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Packing of Certain Mesh Derived Architectures

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Abstract

An *H-packing* of a graph G is a set of vertex disjoint subgraphs of G, each of which is isomorphic to a fixed graph H. An *F-packing* is a natural generalization of H-packing concept. For a given family F of graphs, the problem is to identify a set of vertex-disjoint subgraphs of G, each isomorphic to a member of F [5]. In this paper we have investigated that *Extended Mesh* [7] $EX(m,n)$ for $m \geq 2$, $n \geq 2$, *Mesh* $M(m,n)$ for $m \geq 2$, $n \geq 2$ and *Enhanced Mesh* [7] $EN(m,n)$ for $m \geq 2$, $n \geq 2$ admits a perfect packing and thus determine their packing numbers.

Keywords: *H-packing, F-packing, Perfect packing, Mesh, Extended Mesh, Enhanced Mesh*

1 Introduction and Terminology

Producing patterns with thin films of silicon to form nanomesh structures reduces their thermal conductivity without compromising their good electrical properties [4]. Further arranging molecules themselves into predictable patterns on silicon chips could lead to microprocessors with much smaller circuit elements [13].

Mathematically, assembling in predictable patterns is equivalent to packing in graphs. When H is a connected graph with at least three vertices, Kirkpatrick and Hell proved that the maximum H-packing problem is NP-complete [10].

An ***H-packing*** of a graph G is a set of vertex disjoint subgraphs of G, each of which is isomorphic to a fixed graph H. From the optimization point of view, maximum H-packing problem is to find the maximum number of vertex disjoint copies of H in G called the ***packing number*** denoted by $\lambda(G, H)$. For our convenience $\lambda(G, H)$ is sometimes represented as λ.

An H-packing in G is called ***perfect*** if it covers all the vertices of G.

An ***F-packing*** is a natural generalization of H-packing concept. For a given family F of graphs, the problem is to identify a set of vertex-disjoint subgraphs of G, each isomorphic to a member of F. The F-packing problem is to find an F-packing in a graph G that covers the maximum number of vertices of G. Throughout this paper a path on n vertices is denoted by P_n.

2 Literature Survey

The problem of covering the vertices of a given graph with a maximum number of disjoint copies of the complete graph on two vertices, K_2 is called the *maximum matching problem.* Due to the fruitful results of matching theory, similar results hold for the analogous problem using a graph other than K_2. The problem of covering a graph with copies of graphs other than K_2 alone is called the *graph packing problem.*

Results from packing theory have useful applications to code optimization [3], clustering [9], and component placing [1]. Hell and Kirkpatrick considered one such application, where they mention that their interest in the study of generalized matchings stemmed from the study of scheduling examination periods [2]. H-Packing, is of practical interest in the areas of scheduling [11], wireless sensor tracking [12], wiring-board design [10] and many others.

The mesh networks have been recognized as versatile interconnection networks for parallel computing. This is mainly due to the fact that these families of networks have topologies which reflect the communication pattern of a wide variety of natural problems. [6].

In this paper we find the perfect packing for certain mesh derived architectures such as Extended Mesh, Mesh and Enhanced Mesh with P_3 and P_4 and hence determine their packing numbers.

3 Definitions

Definition 3.1. An $m \times n$ mesh is a graph $M(m, n)$ with vertex set $V = \{(i, j) : 1 \leq i \leq m, 1 \leq j \leq n\}$ and edge set $E = \{((i, j), (i, j+1)) : 1 \leq i \leq m, 1 \leq j \leq n-1\} \cup \{((i, j), (i+1, j)) : 1 \leq i \leq m-1, 1 \leq j \leq n\}$ [8].

Definition 3.2. The extended mesh $EX(m, n)$ is a $m \times n$ mesh in which every 4-cycle is made into a complete graph. There are mn vertices in an extended mesh and we denote each vertex as (i, j) for $i = 1, 2, ..., m$ and $j = 1, 2, ..., n$ [7].

Definition 3.3. An enhanced mesh $EN(m, n)$ is obtained by replacing each 4-cycle of $M(m, n)$ by a wheel, the hub of the wheel being a new vertex. Let $h_{ij}, 1 \leq i \leq m-1, 1 \leq j \leq n-1$ be the hub vertices [8].

