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SIGNED PRODUCT CORDIAL LABELING OF COMB RELATED ARCHITECTURES

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Abstract

A vertex labeling of a graph G, $f : V(G) \to \{-1, +1\}$ with induced edge labeling $f^* : E(G) \to \{+1, -1\}$ defined by $f^*(uv) = f(u) f(v)$ is signed product cordial labeling if $|v_f(1) - v_f(-1)| \leq 1$ and $|e_f{}^*(1) - e_f{}^*(-1)| \leq 1$, where $v_f(i)$ and $e_f{}^*(j)$ are respectively the number of vertices labeled with i and the number of edges labeled with j; $i, j \in \{+1, -1\}$. A graph G is signed product cordial if it admits signed product cordial labeling. In this paper we investigate signed product cordiality of comb, split graph of comb, square graph of comb and shadow graph of comb.
Key Words : Signed product cordial, Comb, Splitting graph, Shadow graph, Square graph.

1 Introduction

Most graph labeling methods trace their origin to one introduced by Rosa in 1967. Labeled graph are becoming an increasingly useful family of mathematical models for a broad range of application. According to Beineks and Hegde [6] graph labeling serves as a frontier between number theory and structure of graph. A detail study of variety of applications of graph labeling is given by Bloom and Golomb [1]. A dynamic survey of graph labeling is published and updated every year by Gallian [3].

A *graph labeling* is an assignment of integers to the vertices or edges, or both, subject to certain conditions.

A graph $G = (V, E)$ is called *signed cordial* if it is possible to label the edges with the number from the set $N = \{+1, -1\}$ in such a way that at each vertex v, the algebraic product of the labels of the edges incident with v is either $+1$ or -1 and the inequalities $|v_f(+1) - v_f(-1)| \leq 1$ and $|e_{f^*}(+1) - e_{f^*}(-1)| \leq 1$ are also satisfied, where $v_f(i)\,,\ i \in \{+1, -1\}$ and $e_{f^*}(j)\,,\ j \in \{+1, -1\}$ are respectively the number of vertices labeled with i and the number of edges labeled with j. A graph is called *signed-cordial* if it admits a signed-cordial labeling.

A vertex labeling of graph G , $f : V(G) \to \{-1, +1\}$ with induced edge labeling $f^* : E(G) \to \{+1, -1\}$ defined by $f^*(uv) = f(u)f(v)$ is *signed product cordial labeling* if $|v_f(1) - v_f(-1)| \leq 1$ and $|e_{f^*}(1) - e_{f^*}(-1)| \leq 1$, where $v_f(i)$ and $e_{f^*}(j)$ are respectively the number of vertices labeled with i and the number of edges labeled with j. A graph G is *signed product cordial* if it admits signed product cordial labeling.

For each vertex v of a graph G, take a new vertex v'. Join v' to all the vertices of G adjacent to v. The graph $S(G)$ thus obtained is called *splitting graph* of G.

A *caterpillar* or *caterpillar tree* is a tree in which all the vertices are within distance 1 of a central path.

A *Comb* is a caterpillar in which each vertex in the path is joined to exactly one pendant vertex.

The *Shadow graph* $D_2(G)$ of a connected graph G is obtained by taking two copies of G say G' and G'', then join each vertices u' in G' to the neighbours of the corresponding vertices u'' in G''.

For a simple connected graph G the *square of graph G* is denoted by G^2 and defined as the graph with the same vertex set as of G and two vertices are adjacent in G^2 if they are at a distance 1 or 2 apart in G.

2 Literature Survey

The concept of cordial graph was introduced by Cahit[2] . Harary introduced S-Cordiality with the first letter of Signed Cordiality.

Devaraj et al.[4] proved that the Petersen graph, complete graph, book graph, Jahangir graph and flower graph are signed cordial. The concept of signed product cordial labeling was introduced by Baskar Babujee[5]. P.Lawrence et al.[11] proved that the arbitrary super subdivision of some graphs is signed product cordial. Santhi et al.[7],[8],[9] proved that flower graph, Binary tree, k-square graphs, cycle related graphs, some star and bistar related graphs are signed product cordial. They have also proved that every signed product cordial labeling is a total signed product cordial labeling. P.P.Ulaganathan et al. [12] proved that duplicate graphs of Bistar, Double Star and Triangular Ladder graphs are signed product cordial. P.Lawarence et al.[10] have investigated Face and Total face signed product cordial labeling of planar graphs.

