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An Arithmetic Operations of Icosagonal Fuzzy Number using Alpha cut

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Abstract

In this paper, a new form of fuzzy number named as Icosagonal fuzzy number is introduced, because it is not possible to restrict the membership function to any specific form. An Alpha cut of Icosagonal fuzzy number is defined and fundamental arithmetic operations are executed using interval arithmetic of Alpha cut and it is also elucidated by numerical examples.

AMS Subject Classification: 06D72

Key Words: fuzzy number, icosagonal fuzzy number, arithmetic operations, α-cut.

1 Introduction

Fuzzy numbers have been introduced by L.A.Zadeh [1, 2]. It is to deal with imprecise numerical quantities in a practical way. Many types of fuzzy sets are defined to clear the vagueness of the existing problems. Fuzzy set theory allows the gradual assessment of the membership of elements in a set which is recounted in the interval $[0, 1]$. In the literature, interval arithmetic was first suggested by Dwyer in 1951 by means of Zadeh's extension principle, the usual arithmetic operations on real numbers can be extended to the fuzzy numbers [3]. The same was developed by Moore [4]. To give out imprecise in real life situation, many researches used triangular and trapezoidal fuzzy numbers. In this paper, we propose Icosagonal fuzzy number with its membership functions. We

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expound basic arithmetic operations on Icosagonal fuzzy number using arithmetic interval of α-cuts and is explicated with numerical examples.

2 Preliminaries

In this section, we give the preliminaries that are required for this study.

Definition 2.1. *A fuzzy set A of the real line with $\mu_A(x) : R \to [0,1]$ is called fuzzy number where $\mu_A(x)$ is membership function if*

(i) *A must be normal and convex fuzzy set*

(ii) *The support of A must be bounded*

(iii) *$\alpha.A$ must be closed interval for every $\alpha \in [0,1]$*

Definition 2.2. [5] *Let X be a non empty set. A fuzzy set A of X is defined a s $A = \{(x, \mu_A(x)) : x \in X\}$ where $\mu_A(x)$ is membership function which maps each elements of X to a value between 0 and 1.*

Definition 2.3. *The support of a fuzzy set A defined on X is a crisp set defined as $support(A) = \{x \in X : \mu_A(x) > 0\}$*

Definition 2.4. *An α-cut of fuzzy set A is crisp set defined as ${}^{\alpha}[A] = \{x \in X \mid \mu_A(x) \geq \alpha\}$*

Definition 2.5. *A fuzzy set A is a convex fuzzy set iff each of its α-cut ${}^{\alpha}A$ is a convex set.*

Definition 2.6. [6] *A fuzzy number $A = (r_1, r_2, r_3)$ is said to be a triangular fuzzy number if its membership function is given by (where $r_1 \leq r_2 \leq r_3$)*

$$\mu_A(x) = \begin{cases} \frac{x-r_1}{r_2-r_1}, \ for \ r_1 \leq x \leq r_2 \\ \frac{r_3-x}{r_3-r_2}, \ for \ r_2 \leq x \leq r_3 \\ 0, \ for \ x > r_3 \end{cases}$$

Definition 2.7. *A fuzzy number $A = (r_1, r_2, r_3, r_4)$ is said to be a trapezoidal fuzzy number if its membership function is given by (where $r_1 \leq r_2 \leq r_3 \leq r_4$)*

$$\mu_A(x) = \begin{cases} 0, \ for \ x < r_1 \\ \frac{x-r_1}{r_2-r_1}, \ for \ r_1 \leq x \leq r_2 \\ 1 \ for \ r_2 \leq x \leq r_3 \\ \frac{r_4-x}{r_4-r_3}, \ for \ r_3 \leq x \leq r_4 \\ 0, \ for \ x > r_4 \end{cases}$$

