Stokes Flow through a Porous Sphere in a Cell Surface with Zero Spin Conditions

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Abstract

This paper deals with the problem of creeping flow of an incompressible micropolar fluid through a porous sphere. The flow inside the porous sphere governed by Brinkman equation for the micropolar fluid and outside the porous sphere governed by Stokes equation for the clear fluid. The stream function solutions of the governing equations are acquired in terms of modified Bessel functions and Gegenbauer functions. On the hypothetical cell surface the Happels condition is assumed with a uniform velocity condition. Drag force and drag coefficient are computed numerically. The variation of drag coefficient with permeability parameter, viscosity ratio, and the volume fraction are demonstrated graphically.

Key Words and Phrases: Brinkman equation, Stokes function, Cell model, Permeability, Micropolar fluid, Viscous fluid, Bessels function, Drag force, Volume fraction.

1 Introduction

Many process and phenomena in Science and Engineering are involved the motion of flow fluids through porous media. Micropolar fluids are fluids with microstructure. Micropolar fluids[1] can also
personate fluids consisting of rigid, random oriented particles restricted in a very small volume element and the elements are rotate about the centre derived by microrotation vector. Micropolar fluids are support body couples and exhibit microrotational vector effects. The examples of Micropolar fluids are certain additive, some polymeric fluids and also animal blood. Commonly on geophysical and biochemical environment and also have engineering applications like sedimentation, fluidization, petroleum industry and lubricant problem, etc. The flow inside the sphere using Brinkman equation Qin and Kaloni studied the flow of an incompressible Newtonian fluid over a porous sphere, they obtained the force of hydrodynamic. D.Srinivasacharya and I.Rajyalakshmi obtained the stokes flow of a micropolar fluid over a porous sphere also they shown the flow pattern, depends upon the various value of micropolar parameters, permeability parameter and the coupling number. Zlatonovski has considered well thought-out the creeping flow with an incompressible Newtonian fluid over a porous prolate spheroidal particle with axi-symmetric by applying Brinkman model. D.Srinivasacharya have evaluated the stokes flow of fluid through a porous approximate sphere. Creeping flow over a porous particles effectively modelled by assuming the cell model technique as represented in the book of Happel and Brenner. In the present era the cell model techniques is most helpful equipment for the creeping flow within porous medium for being the most saturated assemblage from the particles. The spherical motion of a porous sphere within a spherical container calculated by D.Srinivasacharya. He obtained the stream functions and drag force acting on the body. Stokes flow past within a group of porous spherical particles with the help of Mehta-Morse condition evaluated by Satya Deo and Pankaj Shukla. They calculated the stream function and the drag force acting on the body.

This paper we consider the problem of creeping flow of an incompressible micropolar fluid through a porous spherical particle. The flow of inside the porous sphere governed by Brinkman equation and outside region Stokes equation for the clear fluid. The stream function solutions of the governing equations are acquired in terms of modified Bessel functions and Gegenbauer functions. On the hypothetical cell surface the Happels condition is assumed with an uniform velocity. Drag force and drag coefficient are computed...
numerically. The variation of drag coefficient with permeability parameter and volume fraction are demonstrated graphically using Mathematica.

2 Mathematical formulation of the problem

Consider the bounded sphere which is axi-symmetric and incompressible viscous fluid over the porous sphere with uniform velocity $U$ along with $z$ direction. Also the permeable medium is assumed to be homogeneous and isotropic.

The governing equation of the creeping flow for the region(1) within the cell surface is given by

$$\text{div} \tilde{q}^{(1)} = 0 \quad (1)$$

$$\mu \nabla^2 \tilde{q}^{(1)} = \nabla \tilde{p}^{(1)} \quad (2)$$

Here $\tilde{q}^{(1)}$ is velocity, $\mu$ is coefficient of viscosity and $\tilde{p}^{(1)}$ is the hydrostatical pressure outside the porous sphere. From the interior region(2), the porous sphere governed by the Brinkman equation on the cell surface are given by

$$\text{div} \tilde{q}^{(2)} = 0 \quad (3)$$

$$\frac{\mu}{\kappa} \tilde{q}^{(2)} + \nabla \tilde{p}^{(2)} - \kappa \nabla \times \tilde{\omega}^{(2)} + (\mu + \kappa) \nabla \times \nabla \times \tilde{q}^{(2)} = 0 \quad (4)$$

$$-2\kappa \tilde{\omega}^{(2)} + \kappa \nabla \times \tilde{\omega}^{(2)} - \gamma \nabla \times \nabla \times \tilde{\omega}^{(2)} + (\alpha + \beta + \gamma) \nabla (\nabla \cdot \tilde{\omega}^{(2)}) = 0 \quad (5)$$