3 Main Results

Theorem 1. *The comb graph admits signed product cordial labeling*

Proof. Let G be a comb. Let $\{v_i/1 \leq i \leq n\}$ and $\left\{v_i^{'}/1 \leq i \leq n\right\}$ be the set of vertices of comb in which $\left\{v_i^{'}/1 \leq i \leq n\right\}$ are the pendent vertices. Define $f : V(G) \rightarrow \{+1, -1\}$ as follows:

$$f(v_i) = +1; 1 \leq i \leq n$$
$$f(v_{i'}) = -1; 1 \leq i \leq n$$

Clearly $v_f(+1) = v_f(-1) = n$. Therefore $|v_f(+1) - v_f(-1)| = 0 \leq 1$. The induced edge labels are given as follows:

$$f(v_i v_{i+1}) = +1 \text{ and } f\left(v_i v_i^{'}\right) = -1.$$

Thus $e_{f^*}(+1) = n - 1$ and $e_{f^*}(-1) = n$.
Clearly $|e_{f^*}(+1) - e_{f^*}(-1)| = 1 \leq 1$.
Hence the comb is signed product cordial.

□

Theorem 2. *The splitting graph of comb admits signed product cordial labeling.*

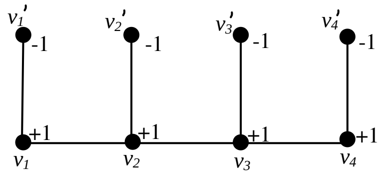

Figure 1: Comb Graph

Proof. Let G be the split graph of comb. Let $\{v_i/1 \leq i \leq n\}$ and $\left\{v_i^{'}/1 \leq i \leq n\right\}$ be the set of vertices of a comb in which $\left\{v_i^{'}/1 \leq i \leq n\right\}$ are the pendent vertices. Let $\{u_i/1 \leq i \leq n\}$ and $\left\{u_i^{'}/1 \leq i \leq n\right\}$ are the newly added vertices. Define $f : V(G) \rightarrow \{+1, -1\}$ as follows:

$$f(v_i) = f\left(u_i^{'}\right) = +1; 1 \leq i \leq n$$
$$f\left(v_i^{'}\right) = f(u_i) = -1; 1 \leq i \leq n$$

Clearly $v_f(+1) = v_f(-1) = 2n$. Therefore $|v_f(+1) - v_f(-1)| = 0 \leq 1$

The induced edge labels are given as follows:

$$f(v_i v_{i+1}) = f\left(v_i^{'} u_i\right) = f\left(v_i u_i^{'}\right) = +1$$
$$f\left(v_i v_i^{'}\right) = f(v_i u_{i+1}) = f(u_i v_{i+1}) = -1$$

Thus $e_{f^*}(+1) = 3n - 1$ and $e_{f^*}(-1) = 3n - 2$.
Clearly $|e_{f^*}(+1) - e_{f^*}(-1)| = 1 \leq 1$.
Hence the split graph of comb is signed product cordial. □

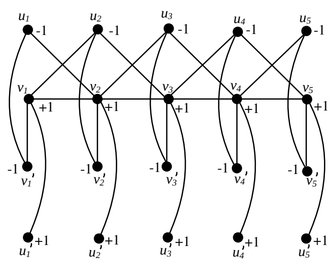

Figure 2: Split graph of comb

Theorem 3. *The shadow graph of comb admits signed product cordial labeling.*

Proof. Consider two copies of comb G_1 and G_2. Let $\{v_i/1 \le i \le n\}$ and $\left\{v_i^{'}/1 \le i \le n\right\}$ be the set of vertices of G_1 Let $\{u_i/1 \le i \le n\}$ and $\left\{u_i^{'}/1 \le i \le n\right\}$ are the set of vertices of G_2 and let G be the shadow graph of comb. Define $f : V(G) \to \{+1, -1\}$ as follows:

$$f(v_i) = f\left(u_i^{'}\right) = +1, 1 \le i \le n;$$
$$f\left(v_i^{'}\right) = f(u_i) = -1, 1 \le i \le n$$

