Rough Neutrosophic Sets In Medical Diagnosis

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Abstract

Neutrosophy is the base of neutrosophic logic, neutrosophic set, neutrosophic probability etc., The concept of rough neutrosophic set is an essential tool for dealing with uncertainties free from the shortcomings that affect the existing methods. In this paper, tangent logarithmic distance and cosecant similarity measure between rough neutrosophic sets are proposed and some of its properties are discussed herein. Utilization of medical diagnosis is presented to find out the disease impacting the patient.

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Key Words and Phrases: Rough neutrosophic set, cosecant similarity measure, tangent logarithmic distance, medical diagnosis.

1 Introduction

Neutrosophic set (generalization of fuzzy sets, intuitionistic fuzzy sets and so on) defined by Florentin Smarandache (see [1]) has capability to deal with uncertainty, imprecise, incomplete and inconsistent information which exists in real world from philosophical
point of view. Said Broumi and Florentin Smarandache (see [4]) developed several similarity measures of neutrosophic sets.

In 1982, Pawlak (see [3]) introduced the concept of rough set (RS), as a formal tool for modeling and processing incomplete information in information systems. There are two basic elements in rough set theory, crisp set and equivalence relation, which constitute the mathematical basis of rough sets. The basic idea of rough set is based upon the approximation of sets by a pair of sets known as the lower approximation and the upper approximation of a set. Here, the lower and upper approximation operators are based on equivalence relation. Nanda and Majumdar (see [2]) examined fuzzy rough sets. Broumi et al (see [5]) introduced rough neutrosophic sets. Surapati Pramanik and Kalyan Mondal (see [7]) introduced cotangent similarity measure of rough neutrosophic sets.

In this paper, by using the notion of rough neutrosophic set, it was provided an exemplary for medical diagnosis. In order to make this, different types of methods are executed.

2 Preliminaries

2.1 Definition (see [5])

Let U be a non-null set and R be an equivalence relation on U. Let P be a neutrosophic set in U with the membership function $T_P$, indeterminacy function $I_P$ and non-membership function $F_P$. The lower and the upper approximations of P in the approximation (U, R) denoted by $\overline{N}(P)$ and $\overline{N}(P)$ are respectively defined as follows:

$$\overline{N}(P) = \{x | T_{\overline{N}(P)}(x), I_{\overline{N}(P)}(x), F_{\overline{N}(P)}(x)) / y \epsilon [x]_R, x \epsilon U \}$$

$$\overline{N}(P) = \{x | T_{\overline{N}(P)}(x), I_{\overline{N}(P)}(x), F_{\overline{N}(P)}(x)) / y \epsilon [x]_R, x \epsilon U \}$$

where $T_{\overline{N}(P)}(x) = \vee_{y \epsilon [x]_R} T_P(y); I_{\overline{N}(P)}(x) = \bigvee_{y \epsilon [x]_R} I_P(y); F_{\overline{N}(P)}(x) = \bigvee_{y \epsilon [x]_R} F_P(y)$

$T_{\overline{N}(P)}(x) = \bigvee_{y \epsilon [x]_R} T_P(y); I_{\overline{N}(P)}(x) = \vee_{y \epsilon [x]_R} I_P(y); F_{\overline{N}(P)}(x) = \bigwedge_{y \epsilon [x]_R} F_P(y)$

So, $0 \leq T_{\overline{N}(P)}(x) + I_{\overline{N}(P)}(x) + F_{\overline{N}(P)}(x) \leq 3$ and $0 \leq T_{\overline{N}(P)}(x) + I_{\overline{N}(P)}(x) + F_{\overline{N}(P)}(x) \leq 3$

where $\vee$ and $\bigwedge$ mean “max” and “min” operators respectively, $T_P(y), I_P(y)$ and $F_P(y)$ are the membership, indeterminacy and non-membership of y with respect to P. It is easy to see that $\overline{N}(P)$ and $\overline{N}(P)$ are two
neutrosophic sets in U, thus the NS mappings \( N, \overline{N} : N(U) \rightarrow N(U) \) are respectively, referred to as the lower and upper rough neutrosophic set approximation operators, and the pair \( (\overline{N}(P), N(P)) \) is called the rough neutrosophic set in \((U,R)\).