3 Icosagonal fuzzy number

In this section a new form of fuzzy number named Icosagonal fuzzy number is introduced which can be more useful in solving many decisions making problems.
A fuzzy number $A_{Icos} = (r_1, r_2, \ldots, r_{19}, r_{20})$ is said to be Icosagonal fuzzy number if its membership function is given by $(where\ 0 \leq k_1 \leq k_2 \leq k_3 \leq k_4 \leq 1)$

$$\mu_{A_{Icos}}(x) = \begin{cases} 0,\ for\ x < r_1 \\ k_1(\frac{x-r_1}{r_2-r_1}),\ for\ r_1 \leq x \leq r_2 \\ k_1\ for\ r_2 \leq x \leq r_3 \\ k_1 + (k_2-k_1)(\frac{x-r_3}{r_4-r_3}),\ for\ r_3 \leq x \leq r_4 \\ k_2\ for\ r_4 \leq x \leq r_5 \\ k_2 + (k_3-k_2)(\frac{x-r_5}{r_6-r_5}),\ for\ r_5 \leq x \leq r_6 \\ k_3\ for\ r_6 \leq x \leq r_7 \\ k_3 + (k_4-k_3)(\frac{x-r_7}{r_8-r_7}),\ for\ r_7 \leq x \leq r_8 \\ k_4\ for\ r_8 \leq x \leq r_9 \\ k_4 + (1-k_4)(\frac{x-r_9}{r_{10}-r_9}),\ for\ r_9 \leq x \leq r_{10} \\ 1\ for\ r_{10} \leq x \leq r_{11} \\ k_4 + (1-k_4)(\frac{r_{12}-x}{r_{12}-r_{11}}),\ for\ r_{11} \leq x \leq r_{12} \\ k_4\ for\ r_{12} \leq x \leq r_{13} \\ k_3 + (k_4-k_3)(\frac{r_{14}-x}{r_{14}-r_{13}}),\ for\ r_{13} \leq x \leq r_{14} \\ k_3\ for\ r_{14} \leq x \leq r_{15} \\ k_2 + (k_3-k_2)(\frac{r_{16}-x}{r_{16}-r_{15}}),\ for\ r_{15} \leq x \leq r_{16} \\ k_2\ for\ r_{16} \leq x \leq r_{17} \\ k_1 + (k_2-k_1)(\frac{r_{18}-x}{r_{18}-r_{17}}),\ for\ r_{17} \leq x \leq r_{18} \\ k_1\ for\ r_{18} \leq x \leq r_{19} \\ k_1(\frac{r_{20}-x}{r_{20}-r_{19}}),\ for\ r_{19} \leq x \leq r_{20} \\ 0,\ for\ x > r_{20} \end{cases}$$

3.1 Arithmetic operations on Icosagonal fuzzy number

The addition, subtraction and multiplication of two Icosagonal fuzzy numbers, say $A_{Icos} = (r_1, r_2, \ldots, r_{20})$ and $B_{Icos} = (s_1, s_2, \ldots, s_{20})$ are given below.

(i) Addition: $A_{Icos} + B_{Icos} = (r_1 + s_1, r_2 + s_2, \ldots, r_{20} + s_{20})$.
Example: Let $A_{Icos} = (1, 2, 3, 4, 5, 7, 9, 10, 11, 13, 15, 16, 18, 19, 20, 22, 24, 26, 28, 30)$ and $B_{Icos} = (1, 3, 4, 5, 6, 8, 9, 12, 14, 15, 17, 21, 23, 25, 26, 27, 28, 29, 30, 31)$. Then $A_{Icos}+B_{Icos} = (2, 5, 7, 9, 11, 15, 18, 22, 25, 28, 32, 37, 41, 44, 46, 49, 52, 55, 58, 61)$

(ii) Subtraction: $A_{Icos} - B_{Icos} = (r_1 - s_1, r_2 - s_2, \dots, r_{20} - s_{20})$.
Example: Let $A_{Icos} = (1, 3, 5, 7, 8, 10, 12, 14, 15, 16, 18, 20, 24, 25, 27, 29, 31, 36, 39, 40)$ and $B_{Icos} = (0, 1, 2, 3, 4, 5, 7, 8, 9, 11, 12, 15, 17, 19, 20, 21, 22, 24, 25, 28)$. Then $A_{Icos} - B_{Icos} = (1, 2, 3, 4, 4, 5, 5, 6, 6, 5, 6, 5, 7, 6, 7, 8, 9, 12, 14, 12)$.

