International Journal of Pure and Applied Mathematics
Volume 120 No. 8 2018, 19-28
ISSN: 1314-3395 (on-line version)
url: http://www.acadpubl.eu/hub/
Special Issue
http://www.acadpubl.eu/hub/

On Cordial labeling of double duplication of graphs

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Abstract

Let G (V, E) be a simple undirected graph where V, E are its vertex set and edge set respectively. Consider a labeling where f is a function from the vertices of G to $\{0, 1\}$ and for each edge xy assign the label $|f(x) - f(y)|$. Then f is called cordial of G if the number of vertices labeled 0 and the number of vertices labeled 1 differs by at most 1 and the number of edges labeled 0 and the number of edges labeled 1 differs by at most 1.

In this paper, we proved the existence of cordial labeling of double duplication of vertices by edges of a twig graph T_n for $n \geq 1$, ladder graph L_n for $n \geq 2$, star graph $K_{1,n}$ for $n \geq 2$ and helm graph H_m for $m \geq 3$. Further, double duplication of all vertices by edges of a graph G is cordial if G is cordial.

Key Words: graph, labeling, cordial, function, duplication.
AMS Subject Classification: 05C78

1 Introduction

The concept of graph labeling was introduced by Rosa in 1967. A graph labeling is an assignment of integers to the vertices or edges, or both subject to certain conditions. Cordial labeling was introduced by Cahit in 1987. A graph G is cordial if there is vertex labeling, f be a function from the vertices of G to $\{0, 1\}$

and for each edge xy assign the label $|f(x) - f(y)|$. Then f is called cordial of G if the number of vertices labeled 0 and the number of vertices labeled 1 differs by at most 1 and the number of edges labeled 0 and the number of edges labeled 1 differs by at most 1.
The concept of double duplication of a graph is introduced in [?].
In this paper, we proved the existence of cordial labeling of double duplication of all vertices by edges of a twig graph T_n for $n \geq 1$, ladder graph L_n for $n \geq 2$, star graph $K_{1,n}$ for $n \geq 2$ and helm graph H_m for $m \geq 3$. Further, double duplication of all vertices by edges of a graph G is cordial if G is cordial.
Definition 1.1: The double duplication of a vertex v_k by an edge $e = \{v_k^{'}v_k^{''}\}$ in a graph G produces a graph $G^{'}$ in which $N(v_k^{'}) = \{v_k, v_k^{''}\}$ and $N(v_k^{''}) = \{v_k, v_k^{'}\}$. Again dulpication of a vertex $v_k, v_k^{'}$ and $v_k^{''}$ by the edges $e^{'} = \{u^1u^2, u^3u^4, u^5u^6\}$ respectively in $G^{'}$ produces a new graph $G^{''}$ such that
$N(u^1) = \left\{v_k^{'}, u^2\right\}, N(u^2) = \left\{v_k^{'}, u^1\right\}, N(u^3) = \left\{v_k^{''}, u^4\right\},$
$N(u^4) = \left\{v_k^{''}, u^3\right\}, N(u^5) = \{v_k, u^6\}$ and $N(u^6) = \{v_k, u^5\}$.

2 Main Results

Theorem 1. *The double duplication of all vertices by edges of a star graph $K_{1,n}, n \geq 2$ admits cordial labeling.*

Proof. Let G be a graph obtained by double duplication of all vertices by edges of a star graph $K_{1,n}, n \geq 2$.
The vertex set and edge set of G are defined as $V = V_1 \bigcup V_2 \bigcup V_3$ where,
$V_1 = \{v_1, v_2 ... v_{n+1}\}, V_2 = \{a_1, a_2 ... a_{2n+2}\},$
$V_3 = \{b_1, b_2 ... b_{6n+6}\}$ and $E = E_1 \bigcup E_2 \bigcup E_3 \bigcup E_4 \bigcup E_5 \bigcup E_6 \bigcup E_7 \bigcup E_8 \bigcup E_9$
where, $E_1 = \{v_1v_i, 2 \leq i \leq n+1\}, E_2 = \{v_ia_{2i-1}, 1 \leq i \leq n+1\},$
$E_3 = \{v_ia_{2i}, 1 \leq i \leq n+1\}, E_4 = \{a_ib_{2i-1}, 1 \leq i \leq 2n+2\},$
$E_5 = \{a_ib_{2i}, 1 \leq i \leq 2n+2\}, E_6 = \{b_{2i-1}b_{2i}, 1 \leq i \leq 3n+3\},$
$E_7 = \{v_ib_{4n+4+2i}, 1 \leq i \leq n+1\}, E_8 = \{v_ib_{4n+3+2i}, 1 \leq i \leq n+1\},$
$E_9 = \{a_{2i-1}a_{2i}, 1 \leq i \leq n+1\}$
with $|V(G)| = 9n+9$ and $|E(G)| = 13n+12$.
We define a binary vertex labeling $f : V \to \{0,1\}$ as follows:

