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# Odd Graceful Labeling On Revised Sunflower Graph

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**Abstract**

In 1991, Gnanajothi [3] introduced a labeling method called *odd graceful labeling*. A graph $G$ with $q$ edges is said to be odd graceful if there is an injection $f$ from $V(G)$ to the set $\{0, 1, 2, ..., (2q-1)\}$ such that resulting edge labels are $\{1, 3, 5, ..., (2q-1)\}$. In this paper, we prove that the revised sunflower graph, obtained by attaching Dutch Windmill graphs at every vertex of the cycle $C_k$, where $k \equiv 0$ $(mod\ 4)$ is odd graceful.

**Key Words**: Odd graceful labeling, Cycle, Dutch Windmill graph, Revised sunflower graph.
**A.M.S Subject classification:** 05C78

## Introduction

The first graph labeling method is the graceful labeling introduced by Rosa [7] in 1967. The graceful labeling of a graph $G$ with $q$ edges is an injection $f$ from the vertices of $G$ to the set $\{0, 1, 2, ..., q\}$ such that when each edge $xy$ is assigned the label $|f(x) - f(y)|$, the resulting edges are distinct. In 1991, Gnanajothi

[3] introduced the odd graceful labeling. An odd-graceful labeling is an injection $f$ from $V(G)$ to $\{0, 1, 2, ..., (2q-1)\}$ such that, when each edge $xy$ is assigned the label or weight $|f(x) - f(y)|$, the resulting edge labels are $\{1, 3, 5, ..., (2q-1)\}$. Gnanajothi [3] proved that every cycle graph is odd graceful if and only if $n$ is even.

Moussa, Badr [6] proved odd graceful labeling of crown graphs. Govindarajan and Srividya [4] proved odd graceful labeling of every odd cycle $C_n, n \geq 7$ with with parallel $P_k$ chords for $k = 2, 4$ after the removal of two edges from the cycle $C_n$. Vaidya [9] proved that the graph obtained by joining two copies of even cycle $C_n$ with path $P_k$ and two copies of even cycle $C_n$ sharing a common edge are odd graceful graphs. In addition, he also proved that the splitting graph of $K_{1,n}$ as well as the tensor product of $K_{1,n}$ and $P_2$ admits odd graceful labeling. Sushant Kumar Rout *et al.* [8] proved that the graph obtained by joining a cycle $C_{12}$ with some star graphs $S_{1,r}$ keeping two, three and five vertices gap between pair of vertices of the cycle admits odd graceful labeling. Badr [1] proved that the revised friendship graphs $F(kC_8)$, $F(kC_{12})$, $F(kC_{16})$ and $F(kC_{20})$ are odd graceful, where $k$ is any positive integer. Jeba Jesintha, K. Ezhilarasi Hilda [5] proved that all subdivided shell flower graphs are odd graceful. For an exhaustive survey on odd graceful labeling refer to the dynamic survey by Gallian [2].

Graph labeling is an active area of research in graph theory which has rigorous applications in coding theory, communication networks, optimal circuit's layouts and graph decomposition problems.

In this paper, we prove that the revised sunflower graph obtained by attaching Dutch Windmill graphs at every vertex of the cycle $C_k$, $k \equiv 0$ $(mod\ 4)$ is odd graceful.

**Definition 1.1**

The **windmill graph** $[W_n^{(m)}]$ is the graph obtained by taking $m$ copies of complete graph with a vertex in common See Figure 1.

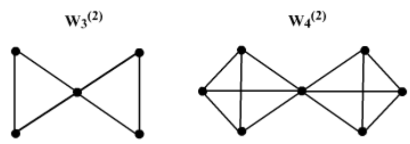


**Figure 1 : The windmill graphs $W_3^{(2)}$ and $W_4^{(2)}$**

**Definition 1.2**

The **Dutch windmill graph** $[D_n^{(m)}]$ , also called a friendship graph, is the graph obtained by taking $m$ copies of the cycle graph $C_n$ with a vertex in common. See Figure 2.

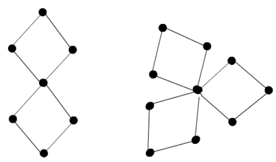

**Figure 2 : The Dutch windmill graphs $D_4^{(2)}$ and $D_4^{(3)}$**

**Main Result**

In this section,we prove that the revised sunflower graph obtained by attaching Dutch Windmill graphs at every vertex of the cycle $C_k$, $k \equiv 0$ $(mod\ 4)$ is odd graceful.

**Theorem**

The revised sunflower graph obtained by attaching Dutch Windmill graphs $[D_4^{(2)}]$ at every vertex of the cycle $C_k$ where $k \equiv 0$ $(mod\ 4)$ admits odd graceful labeling.

