ON $W^{\mu_1\mu_2} - R_0$ SPACES IN SUPRA BITOPOLOGICAL ORDERED SPACES

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November 4, 2018

Abstract

In this chapter we define some new separation axioms of type $W^{\mu_1\mu_2} - R_0$. We also discuss the inter-relationships among these separation properties along with several counter examples.

Key Words: weakly $-\mu_1\mu_2 - \alpha - R_0$, weakly $-\mu_1\mu_2 - \beta - R_0$, weakly $-\nu_1\nu_2 - semi - R_0$, weakly $-\nu_1\nu_2 - \beta - R_0$.

1 INTRODUCTION


1-PREREQISITES

DEFINITION1.1 [1, 2]. A sub family $\mu$ of Z is said to be supra topology on Z if
1. Z, Φ ∈ µ

2. If Bi ∈ µ for all i, j, then ∪Bi ∈ µ. (Z, µ) is called a supra topological space. The elements of are called supra open sets in (Z, µ) and complement of supra open set I called supra closed sets and it is denoted by µc.

**DEFINITION 1.2.**[3]

1. The supra closure of a set E is denoted by clµ(E) and defined as
   clµ(E) = ∪F:F is supra closed and E ⊆ F

2. The supra interior of a set E is denoted by intµ(E) and defined as
   intµ(E) = G:G is supra open and E ⊇ G

**DEFINITION 1.3.** A supra bitopological ordered space [6] is (Z, µa, µb, ≤), where µa and µb are supra topologies on Z and ≤ is a partial order on Z.

Let (Z, µa, µb, ≤) be a supra bitopological ordered space and E be a subset of Z. Each element of µi are called supra µi-open sets in (Z, µa, µb, ≤). The complements are called supra µi-closed sets.

Let (Z, µa, µb, ≤), be a supra bitopological ordered space [6]. For any z ∈ Z, [z, →] = {y ∈ Z/z ≤ y} and [←, z] = {y ∈ Z/y ≤ z} [7]. A subset E of a supra bitopological ordered space (Z, µa, µb, ≤), is said to be increasing [7] if E = iµ1µ2(E) and decreasing [7] if E = dµ1µ2(E), where iµ1µ2(E) = ∪{E ∈ E} and dµ1µ2(E), = ∪{E ∈ E}. A subset of a supra bitopological ordered space (Z, µa, µb, ≤) is said to be balanced [7] if it is both increasing and decreasing.

**DEFINITIONS 1.4.**

**Definition 2.1.** Subset E of a supra bitopological space (Z, µa, µb, ≤), is named as a

1. If E ⊆ µa cl(µb int(E)) it is defined as µaµb semi open where it is stated as µaµb semi closed if µa int(µb cl(E)) ⊆ E.

2. µaµb - pre open if E ⊆ µb int(µa cl(A)) and µaµb pre closed if µb cl(µa int(E)) ⊆ E.

3. µaµb − α−open if E ⊆ µa int(µb cl(a int(E))).
4. \( \mu_a \mu_b \) semi preopen if \( E \subseteq \mu_a \text{cl} (\mu_b \text{int} (\mu_a \text{cl} (E))) \).

We have used the following supra bitopological spaces in this paper.