$f(v_1) = 1$

$f(v_i) = \begin{cases} 0, i \equiv 0(mod2) \\ 1, i \equiv 1(mod2), 2 \leq i \leq n+1. \end{cases}, f(a_{2i-1}) = f(a_{2i}) = \begin{cases} 0, i \equiv 1,3(mod4) \\ 1, i \equiv 0,2(mod4), 1 \leq i \leq n+1. \end{cases}$

$f(b_i) = \begin{cases} 0, i \equiv 1,2(mod4) \\ 1, i \equiv 0,3(mod4), 1 \leq i \leq 4n+4. \end{cases}, f(b_j) = \begin{cases} 0, j \equiv 1,3(mod4) \\ 1, j \equiv 0,2(mod4), 4n+5 \leq j \leq 6n+6. \end{cases}$

The induced edge labeling $f^* : E(G) \to \{0,1\}$ is defined by,

$$f^*(u_iu_j) = \begin{cases} 0, if \quad f(u_i) \text{ and } f(u_j) \text{ receive the same label} \\ 1, if \quad f(u_i) \text{ and } f(u_j) \text{ does not receive the same label.} \end{cases}$$

The vertex and edge condition of a double duplication of all vertices by edges of a star graph $K_{1,n}, n \geq 2$ are as follows:

n	$v_f(0)$	$v_f(1)$	$\lvert v_f(0) - v_f(1) \rvert$	$e_f(0)$	$e_f(1)$	$\lvert e_f(0) - e_f(1) \rvert$
$n \equiv 0(mod2)$	$\frac{9n+10}{2}$	$\frac{9n+8}{2}$	1	$\frac{13n+12}{2}$	$\frac{13n+12}{2}$	0
$n \equiv 1(mod2)$	$\frac{9n+9}{2}$	$\frac{9n+9}{2}$	0	$\frac{13n+11}{2}$	$\frac{13n+13}{2}$	1

From the above labeling pattern we infer that the difference between the vertices labeled with 0 and 1 differ by atmost 1 and the edges labeled with 0 and 1 also differ by atmost 0 and 1, hence the double duplication of all vertices by edges of star graph $K_{1,n}, n \geq 2$ admits cordial labeling. □

Theorem 2. *The double duplication of all vertices by edges of a helm graph $H_m, m \geq 3$ admits cordial labeling.*