Remaining constants $\alpha, \beta, \mu, \kappa$ and $\gamma$ are satisfying the following inequalities,

$$3\alpha + \beta + \gamma \geq 0, \quad 2\mu + \kappa \geq 0, \quad \gamma \geq |\beta|, \quad \kappa \geq 0, \quad \gamma \geq 0 \quad (6)$$

3 Solution of the problem

The stream function for both regions are

$$u^{(i)} = -\frac{1}{r^2 \sin \theta} \frac{\partial \psi^{(i)}}{\partial \theta} \quad v^{(i)} = \frac{1}{r \sin \theta} \frac{\partial \psi^{(i)}}{\partial r} \quad i = 1, 2 \quad (7)$$
Assuming the dimensionless variables,
\[ r = a\tilde{r}, \quad \psi^{(i)} = Ua^2\tilde{\psi}^{(i)}, \quad \tilde{p}^{(i)} = \mu\frac{U}{a}\tilde{p}^{(i)}, \quad v\phi^{(i)} = \frac{U}{a}v\phi^{(i)} \] (8)
and dropping tildes subsequently in further analysis. In equation (1) and (5) eliminating the pressure from the resulting equations, we get the following stream functions which satisfy regularity condition for our model is given as,
\[ E^4\psi^{(1)} = 0 \] (9)
\[ E^2(E^2 - \alpha^2)(E^2 - \beta^2)\psi^{(2)} = 0 \] (10)
Here the stokesian stream function operator is \( E^2 \).
From the Eqs.(9) and (10), we get the stream functions,
\[ \psi^{(1)}(r, \zeta) = [A_1r^{-1} + B_1r + C_1r^2 + D_1r^4]G_2(\zeta) \] (11)
\[ \psi^{(2)}(r, \zeta) = \left[A_2r^2 + C_2\sqrt{r}I_{2\alpha}(\alpha r) + D_2\sqrt{r}I_{2\beta}(\beta r)\right]G_2(\zeta) \] (12)
where \( \zeta = \cos\theta, \) \( G_2(\zeta) \) being the Gegenbauer function.
The microroration component of the porous sphere is given by,
\[ v_\phi^{(2)} = \frac{1}{r\sin\theta} \left[C_2A_\alpha\sqrt{r}I_{2\alpha}(\alpha r) + D_2A_\beta\sqrt{r}I_{2\beta}(\beta r)\right]G_2(\zeta) \] (13)
where \( n^2 = \frac{k(2\mu + k)}{(\mu + k)} \), \( N = \frac{k}{(\mu + k)} \) is the coupling number \( (0 \leq N \leq 1) \) and \( \alpha^2 \) is the micropolar parameter.

4 Boundary Conditions

The dimensionless conditions on the boundary \( r = 1 \) in terms of the stream functions are,
1. Continuity of normal velocity components
\[ \psi^{(1)} = \psi^{(2)} \] (14)
2. Continuity of tangential velocity
\[ \psi_r^{(1)} = \psi_r^{(2)} \] (15)
3. Continuity of tangential stress
\[ \tau_{r\theta}^{(1)} = \tau_{r\theta}^{(2)} \] (16)
4. Continuity of pressure and micro-rotation are given by

\[ p^{(1)} = p^{(2)} \]  \hspace{1cm} (17)

\[ v^{(1)}_\phi = 0 \]  \hspace{1cm} (18)

5. On the outer cell surface at \( r = l \), the condition of impenetrability leads to

\[ v^{(1)}_r = \cos \theta \]  \hspace{1cm} (19)

\[ \tau^{(1)}_\theta = 0 \]  \hspace{1cm} (20)

Using the boundary conditions from (14) and (20) in Eqs.(11) and (13), we get the unknowns \( A_1, B_1, C_1, D_1, A_2, C_2, D_2 \) using Mathematica software.

