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Drag on a Porous Sphere Embedded in Micropolar Fluid with Non-Zero Spin Boundary Condition

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Abstract

This paper examines an analytical study of creeping flow of an incompressible micropolar fluid past a porous sphere with non homogeneous boundary condition for microrotation vector. The Brinkman equation and Stokes equation are used for the flow inside and outside the porous sphere respectively in their stream function formulations. The drag experienced by porous sphere is evaluated and its variation with respect to material parameter is studied. Results are validated with past known cases.

Keywords: Micropolar fluid, Gegenbauer functions, Modified Bessel functions, Drag force.

1 Introduction

Micropolar fluids are nothing but, a fluid with microstructure. Micropolar fluids can also personate fluids consisting of rigid, random oriented particles are suspended in a viscous medium. Eringen's [2] model of micro fluids, understanding with a class of fluids which shows undoubtful microscopic effects which raises from the

local construction and micro-motions of the fluid particles. Micropolar fluids comes under these fluids which can support couple stress and body couples. Thus, distinct particles of complex fluids comprise polymeric suspensions, blood of animal, liquid crystals, ferrofluids, blood flows and bubbly fluids.
The problem of Stokes flow through a swarm of porous approximately spheroidal particles by applying boundary condition namely kuwabara, on the cell surface have been discussed by Deo and Gupta[1]. Gupta and Deo [3] solved analytically the problem of Stokes flow of micropolar fluid past a porous sphere with non-zero boundary condition for microrotations by assuming the flow is uniform. Drag on sphere in micropolar fluids with non-zero boundary conditions for microrotations has been solved by Haffmann et al. [4]. He found out that resistance force experienced on a solid sphere moving with uniform velocity in the case of micropolar fluids. Qin and Kaloni [5] investigated the result of Brinkman's equation Cartesian tensor in the porous media and by applying this solution they obtained that a porous sphere exerted the force of hydrodynamic.

This paper concerns the problem of an analytical study of creeping flow of an incompressible micropolar fluid past a porous sphere with non homogeneous boundary condition for microrotation vector. The Brinkman equation and Stokes equation are used for the flow inside and outside the porous sphere respectively in their stream function formulations. The drag experienced by porous sphere is evaluated and its variation with respect to material parameter is studied. Results are validated with past known cases.

2 Mathematical Formulation

Consider the problem of an incompressible micropolar fluid past a porous sphere of radius $'a'$ in an unbounded medium with origin at the center of the sphere and uniform velocity U moves along the positive $z-$axis. The region of exterior and interior of porous sphere is indicated by $i = 1$ and $i = 2$ respectively. The equation of motions for outside region of porous sphere is governed by Stokes equation as follows

$$div \quad v^{(1)} = 0 \tag{1}$$

$$-\nabla\vec{p}^{\,(1)}+\kappa\nabla\times\vec{\omega}^{(1)}-(\mu_1+\kappa)\nabla\times\nabla\times\vec{v}^{\,(1)}=0 \tag{2}$$

$$-2\kappa\vec{\omega}^{(1)}+\kappa\nabla\times\vec{v}^{(1)}-\gamma\nabla\times\nabla\times\vec{\omega}^{(1)}+(\alpha+\beta+\gamma)\nabla(\nabla.\,\vec{\omega}^{(1)})=0 \tag{3}$$

The equation of motions for inside region of porous sphere is governed by Brinkman equation as follows

$$\mu_e\nabla^2v^{(2)}-\frac{\mu_2}{k}v^{(2)}=\nabla p^{(2)} \tag{4}$$

$$div\ \ v^{(2)}=0 \tag{5}$$

where $\vec{v}^{(1)}$ denotes velocity vector, $\vec{\omega}^{(1)}$ denotes microrotation vector, $\vec{p}^{(1)}$ denotes fluid pressure ,k denotes permeability of porous medium. The material constants $(\mu_1,\kappa)$are viscosity coefficients and (α,β,γ) are gyroviscosity coefficients which satisfy the following inequalities

$$3\alpha+\beta+\gamma\geq 0,\quad 2\mu_1+\kappa\geq 0,\quad \gamma\geq|\beta|\,,\quad \kappa\geq 0,\quad \gamma\geq 0 \tag{6}$$

Velocity components for both the regions interms of stream functions are given by

$$v_r^{(i)}=-\frac{1}{r^2sin\theta}\frac{\partial\psi^{(i)}}{\partial\theta},\qquad v_\theta^{(i)}=\frac{1}{rsin\theta}\frac{\partial\psi^{(i)}}{\partial r} \tag{7}$$

By eliminating pressure from equation (2) to (7) we obtain the following differential equation as

$$E^4(E^2-m^2)\psi^{(1)}=0 \tag{8}$$

$$E^2(E^2-\alpha^2)\psi^{(2)}=0 \tag{9}$$

where $E^2=\frac{\partial^2}{\partial r^2}+\frac{1-\zeta^2}{r^2}\frac{\partial^2}{\partial\zeta^2}$, $\zeta=cos\theta$, $m^2=\frac{k(2\mu_1+\kappa)}{\gamma(\mu_1+\kappa)}a^2$, $\eta^2=\frac{a^2}{k}$ and $N=\frac{\kappa}{\mu_1+\kappa}$ is the coupling number $(0\leq N<1)$