Proof. Let G be a graph obtained by double duplication of all vertices by edges of a helm graph $H_m, m \geq 3$.
The vertex set and edge set of G are defined as $V = V_1 \bigcup V_2 \bigcup V_3$
where, $V_1 = \{v_1, v_2 ... v_{m+1}, ..., v_{2m+1}\}, V_2 = \left\{v_1^{'}, v_2^{'} ... v_{4m+2}^{'}\right\}, V_3 = \left\{v_1^{''}, v_2^{''} ... v_{12m+6}^{''}\right\}$
and $E = E_1 \bigcup E_2 \bigcup E_3 \bigcup E_4 \bigcup E_5 \bigcup E_6 \bigcup E_7 \bigcup E_8 \bigcup E_9 \bigcup \{v_2 \quad v_{m+1}\}$
where, $E_1 = \{v_1 v_i, 2 \leq i \leq m+1\}, E_2 = \{v_i v_{m+i+1}, 2 \leq i \leq m+1\}$,
$E_3 = \left\{v_i v_{2i-1}^{'}, 1 \leq i \leq 2m+1\right\}, E_4 = \left\{v_i v_{2i}^{'}, 1 \leq i \leq 2m+1\right\}$,
$E_5 = \left\{v_i^{'} v_{2i-1}^{''}, 1 \leq i \leq 4m+2\right\}, E_6 = \left\{v_i^{'} v_{2i}^{''}, 1 \leq i \leq 4m+2\right\}$,
$E_7 = \left\{v_{2i-1}^{''} v_{2i}^{''}, 1 \leq i \leq 6m+3\right\}, E_8 = \{v_i v_{i+1}, 2 \leq i \leq m\}$,
$E_9 = \left\{v_{2i-1}^{'} v_{2i}^{'}, 1 \leq i \leq 2m+1\right\}$
with $|V(G)| = 18m + 9$and $|E(G)| = 27m + 12$.
We define a binary vertex labeling $f : V \to \{0, 1\}$ for the following cases:
Case(i): $m \equiv 1(mod2)$

$$f(v_1) = 1, f(v_i) = \begin{cases} 0, i \equiv 1, 2(mod4) \\ 1, i \equiv 0, 3(mod4), 2 \leq i \leq m+1. \end{cases} , f(v_i) = \begin{cases} 0, i \equiv 1(mod2) \\ 1, i \equiv 0(mod2), m+2 \leq i \leq 2m+1. \end{cases}$$

Consider $f(v_1{'}) = 0, f(v_2{'}) = 0$

$$f(v'_{2i-1}) = \begin{cases} 0, i \equiv 0, 3(mod4) \\ 1, i \equiv 1, 2(mod4), 2 \leq i \leq m+1. \end{cases} , \quad f(v'_{2i}) = \begin{cases} 0, i \equiv 1, 2(mod4) \\ 1, i \equiv 0, 3(mod4), 2 \leq i \leq m+1. \end{cases}$$

$$f(v'_{2i-1}) = f(v'_{2i}) = \begin{cases} 0, i \equiv 0(mod2) \\ 1, i \equiv 1(mod2), m+2 \leq i \leq 2m+1. \end{cases} ,$$

$$f(v''_{2i-1}) = f(v''_{2i}) = \begin{cases} 0, i \equiv 1, 3(mod4) \\ 1, i \equiv 0, 2(mod4), 1 \leq i \leq 4m+2. \end{cases}$$

$$f(v''_i) = \begin{cases} 0, i \equiv 1(mod2) \\ 1, i \equiv 0(mod2), 8m+5 \leq i \leq 12m+6. \end{cases}$$

Case(ii): $m \equiv 0(mod2)$

$$f(v_1) = 1, f(v_i) = \begin{cases} 0, i \equiv 2, 3(mod4) \\ 1, i \equiv 0, 1(mod4), 2 \leq i \leq m+1. \end{cases}$$

$f(v_i) = \begin{cases} 0, i \equiv 0(mod2) \\ 1, i \equiv 1(mod2), m+2 \leq i \leq 2m+1. \end{cases}$

$f(v'_{2i-1}) = f(v_{2i}') = \begin{cases} 0, i \equiv 0,1(mod4) \\ 1, i \equiv 2,3(mod4), 1 \leq i \leq m+1. \end{cases}$

$f(v'_{2i-1}) = f(v'_{2i}) = \begin{cases} 0, i \equiv 1(mod2) \\ 1, i \equiv 0(mod2), m+2 \leq i \leq 2m+1. \end{cases}$

$f(v''_{2i-1}) = f(v''_{2i}) = \begin{cases} 0, i \equiv 1,3(mod4) \\ 1, i \equiv 0,2(mod4), 1 \leq i \leq 4m+2. \end{cases}$

$f(v''_i) = \begin{cases} 0, i \equiv 1(mod2) \\ 1, i \equiv 0(mod2), 8m+5 \leq i \leq 12m+6. \end{cases}$

