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BIRTH AND GROWTH OF SUMMABILITY AND PPROXIMATION THEORY

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In the beginning of the 19th century, it was found that there were several series in which the concept of ordinary convergence was clearly failed so it become necessary to consider generalized convergence methods.

With the appearance of Cauchy's monumental work 'Course d'Analyse Algebrique in 1821 and Abel's work on binomial series in 1826, the old hazy notion of convergence of infinite series was put on sound foundation. It was, however, observed that there were certain non-convergent series, which particulasrly in Dynamical Astronomy furnished nearly correct results. It is quite true that the attention paid towards divergent series steadily diminished during the first eighty years of the 19th century.

Approximation theory is that area of analysis which, at its core, is concerned with the ability to approximate functions by simpler and more easily calculated functions. It is an area which, like many other fields of analysis, has its primary roots in the mathematics of the 19th century. At the beginning of the 19th century functions were essentially viewed via concrete formulae, series, or as solutions of equations. However largely as a consequence of the claims of Fourier and the results of Dirichlet, the modern concept of a function distinguished by its requisite properties was introduced and accepted. Once a function, and more specifically a continuous function, is defined implicitly rather than explicitly, the birth of approximation theory becomes an inevitable and unavoidable development.

In this talk we shall discus the origin and developments in the field of summability and approximation theory and their utilities in other fields.

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