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Fuzzy Inventory Model with Re-order Level

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Abstract

In this article, we investigate an inventory problem with the help of fuzzy variables in conjunction with credibility theory. Based on expected value criterion and credibility criterion, a fuzzy expected value model and a fuzzy credibility programming model are constructed. The fuzzy expected value model aims to find the optimal order quantity and re-order level value such that the fuzzy expected value of the total cost is minimal. The fuzzy credibility model is used to find the optimal order quantity and re-order level, such that the total cost does not exceed the budget level. An iterative method is developed to calculate the expected value of the fuzzy credibility function and credibility of fuzzy event. The functionality of the developed method has been studied with the help of numerical examples.

Key Words: inventory, economic order quantity, re-order level, fuzzy variable, credibility

1 Introduction

Inventory control plays vital role in production industry to maintain an optimum inventory level to keep the activities of the organization effectively. An optimum inventory level will keep the company's profit at desirable level. The EOQ formula was introduced by Harris[1]. In the initial studies the parameters involved in the EOQ model, such as the demand, the purchasing cost, re-order level are assumed as crisp values or random variables. In real life situations, it is difficult to identify the probability distribution of these variables due to lack of past data. The usual practice is such parameters are often determined by subject experts. In most of the situations, it is likely that values arrived in that manner will be of subjective nature. Hence fuzzy theory, rather than the conventional probability theory is well suited in the study of inventory models.

In order to have more realistic inventory control models the fuzzy set theory was introduced by Zadeh[2] has been used by researchers. Initially Sommer[3] solved real world inventory problems using linguistic statements. Long term inventory policy making through fuzzy decision making models has been studied by Kacprzyk and Staniewski[4]. Following this several EOQ models have been developed using the concept of fuzzy by many researchers including Vujoseric, Petrovic and Petrovic[5] , Lee and Yao[6], Yao,Chang and Su[7] and Yao and Chiang[8]. In all these above mentioned models the parameters were assumed to be triangular fuzzy numbers or trapezoidal fuzzy numbers. Recently, Wang, Tang and Zhao[9] considered a fuzzy economic order quantity inventory model without backordering using credibility theory due to Liu[10]. In this paper, a fuzzy economic inventory model with re-order is considered using credibility theory.

The paper is organized as follows: Section 2 of this paper gives a brief discussion about Credibility Theory and Section 3 describes the model considered in this work along with solution.The last section of this paper gives results of the numerical study.

2 Credibility Theory

The credibility theory introduced by Baoding Liu[10] in branch of mathematics meant for the studying the behavior of fuzzy phenomena. Certain definitions associated with the credibiltiy theory are stated below.

Let Θ be a non empty set, and P the power set of Θ. Each element in P is called an event. In order to present an axiomtotic definition of credibility, it is necessary to assign to each event 'A'a number Cr(A) which indicates the credibility that 'A'will occur. In order to ensure that the number Cr(A) has certain mathematical properties which we expect a credibility to have, we accept the following four axioms.

1. (Normality) Cr(Θ)=1
2. (Monotonicity) Cr{A} $\leq$ Cr{B} whenever A $\subset$ B
3. (Self Dualitity) Cr{A} + Cr{A^c} = 1
4. (Maximality) Cr{$\cup_i A_i$} = Sup_iCr{A_i} for any events {A_i} with Sup_i Cr{A_i} < 0.5.

2.1 Credibility Space

(Θ,P,Cr) is called a credibility space where Θ is a non empty set, P the power set of Θ and Cr is a credibility measure .