The induced edge labeling $f^* : E(G) \to \{0,1\}$ is given by,

$f^*(u_iu_j) = \begin{cases} 0, if \quad f(u_i) \text{ and } f(u_j) \text{ receive the same label} \\ 1, if \quad f(u_i) \text{ and } f(u_j) \text{ does not receive the same label.} \end{cases}$

The vertex and edge condition of a double duplication of all vertices by edges of a helm graph $H_m, m \geq 3$ are as follows:

m	$v_f(0)$	$v_f(1)$	$\lvert v_f(0) - v_f(1) \rvert$	$e_f(0)$	$e_f(1)$	$\lvert e_f(0) - e_f(1) \rvert$
$m \equiv 0,3(mod4)$	$\frac{18m+10}{2}$	$\frac{18m+8}{2}$	1	$\frac{27m+13}{2}$	$\frac{27m+11}{2}$	1
$m \equiv 1,2(mod4)$	$\frac{18m+8}{2}$	$\frac{18m+10}{2}$	1	$\frac{27m+12}{2}$	$\frac{27m+12}{2}$	0

From the above labeling pattern we infer that the difference between the vertices labeled with 0 and 1 differ by atmost 1 and the edges labeled with 0 and 1 also differ by atmost 0 and 1, hence the double duplication of all vertices by edges of helm graph $H_m, m \geq 3$ admits cordial labeling. □

Theorem 3. *The double duplication of all vertices by edges of a ladder graph $L_n, n \geq 2$ admits cordial labeling.*

Proof. Let G be a graph obtained by double duplication of all vertices by edges of a ladder graph $L_n, n \geq 2$.
The vertex set and edge set be defined by $V = V_1 \bigcup V_2 \bigcup V_3$
where, $V_1 = \{v_1, v_2...v_{2n}\}, V_2 = \{u_1, ..., u_{4n}\}, V_3 = \{w_1, ..., w_{12n}\}$ and
$E = E_1 \bigcup E_2 \bigcup E_3 \bigcup E_4 \bigcup E_5 \bigcup E_6 \bigcup E_7 \bigcup E_8 \bigcup E_9 \bigcup E_{10} \bigcup E_{11} \bigcup E_{12}$
where, $E_1 = \{v_iv_{i+1}, 1 \leq i \leq 2n-1\}, E_2 = \{v_iv_{2n+1-i}, 1 \leq i \leq n\}$,
$E_3 = \{v_iu_{2i-1}, 1 \leq i \leq n\}, E_4 = \{v_iu_{2i}, 1 \leq i \leq n\}$,
$E_5 = \{v_iu_{2i-1}, n+1 \leq i \leq 2n\}, E_6 = \{v_iu_{2i}, n+1 \leq i \leq 2n\}$,
$E_7 = \{u_iw_{2i-1}, 1 \leq i \leq 4n\}, E_8 = \{u_iw_{2i}, 1 \leq i \leq 4n\}$,
$E_9 = \{w_{2i-1}w_{2i}, 1 \leq i \leq 6n\}, E_{10} = \{v_iw_{8n+2i-1}, 1 \leq i \leq 2n\}$,
$E_{11} = \{v_iw_{8n+2i}, 1 \leq i \leq 2n\}, E_{12} = \{u_{2i-1}u_{2i}, 1 \leq i \leq 2n\}$
with $|V(G)| = 18n \; and \, |E(G)| = 27n - 2$.
We define a binary vertex labeling $f : V \to \{0,1\}$ for the following cases:

Case(i): $n \equiv 0(mod4)$

$f(v_i) = \begin{cases} 0, i \equiv 0, 1(mod4) \\ 1, i \equiv 2, 3(mod4) \end{cases}, 1 \le i \le n.$, $f(v_i) = \begin{cases} 0, i \equiv 1, 2(mod4) \\ 1, i \equiv 0, 3(mod4) \end{cases}, n+1 \le i \le 2n.$

$f(u_{2i-1}) = f(u_{2i}) = \begin{cases} 0, i \equiv 2, 3(mod4) \\ 1, i \equiv 0, 1(mod4) \end{cases}, 1 \le i \le n.$,

$f(w_{2i-1}) = f(w_{2i}) = \begin{cases} 0, i \equiv 1, 3(mod4) \\ 1, i \equiv 0, 2(mod4) \end{cases}, 1 \le i \le 4n.$