**Proof**: Let $G$ be the revised sunflower graph obtained by attaching Dutch Windmill graphs $[D_4^{(2)}]$ at every vertex of the cycle $C_k$ where $k \equiv 0$ $(mod\ 4)$. Let $|V(G)| = p$ and $|E(G)| = q$. We describe the graph $G$ as follows. The vertices in the cycle $C_k$ in $G$ are denoted as $u_1, u_2, u_3, ..., u_k$ in the clockwise direction. The middle vertices in the Dutch Windmill graph attached at $u_1$ are denoted by $v_{11}, v_{12}, v_{13}, v_{14}$. The middle vertices in the Dutch Windmill graph attached at $u_2$ are denoted by $v_{21}, v_{22}, v_{23}, v_{24}$. Similarly, The middle vertices in the Dutch Windmill graph attached at $u_3$ are denoted by $v_{31}, v_{32}, v_{33}, v_{34}$. In general, The middle vertices in the Dutch windmill graph attached at vertex $u_k$ will denoted by $v_{ij}$ where $i = 1, 2, 3, ..., k$ and $j = 1, 2, ..., n$ ($n$ is the notation used from $[D_n^m]$). The end vertices in Dutch windmill graph is denoted by $w_1, w_2, w_3, ..., w_{2k}$. See Figure 3. The graph $G$ has $p = 7k$ vertices and $q = 9k$ edges.

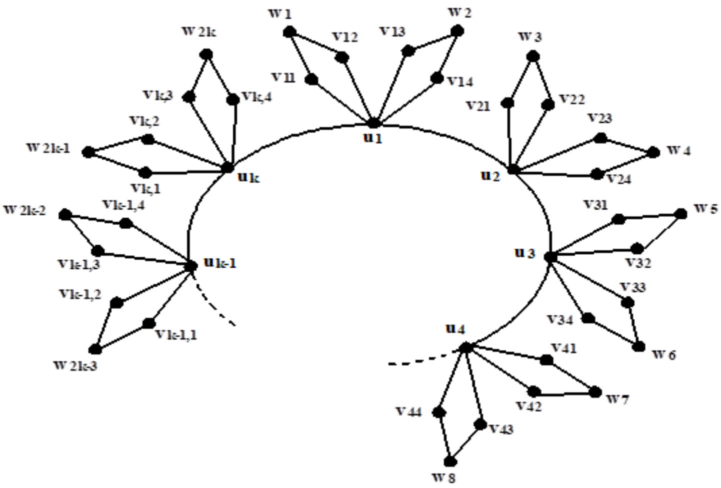


**Figure 3 : Revised Sunflower Graph**

The vertices of revised sunflower graph is labeled by first labeling the vertices of the cycle $C_k$ and then labeling the other vertices of the Dutch Windmill Graph.

The **vertex labels** for the cycle $C_k$ given below.

$$f(u_{2i-1}) = \begin{cases} 5(2i-2), & for \quad 1 \leq i \leq \frac{k}{4} \\ 5(2i-2)+2, & for \quad (\frac{k}{4}+1) \leq i \leq (\frac{k}{2}) \end{cases}$$

$$\text{f}(u_{2i}) \quad = 2q - 10i + 1 \qquad for \quad 1 \leq i \leq \frac{k}{2} \qquad \textbf{(1)}$$

The **vertex labels** for the Dutch windmill graph is as follows

The **vertex labels** for the middle vertices of Dutch Windmill graph is as follows

$$\text{f}(v_{2i-1,j}) \;= (2q-1) - 5(2i-2) - (2j-2), \quad for \quad 1 \leq i \leq (\frac{k}{2}), \;\; 1 \leq j \leq n$$

$$f(v_{2i,j}) = \begin{cases} 2i+4(2i-2)+(2j-2), \\ \quad for \quad 1 \le i \le (\frac{k}{4}), \quad 1 \le j \le n \\ 2i+4(2i-2)+(2j-2)+2, \\ \quad for \quad (\frac{k}{4}+1) \le i \le (\frac{k}{2}), \quad 1 \le j \le n \end{cases}$$

**(2)**

The vertex labels for the end vertices of Dutch Windmill graph is as follows.

f$(w_{4i-3}) = q + k - (2i-2), \qquad for \quad 1 \le i \le (\frac{k}{2})$

f$(w_{4i-2}) = 2p - (2i+2), \qquad for \quad 1 \le i \le (\frac{k}{2})$

$$f(w_{4i-1}) = \begin{cases} p+(2i-2)+(k-3), & for \quad 1 \le i \le (\frac{k}{4}) \\ p+(2i-2)+(k-3)+2, & for \quad (\frac{k}{4}+1) \le i \le (\frac{k}{2}) \end{cases}$$

$$f(w_{4i}) = \begin{cases} nk+(2i-2)+1, & for \quad 1 \le i \le (\frac{k}{4}) \\ nk+(2i-2)+3, & for \quad (\frac{k}{4}+1) \le i \le (\frac{k}{2}) \end{cases}$$

' **(3)**

From the equations (1) to (3) we see that the vertex labels for Revised Sunflower Graph are distinct.

