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GAUSSIAN BEAM IN HIGHLY NONLOCAL NONLINEAR MEDIUM

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Abstract: The spatial solitons are generated due to perfectly balance between the nonlinear effect and the diffraction in optical waveguide, when high energy optical beam propagated through it. When the medium's response of nonlinearity is instantaneous then it is called local nonlinear medium, otherwise it is called nonlocal nonlinear medium. An attempt has been made to investigate the propagation characteristics of Gaussian beam in highly nonlocal nonlinear media. The optical beam propagation has been modeled by well known nonlocal nonlinear Schrödinger equation (NNLSE). The variational method is employed to obtain various first order differential equations showing the variation of free pulse parameter along the propagation distance and critical power for soliton propagation has also been obtained.

AMS Subject Classification: 78A60
Key Words: nonlocal solitons, variational method

1. Introduction

The optical spatial solitons in nonlocal nonlinear media are widely researched theoretically and experimentally [1, 2, 3, 4, 5, 6, 7]. The Nonlocality of optical materials are generally categorized as local, weak nonlocal, general nonlocal, and strong nonlocal. These nomenclature of optical nonlocal materials are based on the relative width of optical beam and length of response function [1]. In case of weak nonlocal the characteristic length of response function is much narrower than the width of the optical beam where as in case of strong nonlocal, the characteristic length of the response is much broader than the width of the optical beam. General nonlocality is the case between weak nonlocality and strong nonlocality. In the recent past, the theoretical research has taken two different type of response function profiles, one is a Gaussian-type response function [2] and the other is an exponential-decay response function [3]. Extensive investigations on strong nonlocal media have given many new phenomena such as large phase shift [1], attraction between out-of-phase solitons [4, 5], attraction between dark solitons [6], and long-range interaction between solitons [7], etc.

2. Mathematical Model

The propagation of a optical beam through nonlocal nonlinear medium is modeled by nonlocal nonlinear Schrödinger equation (NNLSE) of the form

$$i\,\frac{\partial\psi}{\partial z} + \mu\,\frac{\partial^2\psi}{\partial x^2} + \rho\,\psi \int_{-\infty}^{\infty} R(x-\xi)\,I(\xi,z)\,d\xi = 0 \tag{1}$$

where $\psi(x,z)$ is a slowly varying envelop, $\mu = 1/2k$, $\rho = k\eta$, k is the wave number, η is the material constant, z is the longitudinal propagation distance, $R(x)$ is the nonlocal response function, and $I(\xi,z)\,\big(= |\psi(\xi,z)|^2\big)$ is the intensity of the paraxial beam. In present investigation, we consider a quasi-monochromatic partially incoherent beam propagating in isotropic nonlocal kerr media possessing a Gaussian nonlocal response kernel $R(x)$ such that $\int R(x)dx = 1$, i.e.

$$R(x) = \frac{1}{\sqrt{\pi}\sigma}\exp\left(-\frac{x^2}{\sigma^2}\right), \tag{2}$$

where σ is the extent of the nonlocality or the length of response function.
The NNLSE (1) is a nonlinear differential equation which do not have any exact solution, hence the solution is obtained by an approximation technique,

variational analysis [8], which has been used successfully by various authors [1, 9] for solving nonlinear equations. The required Lagrangian density for Eq. (1) is given as

$$L = \frac{i}{2}\left(\psi^* \frac{\partial \psi}{\partial z} - \psi \frac{\partial \psi^*}{\partial z}\right) - \mu \left|\frac{\partial \psi}{\partial x}\right|^2 + \frac{\rho}{2} |\psi|^2 \int_{-\infty}^{\infty} R(x - \xi)\, I(\xi, z)\, d\xi \tag{3}$$

A Gaussian ansatz has been chosen as,

$$\psi(x, z) = A(z) \, \exp\left(-\frac{x^2}{2\, w^2(z)}\right) \, \exp\big(i\, c(z) x^2 + i\, \theta(z)\big) \tag{4}$$

where $A(z)$ is amplitude, $w(z)$ is width, $c(z)$ is phase front curvature, and $\theta(z)$ is phase of the beam.
The average Lagrangian $\langle L \rangle$ has been obtained by inserting ansatz function into Eq. (3).

