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Generalized $\alpha b-$Irresolute Maps in Fuzzy Topological Spaces

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Abstract

The paper reviews the properties of fuzzy topological on Fuzzy generalized $\alpha b-$ open maps, Fuzzy generalized $\alpha b-$closed maps and Fuzzy generalized $\alpha b-$irresolute maps (denoted by fuzzy $g\,\alpha b-$ irresolute maps) and expose of their properties. It will be helpful to further extension of bitopology in fuzzy topological spaces.

AMS Subject Classification: 54A40
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1 Generalized $\alpha b-$Irresolute Maps in Fuzzy Topological Spaces

1.1 Introduction

The legends Pu an Liu [3] introduced the concepts of quasi-coincidence and $q-$ neighbourhoods by which the extension of a function in the fuzzy setting can be very interestingly and effectively carried out. Goguen [7] used the word fuzzy singleton instead of fuzzy point and he accepted the value 1 for the fuzzy singleton and his inclusion is like that of Pu and Liu [3]. Bin shanhna [2] introduced $\alpha-$closed and pre-closed sets in fuzzy topological spaces. Balasubramanian and Sundaram [1] introduced generalized closed sets in fuzzy topological space.

1.2 Generalized αb-Open Maps, Generalized αb-Closed Maps Spaces and Irresolute Maps

This section introduces fuzzy generalized αb-open maps, fuzzy generalized αb-closed maps, fuzzy generalized αb-irresolute maps.

Definition : Let X and Y be the two fuzzy topological spaces. A map $f : (X, \tau) \to (Y, \alpha)$ is called fuzzy generalized αb-open (briefly fuzzy g αb-open) map, if the image of every fuzzy open set in X is fuzzy g αb-open in Y.

Theorem 1.2.1. *'Every fuzzy open map is fuzzy g αb-open' but not conversely.*

Proof: Let $f : (X, \tau) \to (Y, \alpha)$ is a fuzzy open map and π be a fuzzy open set in X, f(π) is fuzzy open in Y. Hence, fuzzy g αb-open in Y. Thus, f is fuzzy g αb- open.

Remark : The converse of the above theorem need not be true as seen from the following example.

Example : Let $X, Y = \{a, b, c\}$, $I = [0, 1]$ and the fuzzy functions.
$\chi, \beta, \gamma : X, Y \to$ I be defined as,

$$\chi(X) \quad = \begin{cases} 1 & if \quad x = b \\ 0 & otherwise. \end{cases}$$

$$\beta(X) \quad = \begin{cases} 1 & if \quad x = a, b \\ 0 & otherwise. \end{cases}$$

$$\gamma(X) \quad = \begin{cases} 1 & if \quad x = b, c \\ 0 & otherwise. \end{cases}$$

with fuzzy topologies $\tau = \{0, 1, \chi, \beta, \gamma\}$ and $\sigma = \{0, 1, \chi\}$ and $f : (X, \tau) \to (Y, \alpha)$ be the identity map. This function is fuzzy $g\alpha b$-open but not fuzzy-open, as the image of a fuzzy open set β in X is β, which is not fuzzy open in Y.

Definition : Let X and Y be the two fuzzy topological spaces. A map $f : (X, \tau) \to (Y, \alpha)$ is called fuzzy g αb-closed map, if the image of every fuzzy closed set in X is fuzzy g αb-closed set in Y.

Theorem 1.2.2. *'Every fuzzy closed map is fuzzy g αb-closed' but not conversely.*

Proof: Let $f : (X, \tau) \to (Y, \alpha)$ be a fuzzy closed map and π be fuzzy closed set in X. Then, f(π) is fuzzy closed in Y. Hence, fuzzy g αb-closed in Y. Thus, f is fuzzy g αb-closed.

Remark : The converse of the above theorem need not be true as seen from the following example.

Example : Let $X, Y = \{a, b, c\}$, $I = [0, 1]$ and the function $\chi, \beta, \gamma : X, Y \to I$ be defined as,

$$\chi(X) \quad = \begin{cases} 1 & if \quad x = b \\ 0 & otherwise. \end{cases}$$

$$\beta(X) \quad = \begin{cases} 1 & if \quad x = a, b \\ 0 & otherwise. \end{cases}$$

$$\gamma(X) \quad = \begin{cases} 1 & if \quad x = c \\ 0 & otherwise. \end{cases}$$

With the fuzzy topologies $\tau = \{0, 1, \gamma\}$ and $\sigma = \{0, 1, \chi, \beta\}$ and $f : (X, \tau) \to (Y, \alpha)$ be the identity map. The function f is then fuzzy g αb-closed but not fuzzy closed, as $f(\gamma) = \gamma$ in X, which is not fuzzy closed in Y.

