known *finite difference time domain method* (FDTD) [3, 4]. It should be noted that the Ψ is a complex quantity, hence can be written as,

$$\Psi = a + i b. \quad (2)$$

Now inserting Eq. (2) into Eq. (1) yields,

$$i \frac{\partial a}{\partial z} - \frac{\partial b}{\partial z} = \nabla^2 a + i \nabla^2 b + \kappa (a^2 + b^2) a + i \kappa (a^2 + b^2) b. \quad (3)$$

After separating real and imaginary part from the Eq. (3), a set of coupled ordinary differential equations (ODEs) are obtained,

$$\frac{\partial a}{\partial z} = \nabla^2 b + \kappa (a^2 + b^2) b, \text{ and} \quad (4)$$

$$\frac{\partial b}{\partial z} = -\nabla^2 a - \kappa (a^2 + b^2) a. \quad (5)$$

These ODEs can be solved by various methods of numerical integration [5], namely, Euler's method, Heun's method or Runge-Kutta method. In this work,

Heun's method is used for which the approximation of the solution $y(z)$ to the equation,

$$\frac{\partial y}{\partial z} = f(y), \quad \text{with } y_{z=0} = y_0, \quad (6)$$

is obtained as

$$y_{n+1} = y_n + \frac{dz}{2}(k_1 + k_2), \quad (7)$$

where,

$$k_1 = f(y_n), \quad \text{and } k_2 = f(y_n + dz k_1). \quad (8)$$

Now applying same procedure to Eq. (4) and Eq. (5), the solution can be obtained as,

$$a_{j+1} = a_j + \frac{dz}{2}(k_{1,a_j} + k_{2,a_j}) \quad (9)$$

$$b_{j+1} = b_j + \frac{dz}{2}(k_{1,b_j} + k_{2,b_j}), \quad (10)$$

where,

$$k_{1,a_j} = f_a(a_j, b_j) \equiv \nabla^2 b_j + \kappa(a_j^2 + b_j^2) b_j. \quad (11)$$

$$k_{1,b_j} = f_b(a_j, b_j) \equiv -\nabla^2 a_j - \kappa(a_j^2 + b_j^2) b_j \quad (12)$$

$$k_{2,a_j} = f_a(a_j + dz k_{1,a_j}, b_j + dz k_{1,b_j}), \quad (13)$$

$$k_{2,b_j} = f_b(a_j + dz k_{1,a_j}, b_j + dz k_{1,b_j}), \quad (14)$$

After the discretization of $\nabla^2 a$ in Eqs. (11), the equation becomes,

$$\nabla^2 a \approx \frac{a_{i-1,j} - 2a_{i,j} + a_{i+1,j}}{(dx)^2}. \quad (15)$$

The boundary conditioned used are,

$$\Psi'(x = 0) = 0 \quad \text{and} \quad \Psi'(x \rightarrow \infty) = 0. \quad (16)$$

and at the other boundary, the condition is set as,

$$a_{M,j} = 0, \quad \text{and} \quad b_{M,j} = 0. \quad (17)$$

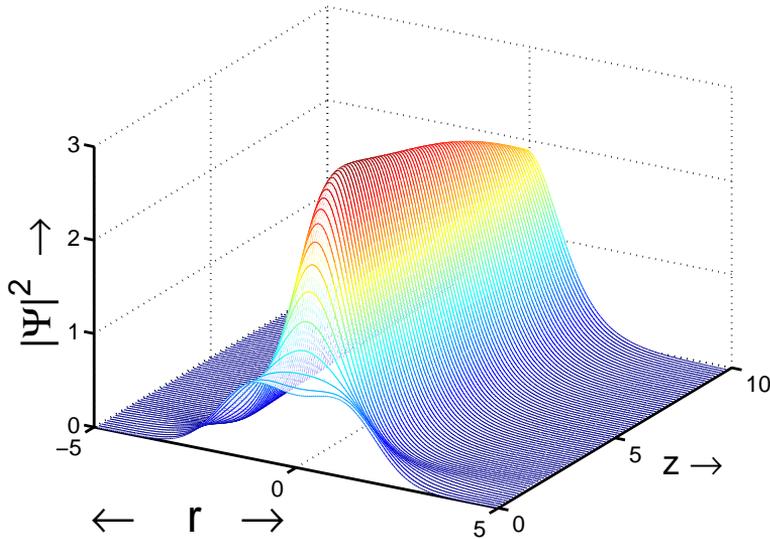


Figure 1: The evolution of a cosh-Gaussian beam in Kerr nonlinear media. $\Psi_0 = 0.881$, $\Omega = 1.0$ and $r_0 = 1.114$.

3. Result and Conclusion

The simulation of FDTD code has been executed with the initial profile of a flat-top beam, which is represented by the cosh-Gaussian ansatz as,

$$\Psi(r, z) = \Psi(0) \cdot \exp\left(-\frac{r^2}{2r_0^2}\right) \cdot \cosh(\Omega \cdot r), \quad (18)$$

where $\Omega(z)$ is the cosh factor, $r_0(z)$ is the width of the beam, r is the radial coordinate of the cylindrical coordinate system and $\Psi(0)$ is the beam amplitude. For this simulation, the initial beam profile is sampled into 1200 points with step size of $dx = 0.05$ and the simulation progresses in z -direction with step size $dz = 0.000001$.

The Fig. 1 has shown the evolution of a cosh-Gaussian beam as it propagates into optical fiber. The beam is initialized with parameters $\Psi_0 = 0.881$, $\Omega = 1.0$ and $r_0 = 1.114$ in the simulation. In conclusion, It is clear from Fig. 1 that the shape of flat-top beam is unstable and it changes into various shapes before becoming sech shape.

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