4 Path Packing of Certain Mesh Derived Architectures

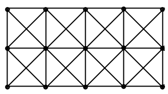
Theorem 1. *The Extended Mesh $EX(m, n)$ for $m \geq 2$, $n \geq 2$ admits a perfect packing with P_3 when $mn \equiv 0\ (mod\ 3)$ and with P_4 when $mn \equiv 0\ (mod\ 4)$ and with P_3 and P_4 when $mn \equiv 1, 2\ (mod\ 3)$.*

Proof. The extended mesh $EX(m, n)$ has mn vertices. When $mn \equiv 0 \pmod{k}$ the total number of vertices of $EX(m, n)$ is a multiple of k and hence a perfect packing exist only when the total number of vertices of the H graph is k or multiples of k.

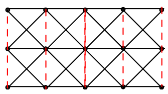
Case 1: $mn \equiv 0 \pmod 3$

The case when $mn \equiv 0 \pmod 3$ implies either m or n is a multiple of three. Fix H as P_3 the path on three vertices. If m is a multiple of three then the extended mesh can be partitioned into $\frac{mn}{3}$ vertical paths P_3 and if n is a multiple of three then the extended mesh can be partitioned into $\frac{mn}{3}$ horizontal paths P_3 as illustrated below. Hence the extended mesh $EX(m, n)$ admits a perfect packing with P_3 where

$$\lambda(G, H) = \frac{mn}{3}$$

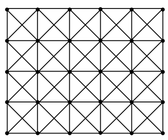


(a) Extended Mesh $EX(3, 5)$

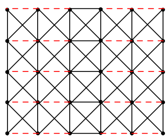


(b) H-Packing of $EX(3, 5)$

Figure 1: When $n = 3k$



(a) Extended Mesh $EX(5, 6)$



(b) H-Packing of $EX(5, 6)$

Figure 2: When $m = 3k$

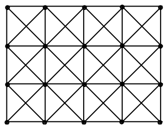
Case 2: $mn \equiv 0 \pmod 4$

The case when $mn \equiv 0 \pmod 4$ then either m or n can be expressed as a multiple of four. Fix H as P_4 the path on four vertices. If m is a multiple of four then the extended mesh can be partitioned into $\frac{mn}{4}$ vertical paths P_4 and if n is a multiple of four then the extended mesh can be partitioned into $\frac{mn}{4}$ horizontal paths P_4 as illustrated below. Hence the extended mesh $EX(m, n)$ admits a perfect packing with P_4 where

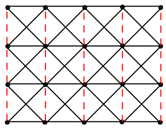
$$\lambda(G, H) = \frac{mn}{4}$$

4

Values for which $mn \equiv 0 \pmod 3$ and $mn \equiv 0 \pmod 4$ the vertices of the graph are packed with copies of P_3.

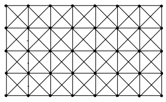


(a) Extended Mesh $EX(4,5)$

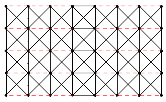


(b) H-Packing of $EX(4,5)$

Figure 3: When $m = 4k$



(a) Extended Mesh $EX(5,8)$



(b) H-Packing of $EX(5,8)$

Figure 4: When $n = 4k$

Case 3: $mn \equiv 1, 2 \pmod 3$

The case when $mn \equiv 1, 2 \pmod 3$ fix the family of graphs to be $F = \{P_3, P_4\}$. When $mn \equiv 1 \pmod 3$ the mn vertices of the extended mesh $EX(m,n)$ can be partitioned into $\{[\frac{mn}{3}] - 1\}$ disjoint paths P_3 and the remaining four vertices can be packed by P_4 . When $mn \equiv 2 \pmod 3$ the mn vertices of the extended mesh $EX(m,n)$ can be partitioned into $\{[\frac{mn}{3}] - 2\}$ disjoint paths P_3 and the remaining eight vertices can be packed by two copies of P_4.

□

The proof of the following result is similar to that of Theorem 1.