$f(w_{2i-1}) = 0,\ f(w_{2i}) = 1, 4n+1 \le i \le 6n.$

Case(ii): $n \equiv 1(mod4)$

$f(v_i) = \begin{cases} 0, i \equiv 0, 1(mod4) \\ 1, i \equiv 2, 3(mod4) \end{cases}, 1 \le i \le n.$, $f(v_i) = \begin{cases} 0, i \equiv 0, 3(mod4) \\ 1, i \equiv 1, 2(mod4) \end{cases}, n+1 \le i \le 2n.$

$f(u_{2i-1}) = f(u_{2i}) = \begin{cases} 0, i \equiv 2, 3(mod4) \\ 1, i \equiv 0, 1(mod4) \end{cases}, 1 \le i \le n.$

$f(u_{2i-1}) = f(u_{2i}) = \begin{cases} 0, i \equiv 1, 2(mod4) \\ 1, i \equiv 0, 3(mod4) \end{cases}, n+1 \le i \le 2n.$

$f(w_{2i-1}) = f(w_{2i}) = \begin{cases} 0, i \equiv 1, 3(mod4) \\ 1, i \equiv 0, 2(mod4) \end{cases}, 1 \le i \le 4n.$

$f(w_{2i-1}) = 0, f(w_{2i}) = 1, , 4n+1 \le i \le 6n.$

Case(iii): $n \equiv 2(mod4)$

$f(v_i) = \begin{cases} 0, i \equiv 1, 2(mod4) \\ 1, i \equiv 3, 0(mod4) \end{cases}, 1 \le i \le n-2.$, $f(v_i) = \begin{cases} 0, i \equiv 0, 1(mod4) \\ 1, i \equiv 2, 3(mod4) \end{cases}, n+1 \le i \le 2n.$

$f(v_{n-1}) = 0, f(v_n) = 1$

$f(u_{2i-1}) = f(u_{2i}) = \begin{cases} 0, i \equiv 0, 3(mod4) \\ 1, i \equiv 1, 2(mod4) \end{cases}, 1 \le i \le n-2.$

$f(u_{2i-1}) = f(u_{2i}) = \begin{cases} 0, i \equiv 2, 3(mod4) \\ 1, i \equiv 0, 1(mod4) \end{cases}, n+1 \le i \le 2n.$

$f(u_{2n-2}) = f(u_{2n-3}) = 1, f(u_{2n}) = f(u_{2n-1}) = 0$

$f(w_{2i-1}) = f(w_{2i}) = \begin{cases} 0, i \equiv 1, 3(mod4) \\ 1, i \equiv 0, 2(mod4) \end{cases}, 1 \le i \le 4n.$

$f(w_{2i-1}) = 0,\ f(w_{2i}) = 1,\ 4n+1 \le i \le 6n.$

Case(iv): $n \equiv 3(mod4)$

$f(v_i) = \begin{cases} 0, i \equiv 1, 2(mod4) \\ 1, i \equiv 0, 3(mod4) \end{cases}, 1 \le i \le n.$, $f(v_i) = \begin{cases} 0, i \equiv 2, 3(mod4) \\ 1, i \equiv 0, 1(mod4) \end{cases}, n+1 \le i \le 2n.$

$f(u_{2i-1}) = \begin{cases} 0, i \equiv 1, 2(mod4) \\ 1, i \equiv 0, 3(mod4) \end{cases}, 1 \le i \le n.$, $f(u_{2i}) = \begin{cases} 0, i \equiv 0, 3(mod4) \\ 1, i \equiv 1, 2(mod4) \end{cases}, 1 \le i \le n.$

$f(u_{2i-1}) = f(u_{2i}) = \begin{cases} 0, i \equiv 0, 1(mod4) \\ 1, i \equiv 2, 3(mod4) \end{cases}, n+1 \le i \le 2n.$, $f(w_{2i-1}) = f(w_{2i}) = \begin{cases} 0, i \equiv 1, 3(mod4) \\ 1, i \equiv 0, 2(mod4) \end{cases}, 1 \le i \le 4n.$