(iii) Multiplication: $A_{Icos} * B_{Icos} = (r_1 s_1, r_2 s_2, \dots, r_{20} s_{20})$.
Example: Let $A_{Icos} = (1, 2, 3, 4, 5, 7, 8, 9, 11, 13, 14, 16, 17, 18, 19, 20, 22, 23, 24, 26)$ and $B_{Icos} = (0, 1, 2, 4, 5, 6, 7, 9, 10, 12, 15, 17, 18, 21, 25, 27, 28, 29, 30, 31)$. Then $A_{Icos} * B_{Icos} = (0, 2, 6, 16, 25, 42, 56, 81, 110, 156, 210, 272, 306, 378, 475, 540, 616, 667, 720, 806)$

4 Alpha Cut

For $\alpha \in [0, 1]$, then α-cut of Icosagonal fuzzy number $A_{Icos} = (r_1, r_2, \dots, r_{20})$ is defined as

$$[A_{Icos}]_\alpha = \begin{cases} [r_1 + (\frac{\alpha}{k_1})(r_2 - r_1), r_{20} - \frac{\alpha}{k_1}(r_{20} - r_{19})], \\ \quad for\ \alpha \in [0, k_1] \\ [r_3 + (\frac{\alpha - k_1}{k_2 - k_1})(r_4 - r_3), r_{18} - (\frac{\alpha - k_1}{k_2 - k_1})(r_{18} - r_{17})], \\ \quad for\ \alpha \in [k_1, k_2] \\ [r_5 + (\frac{\alpha - k_2}{k_3 - k_2})(r_6 - r_5), r_{16} - (\frac{\alpha - k_2}{k_3 - k_2})(r_{16} - r_{15})], \\ \quad for\ \alpha \in [k_2, k_3] \\ [r_7 + (\frac{\alpha - k_3}{k_4 - k_3})(r_8 - r_7), r_{14} - (\frac{\alpha - k_3}{k_4 - k_3})(r_{14} - r_{13})], \\ \quad for\ \alpha \in [k_3, k_4] \\ [r_9 + (\frac{\alpha - k_4}{1 - k_4})(r_{10} - r_9), r_{12} - (\frac{\alpha - k_4}{1 - k_4})(r_{12} - r_{11})], \\ \quad for\ \alpha \in [k_4, 1] \end{cases}$$

4.1 Operations of Icosagonal fuzzy numbers using α-cut

The alpha cut of Icosagonal fuzzy number $[A_{Icos}] = (r_1, r_2, \dots, r_{20})$ for all $\alpha \in [0, 1]$ when $k_1 = \frac{1}{5}, k_2 = \frac{2}{5}, k_3 = \frac{3}{5}, k_4 = \frac{4}{5}$ given by

$$[A_{Icos}]_\alpha = \begin{cases} [r_1 + 5\alpha(r_2 - r_1), r_{20} - 5\alpha(r_{20} - r_{19})],\ for\ \alpha \in [0, 0.2] \\ [r_3 + (5\alpha - 1)(r_4 - r_3), r_{18} - (5\alpha - 1)(r_{18} - r_{17})],\ for\ \alpha \in [0.2, 0.4] \\ [r_5 + (5\alpha - 2)(r_6 - r_5), r_{16} - (5\alpha - 2)(r_{16} - r_{15})],\ for\ \alpha \in [0.4, 0.6] \\ [r_7 + (5\alpha - 3)(r_8 - r_7), r_{14} - (5\alpha - 3)(r_{14} - r_{13})],\ for\ \alpha \in [0.6, 0.8] \\ [r_9 + (5\alpha - 4)(r_{10} - r_9), r_{12} - (5\alpha - 4)(r_{12} - r_{11})],\ for\ \alpha \in [0.8, 1] \end{cases}$$