3 Proposed Definitions with proposition

3.1 Definition

Let \( A = (T_A(x_i), L_A(x_i), E_A(x_i)) \), \( T_A(x_i), L_A(x_i), E_A(x_i) \) and \( B = (T_B(x_i), L_B(x_i), E_B(x_i)) \), \( T_B(x_i), L_B(x_i), E_B(x_i) \) be two rough neutrosophic sets, then the tangent logarithmic distance is defined as

\[
TLD_{RNS}(A,B) = \frac{1}{n+1} \sum |\tan(\log(1 + |T_A(x_i) - T_B(x_i)|) + |L_A(x_i) - L_B(x_i)| + |E_A(x_i) - E_B(x_i)| + |T_A(x_i) - T_B(x_i)| + |L_A(x_i) - L_B(x_i)| + |E_A(x_i) - E_B(x_i)|)| \quad (1)
\]

3.2 Proposition

(i) \( TLD_{RNS}(A,B) \in [-0, 1^+] \); (ii) \( TLD_{RNS}(A,B) = TLD_{RNS}(B,A) \);

(iii) If \( A \subseteq B \subseteq C \) then \( TLD_{RNS}(A,C) \geq TLD_{RNS}(A,B) \) and \( TLD_{RNS}(A,C) \geq TLD_{RNS}(B,C) \) \textbf{Proof}

(i) The proof is straightforward.

(ii) The proof is straightforward.

(iii) It was well known that,

\[
T_A(x_i) \leq T_B(x_i) \leq T_C(x_i); T_A(x_i) \leq T_B(x_i) \leq T_C(x_i); L_A(x_i) \geq L_B(x_i) \geq L_C(x_i); L_A(x_i) \geq L_B(x_i) \geq L_C(x_i); E_A(x_i) \geq E_B(x_i) \geq E_C(x_i); E_A(x_i) \geq E_B(x_i) \geq E_C(x_i),
\]

since \( A \subseteq B \subseteq C \).

Hence,

\[
|T_A(x_i) - T_B(x_i)| \leq |T_A(x_i) - T_C(x_i)|; |T_A(x_i) - T_B(x_i)| \leq |T_A(x_i) - T_C(x_i)|; |L_A(x_i) - L_B(x_i)| \leq |L_A(x_i) - L_C(x_i)|; |T_A(x_i) - T_B(x_i)| \leq |T_A(x_i) - T_C(x_i)|; |E_A(x_i) - E_B(x_i)| \leq |E_A(x_i) - E_C(x_i)|; |F_A(x_i) - F_B(x_i)| \leq |F_A(x_i) - F_C(x_i)|
\]

Here, the tangent logarithmic distance is an increasing function. Therefore,
\[ T_{LDNS}(A, C) \geq T_{LDNS}(A, B) \text{ and } T_{LDNS}(A, C) \geq T_{LDNS}(B, C) \]

### 3.3 Definition

Let \( A = (T_A(x_i), I_A(x_i), F_A(x_i)) \) and \( B = (T_B(x_i), I_B(x_i), F_B(x_i)) \) be two rough neutrosophic sets, then the cosecant similarity measure is defined as

\[
COSEC_{RNS}(A, B) = \frac{1}{m} \sum [\csc\left(\frac{\alpha(x, Y)}{4m}\right)]
\]

where

\[
X = |T_A(x_i) - T_B(x_i)| + |I_A(x_i) - I_B(x_i)| + |F_A(x_i) - F_B(x_i)|
\]

\[
Y = |T_A(x_i) - T_B(x_i)| + |I_A(x_i) - I_B(x_i)| + |F_A(x_i) - F_B(x_i)|
\]

### 3.4 Proposition

(i) \( COSEC_{RNS}(A, B) \in [0, 1] \); (ii) \( COSEC_{RNS}(A, B) = COSEC_{RNS}(B, A) \);

(iii) If \( A \subseteq B \subseteq C \) then \( COSEC_{RNS}(A, C) \leq COSEC_{RNS}(A, B) \) and \( COSEC_{RNS}(A, C) \leq COSEC_{RNS}(B, C) \)

**Proof**

(i) The proof is straightforward.