**EXAMPLES 1.5.**

1. \( \mu_a = \{ \phi, X, \{ p \}, \{ q, r \} \} \;
   \mu_b = \{ \phi, X, \{ p, q \}, \{ p, r \} \}
2. \( \mu_a = \{ \phi, X, \{ p \}, \{ q, r \} \} \;
   \mu_b = \{ \phi, X, \{ q, r \}, \{ p, r \} \}
3. \( \mu_a = \{ \phi, X, \{ p \}, \{ q, r \} \} \;
   \mu_b = \{ \phi, X, \{ p, q \}, \{ q, r \} \}
4. \( \mu_a = \{ \phi, X, \{ p, q \}, \{ p, r \} \} \;
   \mu_b = \{ \phi, X, \{ q, r \}, \{ p, r \} \}
5. \( \mu_a = \{ \phi, X, \{ p, q \}, \{ q, r \} \} \;
   \mu_b = \{ \phi, X, \{ p, q \}, \{ q, r \} \}
6. \( \mu_a = \{ \phi, X, \{ q \}, \{ p, q \}, \{ q, r \} \} \;
   \mu_b = \{ \phi, X, \{ p, q \}, \{ q, r \} \}
7. \( \mu_a = \{ \phi, X, \{ p \}, \{ q, r \} \} \;
   \mu_b = \{ \phi, X, \{ p \}, \{ q, p \}, \{ q, r \} \}
8. \( \mu_a = \{ \phi, X, \{ p \}, \{ q \}, \{ p, q \}, \{ q, r \} \} \;
   \mu_b = \{ \phi, X, \{ p \}, \{ q \} \}
9. \( \mu_a = \{ \phi, X, \{ p \} \} \;
   \mu_b = \{ \phi, X, \{ p \}, \{ q \}, \{ q, r \} \}
10. \( \mu_a = \{ \phi, X, \{ p \}, \{ q \}, \{ p, r \} \} \;
    \mu_b = \{ \phi, X, \{ p \}, \{ q \}, \{ p, r \} \}
11. \( \mu_a = \{ \phi, X, \{ p \}, \{ q \}, \{ p, r \} \} \;
    \mu_b = \{ \phi, X, \{ p \}, \{ q \} \}
12. \( \mu_a = \{ \phi, X, \{ p \}, \{ q, r \} \} \;
    \mu_b = \{ \phi, X, \{ p \}, \{ q \}, \{ p, r \} \}
13. \( \mu_a = \{ \phi, X, \{ p \} \} \;
    \mu_b = \{ \phi, X, \{ p \}, \{ q \} \}

**INCREASING, DECREASING, AND BALANCED TYPE SETS**

We obtain increasing, decreasing, and balanced type sets from the below partial orders

**1.6.** Let \( Z = \{ p, q, r \} \), \( 1 = \{ (p, p), (q, q), (r, r), (p, q), (q, r), (p, r) \} \).
Increasing sets are \( \{ \phi, X, \{ r \}, \{ q, r \} \} \).
Decreasing sets are \( \{ \phi, X, \{ p \}, \{ p, q \} \} \) and Balanced sets are \( \{ \phi, X \} \)
Like that we can calculate increasing, decreasing and balanced sets for remaining partial orders.

**1.7.** Let \( Z = \{ p, q, r \} \), \( 2 = \{ (p, p), (q, q), (r, r), (p, q), (q, r), (p, r) \} \).
**1.8.** Let \( Z = \{ p, q, r \} \), \( 3 = \{ (p, p), (q, q), (r, r), (p, q), (p, r) \} \).
**1.9.** Let \( Z = \{ p, q, r \} \), \( 4 = \{ (p, p), (q, q), (r, r), (p, q), (r, p) \} \).
**1.10.** Let \( Z = \{ p, q, r \} \), \( 5 = \{ (p, p), (q, q), (r, r), (p, r), (q, r) \} \).
1.11 Let $Z = \{p, q, r\}$, $6 = \{(p, p), (q, q), (r, r), (q, p), (p, r), (q, r)\}$.
1.12 Let $Z = \{p, q, r\}$, $7 = \{(p, p), (q, q), (r, r), (q, p)\}$.
1.13 Let $Z = \{p, q, r\}$, $8 = \{(p, p), (q, q), (r, r), (p, r)\}$.
1.14 Let $Z = \{p, q, r\}$, $9 = \{(p, p), (q, q), (r, r), (q, p), (r, p), (q, r)\}$.

**DEFINITIONS 1.15.** (1) $I_{\mu_1 \mu_2}O(Z)$ (resp. $D_{\mu_1 \mu_2}O(Z)$) denotes the collection of all increasing (resp. decreasing, balanced) open subsets of a supra bitopological ordered space $(Z, \mu_a, \mu_b, \leq)$.