Theorem 1.2.3. *A map $f : (X, \tau) \to (Y, \alpha)$ is fuzzy g αb-closed, if and only if for each subset ζ of Y and for each α -open set μ containing $f^{-1}(\zeta\ )$, there is a fuzzy g αb-open set ϵ of Y such that $\zeta \leq \epsilon$ and $f^{-1}(\epsilon) \leq \mu$.*

Proof: Suppose f is fuzzy $g\alpha$b-closed. Let ζ be a subset of Y and μ be an fuzzy α -open set of X, such that $f^{-1}(\zeta) \leq \mu$, $\epsilon = 1 - f(X - \mu)$ is fuzzy g αb-open set containing ζ such that $f^{-1}(\epsilon) \leq \mu$.
Conversely, κ is a fuzzy closed set of X. Then $f^{-1}(Y - f(\kappa)) \leq 1 - \kappa$ and 1- κ is fuzzy open. By hypothesis, there is a fuzzy g αb-open set ϵ of Y, such that $1 - f(\kappa) \leq \epsilon$ and $f^{-1}(\epsilon) \leq 1 - \kappa$. Therefore, $\kappa \leq 1 - f^{-1}(\epsilon)$. Hence, 1- $\epsilon \leq f(\kappa) \leq f\left(X - f^{-1}(\epsilon)\right) \leq 1 - \epsilon$ which implies $f(\kappa) = 1 - \epsilon$. Since, 1- ϵ is fuzzy g αb-closed, $f(\kappa)$ is fuzzy g αb-closed. Hence, f is fuzzy g αb-closed map.

1.3 Generalized αb-Irresolute Maps In Fuzzy Topological Spaces

In the present section, the concept of fuzzy generalized αb-irresolute maps is introduced.

Definition : Let X and Y be the two fuzzy topological spaces. A map $f : (X, \tau) \to (Y, \sigma)$ is said to be fuzzy generalized αb-irresolute map, if the inverse image of every fuzzy g αb-closed set in Y is fuzzy g αb-closed in X.

Example : Let $X, Y = \{a, b, c\}$ and $I = [0, 1]$ and the function $\chi, \beta, \gamma, \delta : X, Y \to I$ be defined as,

$$\chi(X) \quad = \begin{cases} 1 & if \quad x = a \\ 0 & otherwise. \end{cases}$$

$$\beta(X) = \begin{cases} 1 & if \quad x = b \\ 0 & otherwise. \end{cases}$$

$$\gamma(X) = \begin{cases} 1 & if \quad x = a, b \\ 0 & otherwise. \end{cases}$$

$$\delta(X) = \begin{cases} 1 & if \quad x = a, c \\ 0 & otherwise. \end{cases}$$

With the fuzzy topologies $\tau = \{0, 1, \chi, \beta, \gamma\}$ and $\sigma = \{0, 1, \chi, \delta\}$ and $f : (X, \tau) \to (Y, \sigma)$ be the identity map. As inverse image of every fuzzy g αb-closed set in Y is fuzzy g αb-closed set in X under f. Hence, f is fuzzy g αb-irresolute.

Theorem 1.3.1. *If a map $f : (X, \tau) \to (Y, \sigma)$ is fuzzy g αb-irresolute, it is then fuzzy gαb-continuous but not conversely.*

Proof: Let θ be any fuzzy closed set in Y. Then, θ is fuzzy g αb-closed in Y. As f is fuzzy g αb-irresolute, $f^{-1}(\theta)$ is fuzzy g αb-closed in X. Therefore, f is fuzzy g αb-continuous.

Example : Let $X, Y = \{a, b, c\}$ and I = $[0, 1]$ and the function $\chi, \beta, \gamma, \delta$: X,Y$\to$ I be defined as,

$$\chi(X) = \begin{cases} 1 & if \quad x = b \\ 0 & otherwise. \end{cases}$$

$$\beta(X) = \begin{cases} 1 & if \quad x = c \\ 0 & otherwise. \end{cases}$$

$$\gamma(X) = \begin{cases} 1 & if \quad x = b, c \\ 0 & otherwise. \end{cases}$$

$$\delta(X) = \begin{cases} 1 & if \quad x = a \\ 0 & otherwise. \end{cases}$$

With the fuzzy topologies $\tau = \{0, 1, \chi, \beta, \gamma\}$ and $\sigma = \{0, 1, \chi, \gamma\}$ and $f : (X, \tau) \to (Y, \sigma)$ be defined by f(b) = c, f(c) = b and f(a) = a. Then, f is fuzzy g αb-continuous but not fuzzy g αb-irresolute, as the inverse image of fuzzy g αb-open set δ in Y is δ, which is not fuzzy g αb-open in X.