(ii) The proof is straightforward.

(iii) It was well known that,

\[
T_A(x_i) \leq T_B(x_i) \leq T_C(x_i); T_A(x_i) \leq T_B(x_i) \leq T_C(x_i)
\]

\[
I_A(x_i) \geq I_B(x_i) \geq I_C(x_i); I_A(x_i) \geq I_B(x_i) \geq I_C(x_i)
\]

\[
F_A(x_i) \geq F_B(x_i) \geq F_C(x_i); F_A(x_i) \geq F_B(x_i) \geq F_C(x_i)
\]

since \( A \subseteq B \subseteq C \)

Hence,

\[
|T_A(x_i) - T_B(x_i)| \leq |T_A(x_i) - T_C(x_i)|; |T_A(x_i) - T_B(x_i)| \leq |T_A(x_i) - T_C(x_i)|
\]

\[
|I_A(x_i) - I_B(x_i)| \leq |I_A(x_i) - I_C(x_i)|; |I_A(x_i) - I_B(x_i)| \leq |I_A(x_i) - I_C(x_i)|
\]

\[
|F_A(x_i) - F_B(x_i)| \leq |F_A(x_i) - F_C(x_i)|; |F_A(x_i) - F_B(x_i)| \leq |F_A(x_i) - F_C(x_i)|
\]

Here, the cosecant similarity measure is a decreasing function

Therefore,

\( COSEC_{RNS}(A, C) \leq COSEC_{RNS}(A, B) \) and \( COSEC_{RNS}(A, C) \leq COSEC_{RNS}(B, C) \)
4 Methodology

In this section, it was presented an application of rough neutroso-
fic set in medical diagnosis. In a given pathology, suppose S is
a set of symptoms, D is a set of diseases and P is a set of patients
and let Q be a rough neutrosophic relation from the set of patients
to the symptoms. i.e., \( Q(P \rightarrow S) \) and R be a rough neutrosophic
relation from the set of symptoms to the diseases. i.e., \( R(S \rightarrow D) \)
and then the methodology involves three main jobs:
(i) Determination of symptoms
(ii) Formulation of medical knowledge based on rough neutrosophic
sets
(iii) Determination of diagnosis on the basis of various computation
techniques of rough neutrosophic sets.

5 Algorithm

Step 1: The Symptoms of the patients are given to obtain the patient-
symptom relation Q and are noted in Table 1.
Step 2: The medical knowledge relating the symptoms with the set
of diseases under consideration are given to obtain the symptom-
disease relation R and are noted in Table 2.
Step 3: The computation T of the relation of patients and diseases
is found using (1) and (2) and are noted in Table 3 and Table 4
respectively.
Step 4: Finally, minimum value from Table 3 and maximum value
Table 4 of each row were selected to find the possibility of the pa-
tient affected with the respective disease and then it was concluded
that the patient \( P_k \) (k=1,2 and 3) was suffering from the disease \( D_r \)
(\( r=1,2,3 \) and 4).

6 Case study(see [6])

Let there be three patients \( P = \{P_1, P_2, P_3\} \) and the set of symp-
toms \( S = \{S_1 = Temperature, S_2 = Headache, S_3 = Stomachpain,
S_4 = Cough, S_5 = Chestpain\} \). The Rough Neutrosophic Relation
Q \( P \rightarrow S \) is given as in Table 1. Let the set of diseases \( D = \{ D_1 = \text{Viralfever}, D_2 = \text{Malaria}, D_3 = \text{Stomachproblem}, D_4 = \text{Chestproblem} \} \). The Rough Neutrosophic Relation \( R (S \rightarrow D) \) is given as in Table 2.