4.1.1 Addition

Let $A_{Icos} = (r_1, r_2, \ldots, r_{20})$ and $B_{Icos} = (s_1, s_2, \ldots, s_{20})$ be two Icosagonal fuzzy numbers. Let us add the α-cuts of $[A_{Icos}]_\alpha$ and $[B_{Icos}]_\alpha$ of A_{Icos} and B_{Icos} using interval arithmetic as defined below

$$[A_{Icos}]_\alpha + [B_{Icos}]_\alpha = \begin{cases} [r_1 + 5\alpha(r_2 - r_1), r_{20} - 5\alpha(r_{20} - r_{19})] \\ \quad + [s_1 + 5\alpha(s_2 - s_1), s_{20} - 5\alpha(s_{20} - s_{19})], \\ \qquad for\ \alpha \in [0, 0.2] \\ [r_3 + (5\alpha - 1)(r_4 - r_3), r_{18} - (5\alpha - 1)(r_{18} - r_{17})] \\ \quad + [s_3 + (5\alpha - 1)(s_4 - s_3), s_{18} - (5\alpha - 1)(s_{18} - s_{17})], \\ \qquad for\ \alpha \in [0.2, 0.4] \\ [r_5 + (5\alpha - 2)(r_6 - r_5), r_{16} - (5\alpha - 2)(r_{16} - r_{15})] \\ \quad + [s_5 + (5\alpha - 2)(s_6 - s_5), s_{16} - (5\alpha - 2)(s_{16} - s_{15})], \\ \qquad for\ \alpha \in [0.4, 0.6] \\ [r_7 + (5\alpha - 3)(r_8 - r_7), r_{14} - (5\alpha - 3)(r_{14} - r_{13})] \\ \quad + [s_7 + (5\alpha - 3)(s_8 - s_7), s_{14} - (5\alpha - 3)(s_{14} - s_{13})], \\ \qquad for\ \alpha \in [0.6, 0.8] \\ [r_9 + (5\alpha - 4)(r_{10} - r_9), r_{12} - (5\alpha - 4)(r_{12} - r_{11})] \\ \quad + [s_9 + (5\alpha - 4)(s_{10} - s_9), s_{12} - (5\alpha - 4)(s_{12} - s_{11})], \\ \qquad for\ \alpha \in [0.8, 1] \end{cases}$$

4.1.2 Subtraction

Let $A_{Icos} = (r_1, r_2, \ldots, r_{20})$ and $B_{Icos} = (s_1, s_2, \ldots, s_{20})$ be two Icosagonal fuzzy numbers. Let us subtract the α-cuts of $[A_{Icos}]_\alpha$ and $[B_{Icos}]_\alpha$ of A_{Icos} and B_{Icos} using interval arithmetic as defined below

$$[A_{Icos}]_\alpha - [B_{Icos}]_\alpha = \begin{cases} [r_1 + 5\alpha(r_2 - r_1), r_{20} - 5\alpha(r_{20} - r_{19})] \\ - [s_1 + 5\alpha(s_2 - s_1), s_{20} - 5\alpha(s_{20} - s_{19})], \\ \qquad for\ \alpha \in [0, 0.2] \\ [r_3 + (5\alpha - 1)(r_4 - r_3), r_{18} - (5\alpha - 1)(r_{18} - r_{17})] \\ - [s_3 + (5\alpha - 1)(s_4 - s_3), s_{18} - (5\alpha - 1)(s_{18} - s_{17})] \\ \qquad ,\ for\ \alpha \in [0.2, 0.4] \\ [r_5 + (5\alpha - 2)(r_6 - r_5), r_{16} - (5\alpha - 2)(r_{16} - r_{15})] \\ - [s_5 + (5\alpha - 2)(s_6 - s_5), s_{16} - (5\alpha - 2)(s_{16} - s_{15})], \\ \qquad for\ \alpha \in [0.4, 0.6] \\ [r_7 + (5\alpha - 3)(r_8 - r_7), r_{14} - (5\alpha - 3)(r_{14} - r_{13})] \\ - [s_7 + (5\alpha - 3)(s_8 - s_7), s_{14} - (5\alpha - 3)(s_{14} - s_{13})], \\ \qquad for\ \alpha \in [0.6, 0.8] \\ [r_9 + (5\alpha - 4)(r_{10} - r_9), r_{12} - (5\alpha - 4)(r_{12} - r_{11})] \\ - [s_9 + (5\alpha - 4)(s_{10} - s_9), s_{12} - (5\alpha - 4)(s_{12} - s_{11})], \\ \qquad for\ \alpha \in [0.8, 1] \end{cases}$$