Clearly $v_f(+1) = v_f(-1) = 2n$. Therefore $|v_f(+1) - v_f(-1)| = 0 \le 1$.
The induced edge labels are given as follows:

$$f(v_i v_{i+1}) = f(u_i u_{i+1}) = f\left(v_i^{'} u_i\right) = f\left(v_i u_i^{'}\right) = +1.$$
$$f\left(v_i v_i^{'}\right) = f\left(v_i v_i^{'}\right) = f(v_i u_{i+1}) = f(u_i v_{i+1}) = -1.$$

Thus $e_{f^*}(+1) = 4n - 2$ and $e_{f^*}(-1) = 4n - 2$.
Clearly $|e_{f^*}(+1) - e_{f^*}(-1)| = 0 \le 1$.
Hence the shadow graph of comb is signed product cordial. □

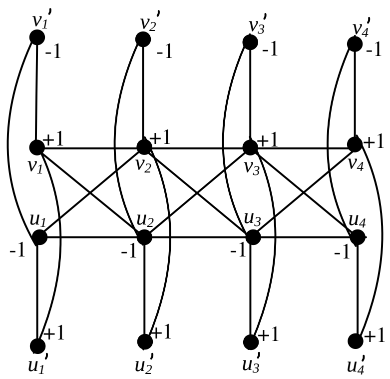

Figure 3: Shadow graph of comb

Theorem 4. *The square graph of comb admits signed product cordial labeling.*

Proof. Let $\{v_i/1 \leq i \leq n\}$ and $\left\{v_i^{'}/1 \leq i \leq n\right\}$ be the set of vertices of G. Let G be a square graph of comb. Define $f : V(G) \rightarrow \{+1, -1\}$ as follows:
Case(i) $n \equiv 0, 2(mod4)$

$$f(v_i) = \begin{cases} +1 & i \ is \ odd \\ -1 & i \ is \ even \end{cases}$$

$$f\left(v_i^{'}\right) = \begin{cases} +1 & 1 \leq i \leq \frac{n}{2} \\ -1 & \frac{n}{2} + 1 \leq i \leq n \end{cases}$$

Case(ii) $n \equiv 3(mod4)$

$$f(v_i) = \begin{cases} -1 & i \ is \ odd \\ +1 & i \ is \ even \end{cases}$$

$$f\left(v_i^{'}\right) = \begin{cases} +1 & 1 \leq i \leq \frac{n-1}{2} \\ -1 & \frac{n+1}{2} \leq i \leq \ n-1 \end{cases}$$

and $f\left(v_n^{'}\right) = +1$.

Case(iii) $n \equiv 1(mod4)$

$$f(v_i) = \begin{cases} -1 & i \ is \ odd \\ +1 & i \ is \ even \end{cases}$$

$$f\left(v_i^{'}\right) = \begin{cases} +1 & 1 \leq i \leq \frac{n+1}{2} \\ -1 & \frac{n+3}{2} \leq i \leq \ n \end{cases}$$

Clearly $v_f(+1) = v_f(-1) = 2n$. Therefore $|v_f(+1) - v_f(-1)| = 0 \leq 1$.
The following table shows that the graph satisfies the condition $|e_{f^*}(+1) - e_{f^*}(-1)| \leq 1$.

S.No	n	Edge labeling
1	$n \equiv 0(mod4)$	$e_{f^*}(+1) = e_{f^*}(-1) + 1$
2	$n \equiv 2(mod4)$	$e_{f^*}(-1) = e_{f^*}(+1) + 1$
3	$n \equiv 1, 3(mod4)$	$e_{f^*}(-1) = e_{f^*}(+1)$

□

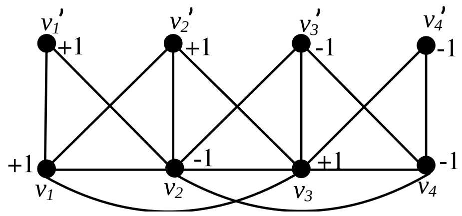

Figure 4: Square graph of comb

4 Remarks

Santhi et al. [7] proved that the every signed product cordial labeling is a Total signed product cordial labeling. Thus comb, split graph of comb, square graph of comb, shadow graph of comb are all total signed product cordial.

5 Conclusion

In this paper we proved that some comb related graph are signed product cordial. Further the signed and signed product cordial of mesh derived architectures are under study.

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