$f(w_{2i-1}) = 0,\ f(w_{2i}) = 1,\ 4n+1 \le i \le 6n.$

The induced edge labeling $f^* : E(G) \to \{0, 1\}$

$$f^*(u_i u_j) = \begin{cases} 0, if \quad f(u_i) \text{ and } f(u_j) \text{ receive the same label} \\ 1, if \quad f(u_i) \text{ and } f(u_j) \text{ does not receive the same label.} \end{cases}$$

The vertex and edge condition of a double duplication of all vertices by edges of a ladder graph $L_n, n \ge 2$ are as follows:

n	$v_f(0)$	$v_f(1)$	$\|v_f(0)-v_f(1)\|$	$e_f(0)$	$e_f(1)$	$\|e_f(0)-e_f(1)\|$
$n\equiv 0(mod4)$	$\frac{27n-2}{2}$	$\frac{27n-2}{2}$	0	$9n$	$9n$	0
$n\equiv 1(mod4)$	$\frac{27n-3}{2}$	$\frac{27n-1}{2}$	1	$9n$	$9n$	0
$n\equiv 2(mod4)$	$\frac{27n-2}{2}$	$\frac{27n-2}{2}$	0	$9n$	$9n$	0
$n\equiv 3(mod4)$	$\frac{27n-1}{2}$	$\frac{27n-3}{2}$	1	$9n$	$9n$	0

From the above labeling pattern we infer that the difference between the vertices labeled with 0 and 1 differ by atmost 1 and the edges labeled with 0 and 1 also differ by atmost 0 and 1, hence the double duplication of all vertices by edges of a ladder graph $L_n, n \geq 2$ admits cordial labeling. □

Theorem 4. *The double duplication of all vertices by edges of a twig graph $T_n, n \geq 1$ admits cordial labeling.*

Proof. Let G be a graph obtained by double duplication of all vertices by edges of a twig graph $T_n, n \geq 1$.
The vertex set and edge set be defined by $V = V_1 \bigcup V_2 \bigcup V_3$
where, $V_1 = \{v_1, v_2...v_{3n+2}\}, V_2 = \{u_1, u_2...u_{6n+4}\}, V_3 = \{w_1, w_2...w_{18n+12}\}$ and $E = E_1 \bigcup E_2 \bigcup E_3 \bigcup E_4 \bigcup E_5 \bigcup E_6 \bigcup E_7 \bigcup E_8 \bigcup E_9 \bigcup E_{10} \bigcup E_{11}$
where, $E_1 = \{v_iv_{i+1}, 1 \leq i \leq n+1\}, E_2 = \{v_iv_{n+2i-1}, 2 \leq i \leq n+1\},$
$E_3 = \{v_iv_{2i+n}, 2 \leq i \leq n+1\}, E_4 = \{v_iu_{2i-1}, 1 \leq i \leq n+2\},$
$E_5 = \{v_iu_{2i}, 1 \leq i \leq n+2\}, E_6 = \{u_iw_{2i-1}, 1 \leq i \leq 6n+4\},$
$E_7 = \{u_iw_{2i}, 1 \leq i \leq 6n+4\}, E_8 = \{w_{2i-1}w_{2i}, 1 \leq i \leq 9n+6\},$
$E_9 = \{v_iw_{12n+7+2i}, 1 \leq i \leq 3n+2\}, E_{10} = \{v_iw_{12n+8+2i}, 1 \leq i \leq 3n+2\}$
$E_{11} = \{u_{2i-1}u_{2i}, 1 \leq i \leq 3n+2\}$
with $|V(G)| = 27n + 18$and $|E(G)| = 39n + 25$.
We define vertex labeling $f : V \to \{0, 1\}$ as follows:

$f(v_i) = \begin{cases} 0, i \equiv 0, 1(mod4) \\ 1, i \equiv 2, 3(mod4), 1 \leq i \leq n+2. \end{cases}$, $f(v_i) = \begin{cases} 1, i \equiv 0(mod2) \\ 0, i \equiv 1(mod2), n+3 \leq i \leq 3n+2. \end{cases}$