4.1.3 Multiplication

Let $A_{Icos} = (r_1, r_2, \ldots, r_{20})$ and $B_{Icos} = (s_1, s_2, \ldots, s_{20})$ be two Icosagonal fuzzy numbers. Let us multiply the α-cuts of $[A_{Icos}]_\alpha$ and $[B_{Icos}]_\alpha$ of A_{Icos} and B_{Icos} using interval arithmetic as defined below

$$[A_{Icos}]_\alpha * [B_{Icos}]_\alpha = \begin{cases} [r_1 + 5\alpha(r_2 - r_1), r_{20} - 5\alpha(r_{20} - r_{19})] \\ * [s_1 + 5\alpha(s_2 - s_1), s_{20} - 5\alpha(s_{20} - s_{19})], \\ \quad for\ \alpha \in [0, 0.2] \\ [r_3 + (5\alpha - 1)(r_4 - r_3), r_{18} - (5\alpha - 1)(r_{18} - r_{17})] \\ * [s_3 + (5\alpha - 1)(s_4 - s_3), s_{18} - (5\alpha - 1)(s_{18} - s_{17})], \\ \quad for\ \alpha \in [0.2, 0.4] \\ [r_5 + (5\alpha - 2)(r_6 - r_5), r_{16} - (5\alpha - 2)(r_{16} - r_{15})] \\ * [s_5 + (5\alpha - 2)(s_6 - s_5), s_{16} - (5\alpha - 2)(s_{16} - s_{15})], \\ \quad for\ \alpha \in [0.4, 0.6] \\ [r_7 + (5\alpha - 3)(r_8 - r_7), r_{14} - (5\alpha - 3)(r_{14} - r_{13})] \\ * [s_7 + (5\alpha - 3)(s_8 - s_7), s_{14} - (5\alpha - 3)(s_{14} - s_{13})], \\ \quad for\ \alpha \in [0.6, 0.8] \\ [r_9 + (5\alpha - 4)(r_{10} - r_9), r_{12} - (5\alpha - 4)(r_{12} - r_{11})] \\ * [s_9 + (5\alpha - 4)(s_{10} - s_9), s_{12} - (5\alpha - 4)(s_{12} - s_{11})], \\ \quad for\ \alpha \in [0.8, 1] \end{cases}$$