2.2 Membership Function

Let ξ be a fuzzy variable defined on the credibility space (Θ,P,Cr). Then its membership function is derived from the credibility measure by

$$\mu(x) = (2Cr\{\xi = x\}) \wedge 1, x \in \Re \tag{1}$$

Let ξ be a fuzzy variable with membership function μ,then for any set B of real numbers we have,

$$Cr\{\xi \in B\} = \frac{1}{2}\left(\sup_{x\in B}\mu(x) + 1 - \sup_{x\in B^c}\mu(x)\right) \tag{2}$$

2.3 Credibility Distribution

The credibility distribution $\Phi : \Re \to [0,1]$ of a fuzzy variable ξ is defined by,

$$\Phi(x) = Cr\{\theta \in \Theta/\xi(\theta) \leq x\} \tag{3}$$

2.4 Expected value

Let ξ be a fuzzy variabe, the the expected value of ξ is defined by,

$$E(\xi) = \int_0^{+\infty} Cr\{\xi \geq r\}dr - \int_{-\infty}^{0} Cr\{\xi \leq r\}dr \tag{4}$$

prvoided that that at least one of the two integrals is finite.
The credibility density function $\phi : \Re \to [0, +\infty)$ of a fuzzy variable ξ is a function such that

$$\begin{gathered} \Phi(x) = \textstyle\int_{-\infty}^{x} \phi(y)dy, \forall x \in \Re \\ \textstyle\int_{-\infty}^{+\infty} \phi(y)dy = 1 \end{gathered} \tag{5}$$

where Φ is the credibility distribution of the fuzzy variable ξ.

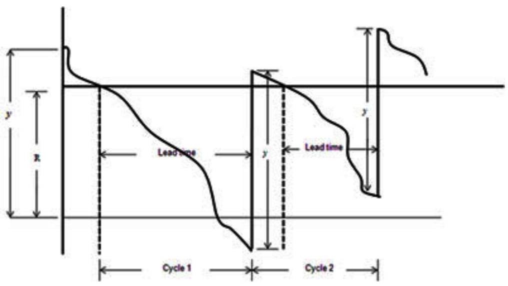
3 Credibilistic EOQ Model

The classical EOQ model in the simplest situation assumes constant demand rate with instantaneous order replenishment with no shortage. Initially it is assumed that the holding cost and the setup cost are deterministic. But in reality these costs are usually affected by several factors and do not remain as deterministic quantities. These kind of situations makes the application of fuzzy theory in the study of inventory models interesting as well as meaningful. Recently Wang, Tang and Zhao[9] considered an EOQ model in the fuzzy sense. Their model assumes no shortage is allowed and the setup cost K and the holding cost h are fuzzy variables in the sense of Liu[10] as described in Section 2. Wang et.al[9] have considered two different solutions for the model proposed by them and illustrated the solution with the help of numerical examples. In this paper, we consider a credibilistic EOQ model that allows shortage of quantity demanded. The model assumes placing of an order with quantity y whenever the inventory drops to a prespecified level R by assuming the demand is a fuzzy variable.

It may be noted that the re-order level R is a function of the lead time between placing and receiving an order. In this work, an optimum policy is identified by determining values of y and R such that the total cost is minimized. It is pertinent to note that the total cost comprises of set up, holding and shortage costs. ln this model we follow three assumptions are needed,

1. During lead time, the unfilled demand is backlogged.
2. A maximum of only one outstanding order is permitted.
3. During Lead time, the demand distribution is not affected by the time.

The scenario suitable for the above described model can be diagramatically represented as below.



In this study it is assumed that $\xi = (a, b, c)$ is a triangular fuzzy variable associated with the demand during the lead time with credibility density function $\phi(x)$, is given by

$$\phi(x) = \begin{cases} \frac{1}{2(b-a)} & if \quad a \leq x \leq b \\ \frac{1}{2(b-a)} & if \quad b \leq x \leq c \\ 0 & otherwise \end{cases} \tag{6}$$

By Liu(2004) the triangular fuzzy variable ξ defined above has the expected value

$$E(\xi) = \frac{a + 2b + c}{4} \tag{7}$$

Now we shall introduce notations need to develope the expression for the total cost function are,
D - Expected demand per unit time; h - holding cost
p - shortage cost per inventory unit; K - setup cost per order

Based on these notations, the elements of the cost function are now determined.

1. Set up cost: The set up cost per unit time is calculated by the product of setup cost per order named,K and the ratio between the expected demand and the inventory D/y. It is given by KD/y.