Theorem 1.3.2. *Let X, Y and Z be the fuzzy topological spaces, then $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \eta)$ be two maps. Their composition $g \circ f : (X, \tau) \to (Z, \eta)$ is fuzzy g αb-continuous, if f is fuzzy g αb-irresolute and g is fuzzy g αb-continuous.*

Proof: Let μ be a fuzzy open set in Z. Consider $(g \circ f)^{-1}(\mu) = \left(f^{-1} \circ g^{-1}\right)(\mu) = f^{-1}(\zeta)$ where $\zeta = g^{-1}(\mu)$ is fuzzy g αb-open in Y as g is fuzzy g αb-continuous. Since, f is fuzzy g αb-irresolute, $f^{-1}(\zeta)$ is fuzzy g αb-open in X. Thus, $(g \circ f)$ is fuzzy g αb-continuous.

Theorem 1.3.3. *Let X, Y and Z be the fuzzy topological spaces and $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \eta)$ be two fuzzy g αb-irresolute maps. Their composition $g \circ f : (X, \tau) \to (Z, \eta)$ is then fuzzy g αb-irresolute.*

Proof: Let μ be fuzzy g αb-open set in Z. Consider $(g \circ f)^{-1}(\mu) = \left(f^{-1} \circ g^{-1}\right)(\mu) = f^{-1}(\zeta)$ where $\zeta = g^{-1}(\mu)$ is fuzzy g αb-open in Y, as g is fuzzy g αb-irresolute. Since, f is fuzzy g αb-irresolute, $f^{-1}(\zeta)$ is fuzzy g αb-open in X. Thus, $(g \circ f)$ is fuzzy g αb-open.

Theorem 1.3.4. *Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \eta)$ be two the maps such that $g \circ f : (X, \tau) \to (Z, \eta)$ is fuzzy g αb-closed map then,*
(i) If f is fuzzy continuous and surjective, then g is fuzzy g αb -closed.
(ii) If g is fuzzy irresolute and injective, f is then fuzzy g αb- closed.

Proof:(i) Let ϵ be a fuzzy closed set of Y. Since $f^{-1}(\epsilon)$ is fuzzy closed in X, $(g \circ f)\left(f^{-1}(\epsilon)\right)$ is fuzzy g αb-closed in Z. Hence, $g(\epsilon)$ is fuzzy g αb-closed in Z. Thus, g is fuzzy g αb-closed.

(ii) Let κ be a fuzzy closed set of X. Thus, $(g \circ f)(\kappa)$ is fuzzy g αb-closed in Z and $g^{-1}(g \circ f)(\kappa)$ is fuzzy g αb-closed in Y. Since, g is injective $f(\kappa) = g^{-1}(g \circ f)(\kappa)$ is fuzzy g αb-closed in Y. Therefore, f is fuzzy g αb-closed.

Theorem 1.3.5. *Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \eta)$ be two the maps such that $g \circ f : (X, \tau) \to (Z, \eta)$ is fuzzy g αb-irresolute, if f is fuzzy g αb-irresolute and g is fuzzy α-irresolute.*

Proof: Let μ be a fuzzy open set in Z. Since every fuzzy open sets are α-open, μ is fuzzy α-open in Z. Now, $(g \circ f)^{-1}(\mu) = \left(f^{-1} \circ g^{-1}\right)(\mu) = f^{-1}(\psi)$, where $\psi = g^{-1}(\mu)$ is fuzzy α-open in Y. As g is fuzzy α-irresolute. Since f is fuzzy g αb-irresolute, $f^{-1}(\psi)$ is fuzzy g αb-open in X. Thus, $(g \circ f)$ is fuzzy g αb-irresolute.

Theorem 1.3.6. *If a map $f : (X, \tau) \to (Y, \sigma)$ is fuzzy g αb-irresolute, it is then fuzzy g αb-continuous, but not conversely.*

Proof: Let ψ be any fuzzy closed set in Y. Then,ψ is fuzzy g αb-closed in Y. As f is fuzzy g αb-irresolute, $f^{-1}(\psi)$ is fuzzy g αb-closed in X. Therefore, f is fuzzy g αb-continuous.