Example 4.1.1: Addition of two Icosagonal fuzzy numbers.
Let $A_{Icos} = (1, 2, 3, 4, 5, 7, 9, 10, 11, 13, 15, 16, 18, 19, 20, 22, 24, 26, 28, 30)$ and $B_{Icos} = (1, 3, 4, 5, 6, 8, 9, 12, 14, 15, 17, 21, 23, 25, 26, 27, 28, 29, 30, 31)$.
For $\alpha \in [0, 0.2]$, we have $[A_{Icos}]_\alpha = [1 + 5\alpha, 30 - 10\alpha]$ and $[B_{Icos}]_\alpha = [1 + 10\alpha, 31 - 5\alpha]$. Thus $[A_{Icos}]_\alpha + [B_{Icos}]_\alpha = [2 + 15\alpha, 61 - 15\alpha]$. Moreover, when $\alpha = 0$, $[A_{Icos}]_\alpha + [B_{Icos}]_\alpha = [2, 61]$ and $\alpha = 0.2$, $[A_{Icos}]_\alpha + [B_{Icos}]_\alpha = [5, 58]$.
For $\alpha \in [0.2, 0.4]$, we have $[A_{Icos}]_\alpha = [2 + 5\alpha, 28 - 10\alpha]$ and $[B_{Icos}]_\alpha = [3 + 5\alpha, 30 - 5\alpha]$. Thus $[A_{Icos}]_\alpha + [B_{Icos}]_\alpha = [5 + 10\alpha, 58 - 15\alpha]$. Moreover, when $\alpha = 0.2$, $[A_{Icos}]_\alpha + [B_{Icos}]_\alpha = [7, 55]$ and $\alpha = 0.4$, $[A_{Icos}]_\alpha + [B_{Icos}]_\alpha = [9, 52]$.
For $\alpha \in [0.4, 0.6]$, we have $[A_{Icos}]_\alpha = [1 + 10\alpha, 26 - 10\alpha]$ and $[B_{Icos}]_\alpha = [2 + 10\alpha, 29 - 5\alpha]$. Thus $[A_{Icos}]_\alpha + [B_{Icos}]_\alpha = [3 + 20\alpha, 55 - 15\alpha]$. Moreover, when $\alpha = 0.4$, $[A_{Icos}]_\alpha + [B_{Icos}]_\alpha = [11, 49]$ and $\alpha = 0.6$, $[A_{Icos}]_\alpha + [B_{Icos}]_\alpha = [15, 46]$.
For $\alpha \in [0.6, 0.8]$, we have $[A_{Icos}]_\alpha = [6 + 5\alpha, 22 - 5\alpha]$ and $[B_{Icos}]_\alpha = [15\alpha, 31 - 10\alpha]$. Thus $[A_{Icos}]_\alpha + [B_{Icos}]_\alpha = [6 + 20\alpha, 53 - 15\alpha]$. Moreover, when $\alpha = 0.6$, $[A_{Icos}]_\alpha + [B_{Icos}]_\alpha = [18, 44]$ and $\alpha = 0.8$, $[A_{Icos}]_\alpha + [B_{Icos}]_\alpha = [22, 41]$.
For $\alpha \in [0.8, 1]$, we have $[A_{Icos}]_\alpha = [3 + 10\alpha, 20 - 5\alpha]$ and $[B_{Icos}]_\alpha = [10 + 5\alpha, 37 - 20\alpha]$. Thus $[A_{Icos}]_\alpha + [B_{Icos}]_\alpha = [13 + 15\alpha, 57 - 25\alpha]$. Moreover, when $\alpha = 0.8$, $[A_{Icos}]_\alpha + [B_{Icos}]_\alpha = [25, 37]$ and $\alpha = 1$, $[A_{Icos}]_\alpha +$

$[B_{Icos}]_\alpha = [28, 32]$. Hence $[A_{Icos}]_\alpha + [B_{Icos}]_\alpha = (2, 5, 7, 9, 11, 15, 18, 22, 25, 28, 32, 37, 41, 44, 46, 49, 52, 55, 58, 61)$.