The complete regular solution for Stokes equations and Brinkman's equations is of the following form

$$\psi^{(1)}(r,\zeta)=\left[r^2+B_1r^{-1}+D_1r+E_1\sqrt{r}K_{\frac{3}{2}}(mr)\right]G_2(\zeta) \tag{10}$$

$$\psi^{(2)}(r,\zeta)=\left[A_2r^2+F_2\sqrt{r}I_{\frac{3}{2}}(\alpha r)\right]G_2(\zeta) \tag{11}$$

$G_2(\zeta)=\frac{1}{2}(1-\zeta^2)$ which is the Gegenbauer function of first kind.

Microrotation for outside the porous sphere is given by

$$v_\phi^{(1)}=\frac{1}{rsin\theta}\left[-D_1^{(1)}r^{-1}+\frac{m^2}{N}E_1^{(1)}\sqrt{r}K_{\frac{3}{2}}(mr)\right]G_2(\zeta) \tag{12}$$

3 Boundary Conditions

Here we apply the following boundary conditions which is used at the interface. The kinematic viscosity of mutual impenetrability at the surface $r = a$ can be taken as

$$\psi^{(1)}(r,\zeta) = \psi^{(2)}(r,\zeta) \tag{13}$$

The continuity of tangential velocity components across the sphere shows that we can take

$$\frac{\partial\psi^{(1)}(r,\zeta)}{\partial r} = \frac{\partial\psi^{(2)}(r,\zeta)}{\partial r} \qquad on \quad r = a \tag{14}$$

The continuity of tangential stress components across the exterior region to be taken as

$$\tau_{r\theta}^{(1)} = \tau_{r\theta}^{(2)} \qquad on \quad r = a \tag{15}$$

Here we use the non-zero spin for the microrotation on the boundary $\omega^{(1)} = \frac{\tau}{2} curl v^{(1)}$
By simplification we get,

$$v_{\phi}^{(1)} = \frac{\tau}{2r sin\theta} E^2\psi^{(1)} \qquad on \quad r = a \tag{16}$$

Continuity of pressure at the boundary of sphere

$$p^{(1)}(r,\zeta) = p^{(2)}(r,\zeta) \qquad on \quad r = a \tag{17}$$

Solving the above preceeding boundary conditions from equations (13) to (17), we get all the values of unknowns which are appearing in stream function equations (10) and (11) by using mathematica software.

4 Determination of Drag Force

The drag force experienced by porous sphere can be evaluated by using the following formula as

$$F = \pi\mu_1 U a \int_0^{\pi} \varpi^3 \frac{\partial}{\partial r}\left(\frac{E^2\psi^{(1)}}{\varpi^2}\right) r d\theta \tag{18}$$

where $\overline{\omega} = rsin\theta$
After integrating (18) we find that

$$F = 2\pi a(2\mu_1 + \kappa)UD_1 \tag{19}$$

Also the non dimensional drag is given by

$$D_N = F/-2\pi\mu_1 Ua \tag{20}$$

5 Special Cases:

Case I: Drag for without spin on boundary($\tau = 0$)
The value of drag force for zero spin turn out as

$$F = \frac{2\pi(2\mu_1+\kappa)\left[12(1+m)(-1+N)\alpha^2(-2 + (2+\alpha^2)\lambda)(\kappa+2\mu_1)(\alpha cosh[\alpha] - sinh[\alpha])\right)}{\Delta'} \tag{21}$$

where

$$\begin{aligned}\Delta' =& (-(-(-2+N)(-3+2(-1+N)\alpha^2)+m(-6-4\alpha^2+N(3+4\alpha^2))\\ &(-2+(2+\alpha^2)\lambda)cosh[\alpha]+(-(-2+N)(6+\alpha^2(6+4N(-1+\lambda)-9\lambda)\\ &+2(-1+N)\alpha^4(-1+\lambda)-6\lambda+m(2N^2\alpha^4-2(6(-1+\lambda)\\ &+2\alpha^4(-1+\lambda)+\alpha^2(-6+9\lambda))+N(6(-1+\lambda)\\ &+\alpha^4(-6+4\lambda)+\alpha^2(-10+13\lambda)))sinh[\alpha]\end{aligned} \tag{22}$$

Here $\lambda = \frac{\mu_2}{\mu_1}$
Case II: Drag on a porous sphere embedded in fluid sphere
If $\kappa \to 0$ i.e, $m \to 0$, $N \to 0$, $\lambda \to 1$, ($\alpha^2 \to \eta^2$)then the micropolar fluid changes into Newtonian fluid. The drag force reduces to

$$F = \frac{-12\pi\mu_1 aU\eta^2(sinh\ \eta - \eta\ cosh\ \eta)}{\eta(3+2\eta^2)cosh\ \eta - 3sinh\ \eta} \tag{23}$$

This result agree from earlier derived Qin and Kaloni[5] result.
Case III: A solid sphere in an unbounded medium
If $\alpha \to \infty$ then the porous sphere becomes solid sphere. In this case the drag force come out as

$$F = -6\pi\mu_1 Ua \tag{24}$$

which is similar result obtained previously by Stokes for the drag past a solid sphere in an unbounded medium.