$f(u_{2i-1}) = f(u_{2i}) = \begin{cases} 0, i \equiv 2, 3(mod4) \\ 1, i \equiv 0, 1(mod4), 1 \leq i \leq n+2. \end{cases}$

$f(u_{2i-1}) = f(u_{2i}) = \begin{cases} 0, i \equiv 0, 2(mod4) \\ 1, i \equiv 1, 3(mod4), n+3 \leq i \leq 3n+2. \end{cases}$

$f(w_{2i-1}) = f(w_{2i}) = \begin{cases} 0, i \equiv 1, 3(mod4) \\ 1, i \equiv 0, 2(mod4), 1 \leq i \leq 7n. \end{cases}$

$f(w_{2i-1}) = 0,\ f(w_{2i}) = 1,\ 7n+1 \leq i \leq 10n+2.$

The induced edge labeling $f^* : E(G) \to \{0, 1\}$ is given by:

$$f^*(u_iu_j) = \begin{cases} 0, if \quad f(u_i) \text{ and } f(u_j) \text{ receive the same label} \\ 1, if \quad f(u_i) \text{ and } f(u_j) \text{ does not receive the same label.} \end{cases}$$

The vertex and edge condition of double duplication of all vertices by edges of a twig graph $T_n, n \geq 1$ are as follows:

n	$v_f(0)$	$v_f(1)$	$\lvert v_f(0)-v_f(1)\rvert$	$e_f(0)$	$e_f(1)$	$\lvert e_f(0)-e_f(1)\rvert$
$n\equiv 0(mod4)$	$\frac{9(3n+2)}{2}$	$\frac{9(3n+2)}{2}$	0	$\frac{39n+24}{2}$	$\frac{39n+26}{2}$	1
$n\equiv 1(mod4)$	$\frac{27n+19}{2}$	$\frac{27n+17}{2}$	1	$\frac{39n+25}{2}$	$\frac{39n+25}{2}$	0
$n\equiv 2(mod4)$	$\frac{9(3n+2)}{2}$	$\frac{9(3n+2)}{2}$	0	$\frac{39n+24}{2}$	$\frac{39n+26}{2}$	1
$n\equiv 3(mod4)$	$\frac{27n+17}{2}$	$\frac{27n+19}{2}$	1	$\frac{39n+25}{2}$	$\frac{39n+25}{2}$	0

From the above labeling pattern we infer that the difference between the vertices labeled with 0 and 1 differ by atmost 1 and the edges labeled with 0 and 1 also differ by atmost 0 and 1, hence the double duplication of all vertices by edges of a twig graph $T_n, n \geq 1$ admits cordial labeling. □

Theorem 5. *If G is a cordial labeling then the double duplication of all vertices by edges of G is also cordial.*

Proof. Let $G = (V, E)$ be an arbitrary graph with n vertices and m edges. Consider a graph $G^{'} = (V^{'}, E^{'})$ which is obtained by double duplication of all vertices by edges of G.
The vetex set and edge set of $G^{'}$ are defined by
$V^{'} = \{v_1, v_2, ..., v_n\} \bigcup \{u_1, u_2, ..., u_{2n}\} \bigcup \{w_1, w_2, ..., w_{6n}\} and$
$E^{'} = E_1 \bigcup E_2 \bigcup E_3 \bigcup E_4 \bigcup E_5 \bigcup E_6 \bigcup E_7 \bigcup E_8 \bigcup E$
where $E_1 = \{v_i u_{2i-1}, 1 \leq i \leq n\}$; $E_2 = \{v_i u_{2i}, 1 \leq i \leq n\}$;
$E_3 = \{u_i w_{2i-1}, 1 \leq i \leq 2n\}$; $E_4 = \{u_i w_{2i}, 1 \leq i \leq 2n\}$;
$E_5 = \{w_{2i-1} w_{2i}, 1 \leq i \leq 3n\}$; $E_6 = \{v_i w_{4n+(2i-1)}, 1 \leq i \leq n\}$;
$E_7 = \{v_i w_{4n+2i}, 1 \leq i \leq n\}$ $and E_8 = \{u_{2i-1} u_{2i}, 1 \leq i \leq n\}$
with $|V'(G')| = 9n \ \ and \ \ |E'(G')| = 12n + m$ where m is the number of edges in G