Now we compute the edge labels for the Revised Sunflower Graph by first computing the edge labels for cycle $C_k$ for $k \equiv 0 \ (mod\ 4)$ as follows:

$$|f(u_{2i-1}) - f(u_{2i})| = \begin{cases} 2(10i-q)-11, & for\ 1 \le i \le (\frac{k}{4}) \\ 2(10i-q)-9, & for (\frac{k}{4}+1) \le i \le (\frac{k}{2}) \end{cases}$$

$$|f(u_{2i+1}) - f(u_{2i})| = \begin{cases} 20i-2q-1, & for\ 1 \le i \le (\frac{k}{4}-1) \\ 20i-2q+1, & for (\frac{k}{4}) \le i \le (\frac{k}{2}-1) \end{cases}$$

$|f(u_1) - f(u_k)| = 2p - k + 1,$ **(4)**

The edge labels for the remaining edges of the Dutch Windmill Graph is computed as follows

$$|f(u_{2i-1}) - f(v_{2i-1,j})| = \begin{cases} (2j-2)-(2q-1)+10(2i-2), \\ for \quad 1 \le i \le (\frac{k}{4}), \quad 1 \le j \le n \\ \\ (2j-(2q-1)+10(2i-2) \\ for \quad (\frac{k}{4}+1) \le i \le (\frac{k}{2}), \quad 1 \le j \le n \end{cases}$$

$$|f(u_{2i}) - f(v_{2i,j})| \quad = \begin{cases} 2q-20i-2j+11, \\ for \quad 1 \le i \le (\frac{k}{4}), \quad 1 \le j \le n \\ \\ 2q-20i-2j+9, \\ for \quad (\frac{k}{4}+1) \le i \le (\frac{k}{2}), \quad 1 \le j \le n \end{cases}$$

**(5)**

$$|f(v_{2i-1,j}) - f(w_{4i-3})| = 9+q-8i-2j-k \quad for \quad 1 \le i \le (\tfrac{k}{2}), \quad 1 \le j \le (\tfrac{n}{2}),$$

$$|f(v_{2i-1,j}) - f(w_{4i-2})| = 2q-8i-2j-2p+13 \quad for \quad 1 \le i \le (\tfrac{k}{2}), \quad (\tfrac{n}{2}+1) \le j \le n,$$

$$|f(v_{2i,j}) - f(w_{4i-1})| = 8i+2j-k-p-5, \quad for \quad 1 \le i \le (\tfrac{k}{2}), \quad 1 \le j \le (\tfrac{n}{2}),$$

$$|f(v_{2i,j}) - f(w_{4i})| = 8i+2j-nk-9, \quad for \quad 1 \le i \le (\tfrac{k}{2}), \quad (\tfrac{n}{2}+1) \le j \le n,$$

**(6)**

From the above computed edge labels we see that the edge labels for the Revised Sunflower Graph are the distinct odd numbers from the set $\{1, 3, 5, ..., (2q-1)\}$.

Hence the Revised Sunflower Graph is odd graceful.
We illustrate the above theorem as follows in Figure 4

**Illustration**

When $k = 8, p = 56, q = 72$

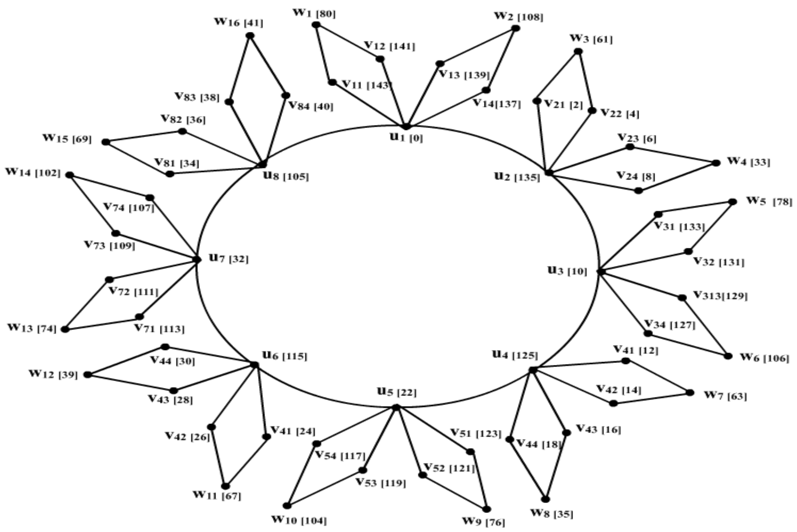


**Figure 4** : Revised Sunflower Graph

**Conclusion**

In this paper,we have proved that the revised sunflower graph obtained by attaching Dutch Windmill graphs at every vertex of the cycle $C_k$, $k \equiv 0$ $(mod\ 4)$ is odd graceful. We further intend to prove the graph $[D_n^2]$ for $n = 5, 6, ...,$ attached at every vertex of the cycle $C_k$ is odd graceful.

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