6 Results and Discussion:

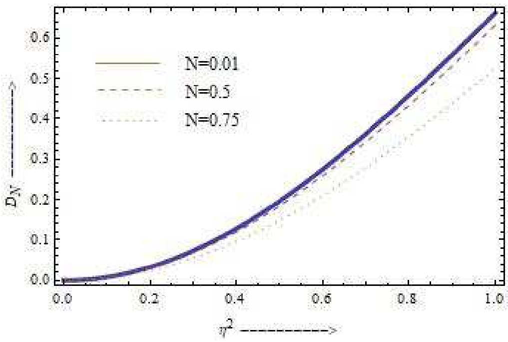

Figure 1: Variation of D_N versus η^2 for various values of N with m=5

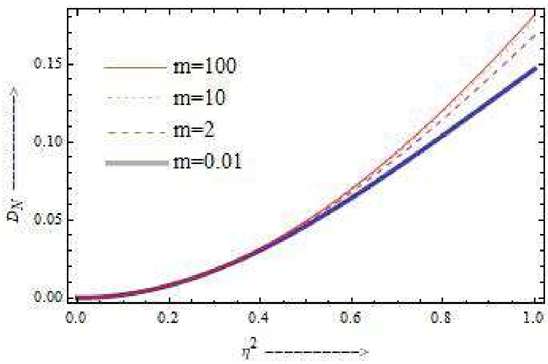

Figure 2: Variation of D_N versus η^2 for various values of m with N=0.5

Figure 1 represents the variation of nondimensional drag D_N with respect to permeability parameter η^2 for various values of coupling number N and $m = 5$. It is noticed that drag increases with increase of permeability parameter η^2 whereas drag decreases while increasing of coupling number N. Also it is found that drag is lesser in the case of micropolar fluid with spin condition than that of Newtonian fluid case. Variation of nondimensional drag

D_N against permeability parameter η^2 is shown in figure 2. It is clear that there is increase in the drag as the micropolar parameter m is increasing.

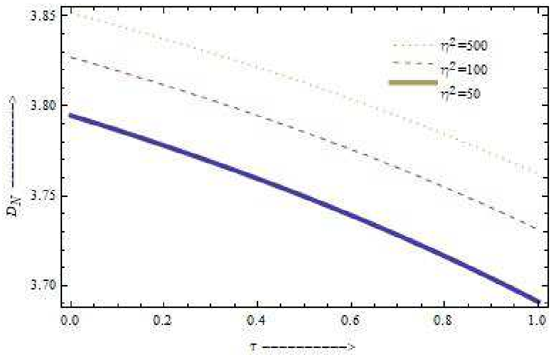

Figure 3: Variation of D_N versus τ for various values of η^2 with m=10, N=0.5

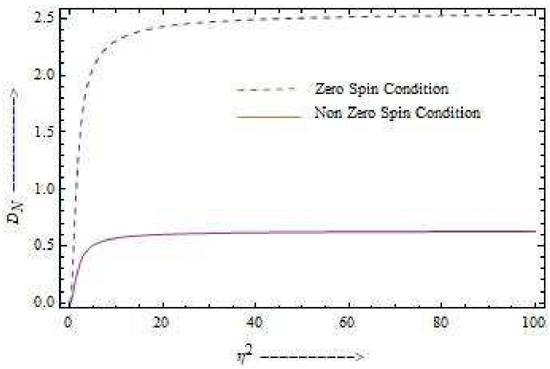

Figure 4: Effect of η^2 on D_N with non-zero and zero spin boundary condition for various values of m when N=0.5

Figure 3 shows that variation of nondimensional drag D_N with respect to parameter τ. It is evident that drag is decreasing as spin parameter τ is increasing and increases with increasing values of permeability parameter. Also impact of spin parameter τ is exceedingly large on drag. Effect of permeability parameter η^2 on nondimensional drag D_N for various values of m with non-zero and zero spin boundary condition is shown in figure 4. It is interesting to note that values of nondimensional drag D_N is lesser in the case of nonzero microrotation than that of zero microrotation case.

7 Conclusion:

The stream function solutions are obtained to flow field equations for steady axisymmetric Stokes flow of micropolar fluid past a porous sphere with non-zero boundary condition. The drag force exerted by a porous sphere embedded in micropolar fluid has been determined. This present model is found to deduce to previous well known results from literature in limiting cases. It is examined that effect of micropolar parameter m, permeability parameter η^2, coupling number N and spin parameter τ control the drag force. Also drag resist greater with zero microrotation vector in comparison of non-zero microrotation vector.

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