Case (i) $n \equiv 0(mod2)$
Since the number of vertices are even, the total number of vertices labeled with 0's and 1's differ by 0 in G.
We define a binary vertex labeling $f : V \rightarrow \{0, 1\}$ we consider the following,
The vertex labeling for G' is defined as follows :
(1) If the vertex set $\{v_1, v_2, ..., v_{n/2}\}$ in G, receives the label 0 then the labeling pattern for the newly added vertices in $G^{'}$ are as follows:

$f(u_i) = 1, 1 \leq i \leq n$

$$f(w_{2i-1}) = f(w_{2i}) = \begin{cases} 0, i \equiv 1, 3(mod4) \\ 1, i \equiv 0, 2(mod4), 1 \leq i \leq n. \end{cases}$$

(2) If the vertex set $\{v_{(n/2)+1}, v_{(n/2)+2}, ..., v_n\}$ in G, receives the label 1 then the labeling pattern for the newly added vertices in $G^{'}$ are as follows:

$f(u_i) = 0, , n + 1 \leq i \leq 2n$

$f(w_{2i-1}) = f(w_{2i}) = \begin{cases} 0, i \equiv 1,3(mod4) \\ 1, i \equiv 0,2(mod4) \end{cases}, n+1 \leq i \leq 2n.$

Continuing the numbering pattern. $f(w_j) = \begin{cases} 0, j \quad is \quad odd \\ 1, j \quad is \quad even \end{cases}, 4n+1 \leq i \leq 6n.$

The induced edge labeling of the newly added edges of G differ by zero.

Case(ii) $n \equiv 1(mod2)$

As n is odd there are two possibilities of vertex labeling namely (n+1)/2 vertices labeled zero and (n-1)/2 vertices labeled one and vice versa

Here we discuss the possibilities:

Sub Case (i): When (n+1)/2 is even labeled zero and (n-1)/2 is odd labeled one

Sub Case (ii): When (n+1)/2 is even labeled one and (n-1)/2 is odd labeled zero

Sub Case (iii): When (n+1)/2 is odd labeled zero and (n-1)/2 is even labeled one

Sub Case (iv): When (n+1)/2 is odd labeled one and (n-1)/2 is even labeled zero

We define a binary vertex labeling $f : V \to \{0, 1\}$ for the following,

(1) If $f(v_i) = 0$, $1 \leq i \leq n$

Then the first duplication of all vertices by edges will produce u_i's where $f(u_i) = 1$, $1 \leq i \leq 2n$. Again the duplication of all vertices(ie., v_i's and u_i's) by edges produces a new set of w_i's where $f(w_i)$ receives the following labeling pattern:

$f(w_{2i-1}) = f(w_{2i}) = \begin{cases} 0, i \equiv 1,3(mod4) \\ 1, i \equiv 0,2(mod4) \end{cases}, 1 \leq i \leq 4n.$

$f(w_i) = \begin{cases} 0, i \equiv 1(mod2) \\ 1, i \equiv 0(mod2) \end{cases}, 4n+1 \leq i \leq 6n.$

(2) If $f(v_i) = 1$, $1 \leq i \leq n$

Then the first duplication of all vertices by edges will produce u_i's where $f(u_i) = 0$, $1 \leq i \leq 2n$. Again the duplication of all vertices(ie., v_i's and u_i's) by edges produces a new set of w_i's where $f(w_i)$ receives the following labeling pattern:

$f(w_{2i-1}) = f(w_{2i}) = \begin{cases} 0, i \equiv 1,3(mod4) \\ 1, i \equiv 0,2(mod4) \end{cases}, 1 \leq i \leq 4n.$

$f(w_i) = \begin{cases} 0, i \equiv 1(mod2) \\ 1, i \equiv 0(mod2) \end{cases}, 4n+1 \leq i \leq 6n.$

The induced edge labeling of the newly added edges of G differ by zero.

Hence G' is cordial labeling if G is cordial. □

3 Conclusion

In this paper we have investigated some new double duplication of graphs. To analyze similar results for other graph families is our future area of research.

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