5 Evaluation of drag force

Drag force calculated by the fluid sphere on the cell surface is given by,

\[ F = \mu \pi u a \int_0^\pi \omega^3 \frac{\partial}{\partial r} \left( \frac{E^2 \psi}{\omega^2} \right) r \, d\theta. \]  \hspace{1cm} (21)

Putting the values \( \omega = r \sin \theta \) and \( E^2(\psi) = \frac{\partial^2 \psi}{\partial \theta^2} + \frac{1}{r^2} \frac{\partial^2 \psi}{\partial r^2} - \frac{\cos \theta}{r^2} \).

in equation (21), we get the drag force,

\[ D = 4\pi \mu u a \, B_1 \]  \hspace{1cm} (22)

Also the non dimensional drag \( D_N \) can be defined as,

\[ D_N = \frac{D}{4\pi \mu u a} \]  \hspace{1cm} (23)

6 Deduction of Known Results:

Case: 1 If \( A_\alpha \to \frac{\eta^2}{2} \) and \( A_\beta \to 0 \), then the drag comes out as,

\[ D_N = - \left[ \frac{\eta^2 \left( \beta \left( 10\lambda \dot{\xi} + \beta^2(3 + 2\lambda \dot{\xi})\cosh \beta \right) - 3(10\lambda \dot{\xi} + \beta^2(1 + 2\lambda \dot{\xi})\cosh \beta) \right)}{\left( \beta \left( 30(-3 + \eta(1 + 4\lambda \dot{\xi}))\lambda \dot{\xi} + \beta^2(-3(1 + 4\lambda \dot{\xi}) + \eta^2(-2 + 3\lambda \dot{\xi} - 3\lambda \dot{\xi} + 2\lambda^2)) \right) \cosh \beta \right)} \right] \]

\[ + \left[ \frac{3(10(3 - \eta^2(-1 + \lambda \dot{\xi}))\lambda \dot{\xi} + \beta^2(1 + 14\lambda \dot{\xi} - \eta^2(4\lambda \dot{\xi} - 5\lambda \dot{\xi} + 4\lambda^2)) \sinh \beta}{\beta \left( 30(-3 + \eta(1 + 4\lambda \dot{\xi}))\lambda \dot{\xi} + \beta^2(-3(1 + 4\lambda \dot{\xi}) + \eta^2(-2 + 3\lambda \dot{\xi} - 3\lambda \dot{\xi} + 2\lambda^2)) \right) \sinh \beta} \right] \]

\hspace{1cm} (24)
This result has been obtained earlier in Davis and Stone [11].

Case: 2 If \( \beta \to \infty \) then the drag comes out as

\[
D_N = 6\pi \mu U a \frac{3 + 2\lambda \frac{1}{3}}{2 - 3\lambda \frac{1}{3} + 3\lambda \frac{1}{3} - 2\lambda^2}
\]  

(25)

where \( \lambda = l^3 \) being the volume fraction. This agree with the result of Happel.J [10]

Case: 3 If \( \lambda \to 0, \beta^2 \to \eta^2 \) then the drag comes out as

\[
D_N = 6\pi \mu U a \frac{\eta^2(\sinh \eta - \eta \cosh \eta)}{\eta(3 + 2\eta^2) \cosh \eta - 3 \sinh \eta}
\]

(26)

which agree with the result derived from Qin and Kaloni[4].

7 Result and Discussion

The drag \( D_N \) is presented graphically for different values of viscosity ratios, permeability parameters, volume fractions are depicted in Figure (1) to (3). From Figure.1, the variation of drag \( D_N \) with

Figure 1: The variation of Drag \( D_N \) with the permeability parameter \( \eta \).
respect to permeability parameter $\eta$ for different values of viscosity ratio $\gamma$ is given. It is observed that the drag is increasing as the permeability parameter is increasing. It is noted that the drag decreases with increasing of viscosity ratio $\gamma$. Here it is almost same for the value of $\gamma < 0.6$, but the drag decreases slowly with increasing of viscosity ratio when $\gamma > 0.6$. The variation of non-dimensional drag $D_N$ with viscosity ratio $\gamma$ as shown in Figure.2. Here the drag slightly decreases with increases of viscosity ratio and also the drag $D_N$ increases with the increasing permeability parameter, then it becomes almost constant for the different values of permeability parameter. Figure 3 shows the drag $D_N$ with volume fraction $\lambda$ for different values of permeability parameter $\eta$. It observed from the figure, the drag is gradually increasing as increasing of the volume fraction, also the drag increases with increasing of the permeability parameter $\eta$. Here it is interesting to note that, the drag almost same for the values of $\lambda < 0.5$. The drag increases slowly with increasing of the volume fraction, when $\lambda > 0.5$.
Figure 3: The variation of Drag $D_N$ with volume fraction $\lambda$.

References