$$\begin{aligned} \langle L \rangle &= \int_{-\infty}^{\infty} L \, dx \\ &= \frac{\sqrt{\pi}\, \rho\, w^2 A^4}{2\sqrt{2\, w^2 + \sigma^2}} - \sqrt{\pi}\, A^2 \left(w\, \frac{\partial \theta}{\partial z} + \frac{w^3}{2} \frac{\partial c}{\partial z} + \frac{\mu\, \left(1 + 4\, c^2 w^4\right)}{2\, w} \right). \end{aligned} \tag{5}$$

The average Lagrangian results in a set of dynamical equations corresponding to different free pulse parameters after applying the Euler-Lagrange equation

$$\frac{\partial \langle L \rangle}{\partial r_j} - \frac{d}{dz}\left(\frac{\partial \langle L \rangle}{\partial \dot{r}_j}\right) = 0, \tag{6}$$

where $r_j = A(z), c(z), w(z), \text{and } \theta(z)$. Now we have substituted Eq. (5) into Eq. (6) to solve it for pulse parameters, which results in several first order ordinary differential equations showing the variation of free pulse parameter along the propagation distance. These equations are:

$$\frac{\partial c}{\partial z} = -4\, \mu\, c^2 + \frac{\mu}{w^4} - \frac{\rho\, P_0}{\sqrt{\pi}\, \left(2\, w^2 + \sigma^2\right)^{3/2}} \tag{7}$$

$$\frac{\partial \theta}{\partial z} = \frac{\rho\, P_0\, \left(5\, w^2 + 2\, \sigma^2\right)}{2\sqrt{\pi}\, \left(2\, w^2 + \sigma^2\right)^{3/2}} - \frac{\mu}{w^2} \tag{8}$$

$$\frac{\partial w}{\partial z} = 4\, \mu\, c\, w. \tag{9}$$

The amplitude $A(z)$ and width $w(z)$ of the beam are related to $\sqrt{\pi} w\, A^2 = P_0$, where P_0 is initial beam power.

The differential of Eq. (9) with respect to z with normalization $y(z) = w/w_0$, where $w_0 = w(0)$, to give

$$\frac{1}{\mu}\frac{\partial^2 y}{\partial z^2} = \frac{4\,\mu}{{w_0}^3 y^3} - \frac{4\,\rho\,P_0 w_0\,y}{\sqrt{\pi}\,\left(2\,{w_0}^2 y^2 + \sigma^2\right)^{3/2}} \equiv F(y). \tag{10}$$

Eq (10) is analogous to the Newton's second law of motion in classical mechanics under the force $F(y)$. This force is a balance between diffractive and refractive forces represented by the first and the second terms of the equation, respectively. If both forces are equal and $y = 1$, we can obtain the critical power for soliton propagation as

$$P_c = \frac{\sqrt{\pi}\,\mu\,\left(2\,{w_0}^2 + \sigma^2\right)^{3/2}}{{w_0}^4 \rho}. \tag{11}$$

As the $F(y)$ is a conservative force, i.e., $F(y) = -dV(y)/dy$, the equivalent potential can be written as,

$$V\left(y\right) = \frac{2\,\mu\,\left(1 - y^2\right)}{{w_0}^3 y^2} - \frac{2\,\rho\,P_0}{\sqrt{\pi}\,w_0}\left(\frac{1}{\sqrt{2\,{w_0}^2 y^2 + \sigma^2}} - \frac{1}{\sqrt{2\,{w_0}^2 + \sigma^2}}\right). \tag{12}$$

3. Result and Conclusion

The present work emphasizes the propagation of spatial solitons in highly nonlocal nonlinear medium taking Gaussian response function and Gaussian optical beam into consideration. This has been modeled using nonlocal nonlinear Schrödinger equation and the approximate solution in the form of various first order differential equations has been obtained using the variational method. Further an equation to estimate the critical power for soliton propagation has also been obtained. These equations will play an important role in explaining Nematic crystal display, Bose-Einstein condensate, etc.

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