Example 4.1.2: Subtraction of two Icosagonal fuzzy numbers.
Let $A_{Icos} = (1, 3, 5, 7, 8, 10, 12, 14, 15, 16, 18, 20, 24, 25, 27, 29, 31, 36, 39, 40)$ and $B_{Icos} = (0, 1, 2, 3, 4, 5, 7, 8, 9, 11, 12, 15, 17, 19, 20, 21, 22, 24, 25, 28)$.
For $\alpha \in [0, 0.2]$, we have $[A_{Icos}]_\alpha = [1 + 10\alpha, 40 - 5\alpha]$ and $[B_{Icos}]_\alpha = [5\alpha, 28 - 15\alpha]$. Thus $[A_{Icos}]_\alpha - [B_{Icos}]_\alpha = [1 + 5\alpha, 12 + 10\alpha]$. Moreover, when $\alpha = 0$, $[A_{Icos}]_\alpha - [B_{Icos}]_\alpha = [1, 12]$ and $\alpha = 0.2$, $[A_{Icos}]_\alpha - [B_{Icos}]_\alpha = [2, 14]$.
For $\alpha \in [0.2, 0.4]$, we have $[A_{Icos}]_\alpha = [3 + 10\alpha, 41 - 25\alpha]$ and $[B_{Icos}]_\alpha = [1 + 5\alpha, 26 - 10\alpha]$. Thus $[A_{Icos}]_\alpha - [B_{Icos}]_\alpha = [2 + 5\alpha, 15 - 15\alpha]$. Moreover, when $\alpha = 0.2$, $[A_{Icos}]_\alpha - [B_{Icos}]_\alpha = [3, 12]$ and $\alpha = 0.4$, $[A_{Icos}]_\alpha - [B_{Icos}]_\alpha = [4, 9]$.
For $\alpha \in [0.4, 0.6]$, we have $[A_{Icos}]_\alpha = [4 + 10\alpha, 33 - 10\alpha]$ and $[B_{Icos}]_\alpha = [2 + 5\alpha, 23 - 5\alpha]$. Thus $[A_{Icos}]_\alpha - [B_{Icos}]_\alpha = [2 + 5\alpha, 10 - 5\alpha]$. Moreover, when $\alpha = 0.4$, $[A_{Icos}]_\alpha - [B_{Icos}]_\alpha = [4, 8]$ and $\alpha = 0.6$, $[A_{Icos}]_\alpha - [B_{Icos}]_\alpha = [5, 7]$.
For $\alpha \in [0.6, 0.8]$, we have $[A_{Icos}]_\alpha = [6 + 10\alpha, 28 - 5\alpha]$ and $[B_{Icos}]_\alpha = [4 + 5\alpha, 25 - 10\alpha]$. Thus $[A_{Icos}]_\alpha - [B_{Icos}]_\alpha = [2 + 5\alpha, 3 + 5\alpha]$. Moreover, when $\alpha = 0.6$, $[A_{Icos}]_\alpha - [B_{Icos}]_\alpha = [5, 6]$ and $\alpha = 0.8$, $[A_{Icos}]_\alpha - [B_{Icos}]_\alpha = [6, 7]$.
For $\alpha \in [0.8, 1]$, we have $[A_{Icos}]_\alpha = [11 + 5\alpha, 28 - 10\alpha]$ and $[B_{Icos}]_\alpha = [1 + 10\alpha, 27 - 15\alpha]$. Thus $[A_{Icos}]_\alpha - [B_{Icos}]_\alpha = [10 - 5\alpha, 1 + 5\alpha]$. Moreover, when $\alpha = 0.8$, $[A_{Icos}]_\alpha - [B_{Icos}]_\alpha = [6, 5]$ and $\alpha = 1$, $[A_{Icos}]_\alpha - [B_{Icos}]_\alpha = [5, 6]$. Hence $[A_{Icos}]_\alpha - [B_{Icos}]_\alpha = (1, 2, 3, 4, 4, 5, 5, 6, 6, 5, 6, 5, 7, 6, 7, 8, 9, 12, 14, 12)$.