Remark : The converse of the theorem need not be true as seen from the following example.

Example : Let $X, Y = \{a, b, c\}$ and I = $[0, 1]$ and the function χ, β, γ: X,Y→ I be defined as,

$$\chi(X) \quad = \begin{cases} 1 & if \quad x = a \\ 0 & otherwise. \end{cases}$$

$$\beta(X) \quad = \begin{cases} 1 & if \quad x = b \\ 0 & otherwise. \end{cases}$$

$$\gamma(X) \quad = \begin{cases} 1 & if \quad x = a, b \\ 0 & otherwise. \end{cases}$$

With the fuzzy topologies $\tau = \{0, 1, \chi\}$ and $\sigma = \{0, 1, \chi, \beta, \gamma\}$ and $f : (X, \tau) \to (Y, \sigma)$ be the identity map. Then, f is fuzzy g α b-continuous but not fuzzy g αb-irresolute, as the inverse image of a fuzzy g αb-open set β in Y is β, which is not fuzzy g αb-open in X.

Theorem 1.3.7. *If a bijection map $f : (X, \tau) \to (Y, \sigma)$ is fuzzy α-open and fuzzy g αb-continuous, f is then fuzzy g αb-irresolute.*

Proof: Let ψ be a fuzzy g αb-closed set in Y, such that $f^{-1}(\psi) \le \varphi$, where φ is a fuzzy α-open set in X. Then,$\psi \le f(\varphi)$ hold. Since, $f(\varphi)$ is fuzzy α-open and ψ is fuzzy g αb-closed set in Y, $bcl(\psi) \le f(\varphi)$ holds. Hence, $f^{-1}(bcl(\psi)) \le \varphi$. Since, f is fuzzy g αb-continuous and $bcl(\psi)$ is closed in Y, $bcl\left(f^{-1}(bcl(\psi))\right) \le \varphi$ and $bcl\left(f^{-1}(\psi)\right) \le \varphi$. Therefore, $f^{-1}(\psi)$ is fuzzy g αb-closed. Thus, f is fuzzy g αb-irresolute.

Remark : The following examples show that no assumption of above theorem can be removed

Example : Let $X, Y = \{a, b, c\}$, $I = [0, 1]$ and the fuzzy functions.
$\chi, \beta, \gamma, \delta$: X,Y → I be defined as,

$$\chi(X) \quad = \begin{cases} 1 & if \quad x = a \\ 0 & otherwise. \end{cases}$$

$$\beta(X) \quad = \begin{cases} 1 & if \quad x = c \\ 0 & otherwise. \end{cases}$$

$$\gamma(X) \quad = \begin{cases} 1 & if \quad x = a, b \\ 0 & otherwise. \end{cases}$$

$$\delta(X) \quad = \begin{cases} 1 & if \quad x = a, c \\ 0 & otherwise. \end{cases}$$

with fuzzy topologies $\tau = \{0, 1, \chi, \beta, \delta\}$ and $\sigma = \{0, 1, \chi, \gamma\}$ and $f : (X, \tau) \rightarrow (Y, \alpha)$ be defined by $f(a) = a = f(c)$ and $f(b) = b$. Then, f is fuzzy g αb-continuous and fuzzy α -open but not bijective and so f is not fuzzy g αb-irresolute.

Example : Let $X, Y = \{a, b, c\}$, $I = [0, 1]$ and the fuzzy functions.
$\chi, \beta, \gamma, \delta$: $X, Y \rightarrow$ I be defined as,

$$\chi(X) \quad = \begin{cases} 1 & if \quad x = a \\ 0 & otherwise. \end{cases}$$

$$\beta(X) \quad = \begin{cases} 1 & if \quad x = b \\ 0 & otherwise. \end{cases}$$

$$\gamma(X) \quad = \begin{cases} 1 & if \quad x = c \\ 0 & otherwise. \end{cases}$$

$$\delta(X) \quad = \begin{cases} 1 & if \quad x = a, c \\ 0 & otherwise. \end{cases}$$

with fuzzy topologies $\tau = \{0, 1, \chi, \gamma, \delta\}$ and $\sigma = \{0, 1, \beta\}$ and $f : (X, \tau) \rightarrow (Y, \alpha)$ be defined by $f(a) = b = f(c)$ and $f(b) = b$. Then, f is fuzzy g αb-continuous, bijective and not fuzzy α -open and so f is not fuzzy g αb-irresolute.

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