**DEFINITIONS 1.16.** A subset $E$ of a supra topological ordered space $(Z, \mu_a, \mu_b, \leq)$ is called an increasing semi-open set (resp. a decreasing semi-open set, a balanced semi-open set). Similarly we define for pre-open, $\alpha$-open and $\beta$-open sets.

**NOTATIONS 1.17.** For a Supra bitopological ordered space $(Z, \mu_a, \mu_b, \leq)$, the following symbols for different types of collections of subsets of $X$ are used.

$I_{\mu_1 \mu_2}SO(Z)$ (resp. $D_{\mu_1 \mu_2}SO(Z)$) denotes the collection of all increasing (resp. decreasing, balanced) semi-open subsets. Similarly we denote $I_{\mu_1 \mu_2}SC(Z)$ (resp. $D_{\mu_1 \mu_2}SC(Z)$), $I_{\mu_1 \mu_2}PO(Z)$ (resp. $D_{\mu_1 \mu_2}PO(Z)$), $I_{\mu_1 \mu_2}PC(Z)$ (resp. $D_{\mu_1 \mu_2}PC(Z)$), $I_{\mu_1 \mu_2}\alpha O(Z)$ (resp. $D_{\mu_1 \mu_2}\alpha O(Z)$) for pre-open, $\alpha$-open and $\beta$-open sets. $I_{\mu_1 \mu_2}\alpha C(Z)$ (resp. $D_{\mu_1 \mu_2}\alpha C(Z)$) denotes the collection of all increasing (resp. decreasing, balanced) $\alpha$-closed subsets. Similarly we denote for pre-closed, $\alpha$-closed and $\beta$-closed sets.

**DEFINITIONS 1.18.** Any supra bitopological space is called a
1. \( R_0 \) \( \{ X \} \subset G \) whenever \( x \in G \subset \mu \\
2. Semi- \( R_0 \) \[ 8 \] if for \( x \in G \in \text{So}(x) \), \( scl(x) \subset G \\
3. Weakly- \( R_0 \) \[ 8 \] if \( \bigcap_{x \in X} \text{cl}(x) = \emptyset \).
4. Weakly- semi- \( R_0 \) \[ 8 \] if \( \bigcap_{x \in X} \text{scl}(x) = \emptyset \).
5. Weakly- \( \alpha R_0 \) \[ 8 \] if \( \bigcap_{x \in X} \text{ac}(x) = \emptyset \).
6. Weakly- \( \beta R_0 \) \[ 8 \] if \( \bigcap_{x \in X} \{ x \} = \emptyset \).
7. Weakly- \( \omega R_0 \) \[ 8 \] if \( \bigcap_{x \in X} \text{p cl}(x) = \emptyset \).

II W-R0 TYPE SPACES IN SUPRA BITOPOLOGICAL ORDERED SPACES

Definition 2.1. A space \((Z, \mu_a, \mu_b, \leq)\) is called a

1. Weakly-\( R^{1112-2}_{\alpha R_0} \) if \( \bigcap_{x \in X} \text{cl}(x) = \phi \).
2. Weakly-\( R^{1112-2}_{\omega R_0} \) if \( \bigcap_{x \in X} \text{p cl}(x) = \phi \).
3. Weakly-\( R^{1112-2}_{\omega R_0} \) if \( \bigcap_{x \in X} \text{p cl}(x) = \phi \).
4. Weakly-\( R^{1112-2}_{\omega R_0} \) if \( \bigcap_{x \in X} \text{p cl}(x) = \phi \).
5. Weakly-\( R^{1112-2}_{\omega R_0} \) if \( \bigcap_{x \in X} \text{p cl}(x) = \phi \).
6. Weakly-\( R^{1112-2}_{\omega R_0} \) if \( \bigcap_{x \in X} \text{p cl}(x) = \phi \).
7. Weakly-\( R^{1112-2}_{\omega R_0} \) if \( \bigcap_{x \in X} \text{p cl}(x) = \phi \).
8. Weakly-\( R^{1112-2}_{\omega R_0} \) if \( \bigcap_{x \in X} \text{p cl}(x) = \phi \).
9. Weakly-\( R^{1112-2}_{\omega R_0} \) if \( \bigcap_{x \in X} \text{p cl}(x) = \phi \).
10. Weakly-\( R^{1112-2}_{\omega R_0} \) if \( \bigcap_{x \in X} \text{p cl}(x) = \phi \).
11. Weakly-\( R^{1112-2}_{\omega R_0} \) if \( \bigcap_{x \in X} \text{p cl}(x) = \phi \).
12. Weakly-\( R^{1112-2}_{\omega R_0} \) if \( \bigcap_{x \in X} \text{p cl}(x) = \phi \).
13. Weakly-\( R^{1112-2}_{\omega R_0} \) if \( \bigcap_{x \in X} \text{p cl}(x) = \phi \).
14. Weakly-\( R^{1112-2}_{\omega R_0} \) if \( \bigcap_{x \in X} \text{p cl}(x) = \phi \).