Example 4.1.3: Multiplication of two Icosagonal fuzzy numbers.
Let $A_{Icos} = (1, 2, 3, 4, 5, 7, 8, 9, 11, 13, 14, 16, 17, 18, 19, 20, 22, 23, 24, 26)$ and $B_{Icos} = (0, 1, 2, 4, 5, 6, 7, 9, 10, 12, 15, 17, 18, 21, 25, 27, 28, 29, 30, 31)$.
For $\alpha \in [0, 0.2]$, we have $[A_{Icos}]_\alpha = [1 + 5\alpha, 26 - 10\alpha]$ and $[B_{Icos}]_\alpha = [5\alpha, 31 - 5\alpha]$. Thus $[A_{Icos}]_\alpha * [B_{Icos}]_\alpha = [25\alpha^2 + 5\alpha, 50\alpha^2 - 440\alpha + 806]$. Moreover, when $\alpha = 0$, $[A_{Icos}]_\alpha * [B_{Icos}]_\alpha = [0, 806]$ and $\alpha = 0.2$, $[A_{Icos}]_\alpha * [B_{Icos}]_\alpha = [2, 720]$. For $\alpha \in [0.2, 0.4]$, we have $[A_{Icos}]_\alpha = [2 + 5\alpha, 24 - 5\alpha]$ and $[B_{Icos}]_\alpha = [10\alpha, 30 - 5\alpha]$. Thus $[A_{Icos}]_\alpha * [B_{Icos}]_\alpha = [50\alpha^2 + 20\alpha, 25\alpha^2 - 270\alpha + 720]$. Moreover, when $\alpha = 0.2$, $[A_{Icos}]_\alpha * [B_{Icos}]_\alpha = [6, 667]$ and $\alpha = 0.4$, $[A_{Icos}]_\alpha * [B_{Icos}]_\alpha = [16, 616]$.
For $\alpha \in [0.4, 0.6]$, we have $[A_{Icos}]_\alpha = [1 + 10\alpha, 22 - 5\alpha]$ and $[B_{Icos}]_\alpha = [3 + 5\alpha, 31 - 10\alpha]$. Thus $[A_{Icos}]_\alpha * [B_{Icos}]_\alpha = [50\alpha^2 + 35\alpha + 3, 50\alpha^2 - 375\alpha + 682]$. Moreover, when $\alpha = 0.4$, $[A_{Icos}]_\alpha * [B_{Icos}]_\alpha = [25, 540]$ and $\alpha = 0.6$, $[A_{Icos}]_\alpha * [B_{Icos}]_\alpha = [42, 475]$.

For $\alpha \in [0.6, 0.8]$, we have $[A_{Icos}]_\alpha = [5 + 5\alpha, 21 - 5\alpha]$ and $[B_{Icos}]_\alpha = [1 + 10\alpha, 30 - 15\alpha]$. Thus $[A_{Icos}]_\alpha * [B_{Icos}]_\alpha = [50\alpha^2 + 55\alpha + 5, 75\alpha^2 - 465\alpha + 630]$. Moreover, when $\alpha = 0.6$, $[A_{Icos}]_\alpha * [B_{Icos}]_\alpha = [56, 378]$ and $\alpha = 0.8$, $[A_{Icos}]_\alpha * [B_{Icos}]_\alpha = [81, 306]$.
For $\alpha \in [0.8, 1]$, we have $[A_{Icos}]_\alpha = [3 + 10\alpha, 24 - 10\alpha]$ and $[B_{Icos}]_\alpha = [2 + 10\alpha, 25 - 10\alpha]$. Thus $[A_{Icos}]_\alpha * [B_{Icos}]_\alpha = [100\alpha^2 + 50\alpha + 6, 100\alpha^2 - 490\alpha + 600]$. Moreover, when $\alpha = 0.8$, $[A_{Icos}]_\alpha * [B_{Icos}]_\alpha = [110, 272]$ and $\alpha = 1$, $[A_{Icos}]_\alpha * [B_{Icos}]_\alpha = [156, 210]$.
Hence $[A_{Icos}]_\alpha * [B_{Icos}]_\alpha = (0, 2, 6, 16, 25, 42, 56, 81, 110, 156, 210, 272, 306, 378, 475, 540, 616, 667, 720, 806)$.

5 Conclusion

Icosagonal fuzzy number was introduced in this paper and also certain arithmetic operations were performed such as addition, subtraction and multiplication. Icosagonal fuzzy number can be applied to the problem which has twenty points in representation. In future, it may be applied in decision making problems as well as in operations research problems.

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