<table>
<thead>
<tr>
<th>( P )</th>
<th>Temperature</th>
<th>Headache</th>
<th>Stomach pain</th>
<th>Cough</th>
<th>Chest pain</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_1 )</td>
<td>((6, .4, .3), (6, .2, 1))</td>
<td>((4, .4, .4), (6, .2, 2))</td>
<td>((5, .3, .2), (7, .1, 2))</td>
<td>((6, .2, 4), (8, .0, 2))</td>
<td>((4, .4, .4), (6, .2, 2))</td>
</tr>
<tr>
<td>( P_2 )</td>
<td>((5, .3, .4), (7, .3, 2))</td>
<td>((5, .5, 3), (7, .3, 2))</td>
<td>((5, .3, 4), (7, .1, 4))</td>
<td>((5, .3, 3), (7, .1, 3))</td>
<td>((5, .3, 3), (7, .1, 3))</td>
</tr>
<tr>
<td>( P_3 )</td>
<td>((6, .4, .4), (8, .2, 2))</td>
<td>((5, .2, 3), (7, .0, 1))</td>
<td>((4, .3, 4), (8, .1, 2))</td>
<td>((6, .1, 4), (8, .1, 2))</td>
<td>((5, .3, 3), (7, .1, 1))</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>R</th>
<th>Viralfever</th>
<th>Malaria</th>
<th>Stomach problem</th>
<th>Chest problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature</td>
<td>((6, .5, .4), (8, .3, 2))</td>
<td>((1, .4, 4), (5, .2, 2))</td>
<td>((3, .4, 4), (5, .2, 2))</td>
<td>((2, .4, 6), (4, .4, 4))</td>
</tr>
<tr>
<td>Headache</td>
<td>((5, .3, 4), (7, .3, 2))</td>
<td>((2, .3, 4), (6, .3, 2))</td>
<td>((2, .3, 3), (4, .1, 1))</td>
<td>((1, .5, 5), (5, .3, 3))</td>
</tr>
<tr>
<td>Stomach pain</td>
<td>((2, .3, 4), (4, .3, 2))</td>
<td>((1, .4, 4), (3, .2, 2))</td>
<td>((4, .3, 4), (6, .1, 2))</td>
<td>((1, .4, 6), (3, .2, 4))</td>
</tr>
<tr>
<td>Cough</td>
<td>((4, .3, 3), (6, .1, 1))</td>
<td>((3, .8, 3), (5, .1, 3))</td>
<td>((1, .6, 6), (3, .4, 4))</td>
<td>((5, .3, 4), (7, .1, 2))</td>
</tr>
<tr>
<td>Chest pain</td>
<td>((2, .4, 4), (4, .2, 2))</td>
<td>((1, .3, 3), (3, .1, 1))</td>
<td>((1, .4, 4), (3, .2, 2))</td>
<td>((4, .4, 4), (6, .2, 2))</td>
</tr>
</tbody>
</table>
Table 3: Tangent logarithmic distance (Using step 3 and step 4)

<table>
<thead>
<tr>
<th></th>
<th>Viral fever</th>
<th>Malaria</th>
<th>Stomach problem</th>
<th>Chest problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>0.0843</td>
<td>0.1195</td>
<td>0.1190</td>
<td>0.1052</td>
</tr>
<tr>
<td>$P_2$</td>
<td>0.0935</td>
<td>0.1163</td>
<td>0.1319</td>
<td>0.1106</td>
</tr>
<tr>
<td>$P_3$</td>
<td>0.0932</td>
<td>0.1210</td>
<td>0.1213</td>
<td>0.1328</td>
</tr>
</tbody>
</table>

Table 4: Cosecant similarity measure (Using step 3 and step 4)

<table>
<thead>
<tr>
<th></th>
<th>Viral fever</th>
<th>Malaria</th>
<th>Stomach problem</th>
<th>Chest problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>0.8232</td>
<td>0.6900</td>
<td>0.7142</td>
<td>0.7924</td>
</tr>
<tr>
<td>$P_2$</td>
<td>0.7867</td>
<td>0.6954</td>
<td>0.6687</td>
<td>0.7240</td>
</tr>
<tr>
<td>$P_3$</td>
<td>0.7908</td>
<td>0.6753</td>
<td>0.7190</td>
<td>0.6624</td>
</tr>
</tbody>
</table>

From Table 3 and Table 4, it is obvious that, if the doctor agrees, then $P_1, P_2 and P_3$ suffers from Viral fever.

7 Conclusion

Our proposed methods are most reliable to handle medical diagnosis problems quiet comfortably. The recommended techniques can invade in other areas such as clustering, image processing etc., In future, we will enhance these methods to other types of neutrosophic sets.

References