Theorem 2. *The Mesh $M(m,n)$ for $m \geq 2$, $n \geq 2$ admits a perfect packing with P_3 when $mn \equiv 0 \pmod 3$ and with P_4 when $mn \equiv 0 \pmod 4$ and with P_3 and P_4 when $mn \equiv 1, 2 \pmod 3$.*

Theorem 3. *The Enhanced Mesh $EN(m,n)$ for $m \geq 2$, $n \geq 2$ admits a perfect packing with P_3 when $mn + (m-1)(n-1) \equiv 0 \pmod 3$ and with P_4 when $mn + (m-1)(n-1) \equiv 0 \pmod 4$ and with P_3 and P_4 when $mn + (m-1)(n-1) \equiv 1, 2 \pmod 3$.*

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Proof. The enhanced mesh $EN(m, n)$ has $mn + (m-1)(n-1)$ vertices. When $mn+(m-1)(n-1) \equiv 0 \pmod k$ the total number of vertices of $EN(m, n)$ is a multiple of k and hence a perfect packing exist only when the total number of vertices of the H graph is k or multiples of k. Construct an Hamiltonian path in the enhanced mesh that traverses through all the vertices of the mesh exactly once as illustrated below.

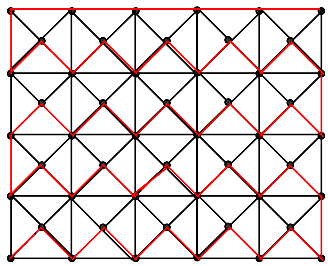

Figure 5: Enhanced Mesh $EN(5, 6)$

Case 1: $mn + (m-1)(n-1) \equiv 0 \pmod 3$

The case when $mn+(m-1)(n-1) \equiv 0 \pmod 3$, the Hamiltonian path can be partitioned into $\frac{mn+(m-1)(n-1)}{3}$ disjoint paths P_3 by removing an edge after every two edges in the Hamiltonian path. Hence the enhanced mesh $EN(m, n)$ admits a perfect packing with P_3 where

$$\lambda(G, H) = \frac{mn + (m-1)(n-1)}{3}$$

Case 2: $mn + (m-1)(n-1) \equiv 0 \pmod 4$

The case when $mn+(m-1)(n-1) \equiv 0 \pmod 4$, the Hamiltonian path can be partitioned into $\frac{mn+(m-1)(n-1)}{4}$ disjoint paths P_4 by removing an edge after every three edges in the Hamiltonian path. Hence the enhanced mesh $EN(m, n)$ admits a perfect packing with P_4 where

$$\lambda(G, H) = \frac{mn + (m-1)(n-1)}{4}$$

Case 3: $mn + (m-1)(n-1) \equiv 1, 2 \pmod 3$

The case when $mn+(m-1)(n-1) \equiv 1 \pmod 3$, the Hamiltonian path is partitioned into disjoint paths P_3 by removing an edge after every two edges of the path until $\{[\frac{mn+(m-1)(n-1)}{3}]-1\}$ copies of P_3 are obtained and the remaining four vertices account for one copy of P_4. When $mn+(m-1)(n-1) \equiv 2 \pmod 3$, the Hamiltonian path is partitioned into disjoint paths P_3 by removing an edge after every two edges of the path until $\{[\frac{mn+(m-1)(n-1)}{3}]-2\}$ copies of P_3 are obtained and the remaining eight vertices account for two copies of P_4.

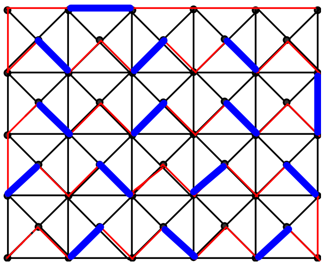

Figure 6: Enhanced Mesh $EN(5,6)$, $mn+(m-1)(n-1) \equiv 2 \pmod 3$

□

5 Conclusion

In this paper we have proved that Extended Mesh $EX(m,n)$ for $m \geq 2$, $n \geq 2$, Mesh $M(m,n)$ for $m \geq 2$, $n \geq 2$ and Enhanced Mesh $EN(m,n)$ for $m \geq 2$, $n \geq 2$ admit perfect packings. Perfect packing of various other mesh derived architectures are under study.

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