2. Expected holding cost: The expected inventory is calculated by the mean value of initial and final expected inventories of a cycle, $y + E\{R - \xi\}$ and E$\{R - \xi\}$ respectively. That is, the expected inventory I is given by,

$$I = \frac{(y + E\{R - \xi\}) + E\{R - \xi\}}{2} = \frac{y}{2} + R - E(\xi) \tag{8}$$

   The average holding cost per unit time thus equals to hI. In this work, the expected inventory level is computed by not considering the cases were $R - E\{\xi\}$ are negative.

3. Expected shortage cost: In the given system shortage cost arises when the quantity demanded ξ exceeds R, the re-order level. That is when $\xi > R$. In this case the expected shortage quantity per cycle is given by,

$$S = \int_R^{+\infty} (x - R)\phi(x)dx = \int_R^{c} (x - R)\phi(x)dx \tag{9}$$

$$= \begin{cases} \frac{a+2b+c}{4} - R & if \quad R < a \\ \frac{(b-R)^2}{4(b-a)} + \frac{c-2R+b}{4} & if \quad a \le R \le b \\ \frac{(c-R)^2}{4(c-b)} & if \quad b \le R \le c \\ 0 & if \quad R > c \end{cases} \tag{10}$$

Here, we have assumed the shortage cost per unit, say p is proportional to quantity. Since p is the shortage cost per unit, clearly the expected shortage cost per cycle is pS. Therefore the shortage cost per unit is pDS/y. Thus we get the total cost function depends on the variables y and R as stated below.

$$TC(y,R) = \begin{cases} \frac{DK}{y} + h\left[\frac{y}{2} + R + \frac{(a+2b+c)}{4}\right] \\ \quad + \frac{PD}{y}\left[\frac{(a+2b+c)}{4} - R\right] & if \quad R < a \\ \frac{DK}{y} + h\left[\frac{y}{2} + R + \frac{(a+2b+c)}{4}\right] \\ + \frac{PD}{y}\left[\frac{(b-R)^2}{4(b-a)} + \frac{(c-2R+b)}{4}\right] & if \quad a \le R \le b \\ \frac{DK}{y} + h\left[\frac{y}{2} + R + \frac{(a+2b+c)}{4}\right] \\ \quad + \frac{PD}{y}\left[\frac{(c-R)^2}{4(c-b)}\right] & if \quad b \le R \le c \\ \frac{DK}{y} + h\left[\frac{y}{2} + R + \frac{(a+2b+c)}{4}\right] & if \quad R > c \end{cases} \tag{11}$$

Optimizing the above bivariate function of the non negative variables y and R is not a staight forward task. Hence it is decided to seek a numerical solution. The package *stats*4 in R statistical computing software has *optim* that can be used for optimizing TC(y,R). Details related to the function *optim* can be obtained in stat.ethz.ch/R-manual/R-devel/library/stats/html/optim.html.

The following table gives optimal values of y and R for different sets of D, K, h, p, a, b and c. The initial values used while calling the function are also reported in the table. It was observed that the optimal values of y and R continued to be robust for different choices of initial values, with increased/decreased number of iterations.

D	K	h	p	a	b	c	y^*	R^*	**TC**	$*$	$**$
1000	100	2	10	0	50	100	319.41	93.61	726.1	(200, 80)	87
750	75	1	7	0	25	50	337.02	46.79	358.81	(200, 40)	69
500	50	0.5	4	10	20	30	316.97	28.41	162.72	(200, 20)	53
250	25	0.25	3	0	15	30	224.75	27.75	59.37	(200, 20)	65
2000	500	10	25	0	150	250	456.51	231.74	5506.78	(1000, 175)	77
10000	20	2	4	0	100	200	449.39	195.51	1089.94	(200, 150)	67
100	5	0.1	1	0	5	10	100.5	8.9	10.4	(50, 7)	61

$*$- Initial Values, $**$- Number of Iterations.

4 Conclusion

In this paper we have derived a fuzzy EOQ model when backordering is involved. For derivation we have considered the demand as a triangular fuzzy variable.The total cost function is optimized and found the optimum value of inventory level and re-order level. Using R statistical computing software we developed a program and it has been tested for different set of numerical values.

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