Theorem 2.2: Every weakly \( R^{1112-2} - R_0 \) space is a weakly \( R_0 \) space. Every weakly \( R_0 \) space is need not be a weakly \( R^{1112-2 - R_0} \) space.

Example 2.3: Let \( Z = \{ p, q, r \} \), \( \mu_a = \{ \phi, X, \{ a \}, \{ a, b \}, \{ b, c \} \} \)
\( \mu_b = \{ \phi, X, \{ a, b \}, \{ a, c \} \} \)
\[ \leq 9 = \{(p, p), (q, q), (r, r), (q, p), (r, p), (q, r)\}. \]

**Theorem 2.4:** Every weakly $i\mu_1\mu_2 - \alpha - R_0$ space is a weakly $i\mu_1\mu_2 - Pre - R_0$ space. Every weakly $i\mu_1\mu_2 - Pre - R_0$ space need not be a weakly $i\mu_1\mu_2 - \alpha - R_0$ space.

**Example 2.5:** Let $Z = \{p, q, r\}$, $\mu_a = \{\phi, X, \{p\}, \{p, q\}, \{q, r\}\}$, $\mu_b = \{\phi, X, \{p, q\}, \{p, r\}\}$

\[ \leq 3 = \{(p, p), (q, q), (r, r), (p, q), (p, r)\}. \]

**Theorem 2.6:** Every weakly $i\mu_1\mu_2 - \alpha - R_0$ space is a weakly $i\mu_1\mu_2 - \beta - R_0$ space. Every weakly $i\mu_1\mu_2 - \beta - R_0$ space need not be a weakly $i\mu_1\mu_2 - \alpha - R_0$ space.

**Example 2.7:** Let $Z = \{p, q, r\}$, $\mu_a = \{\phi, X, \{p, q\}\} \mu_b = \{\phi, X, \{p\}, \{p, q\}, \{q, r\}\}$

\[ \leq 3 = \{(p, p), (q, q), (r, r), (p, q), (p, r)\}. \]

**Theorem 2.8:** Every weakly $i\mu_1\mu_2 - semi - R_0$ space is independent of weakly $i\mu_1\mu_2 - Pre - R_0$ space.

**Proof:** It follows from the following examples

**Example 2.9:** Let $Z = \{p, q, r\}$, $\mu_a = \{\phi, X, \{p, q\}\} \mu_b = \{\phi, X, \{p\}, \{p, q\}, \{q, r\}\}$

\[ \leq 7 = \{(p, p), (q, q), (r, r), (q, p)\}. \]

**Example 2.10:** Let $Z = \{p, q, r\}$, $\mu_a = \{\phi, X, \{p\}, \{q\}, \{p, q\}, \{p, r\}\}$

\[ \mu_b = \{\phi, X, \{p, q\}\}$

\[ \leq 7 = \{(p, p), (q, q), (r, r), (q, p)\}. \]

**Theorem 2.11:** Every weakly $i\mu_1\mu_2 - R_0$ space is independent of weakly $d\mu_1\mu_2 - R_0$ space.

**Proof:** It follows from the following examples

**Example 2.12:** Let $Z = \{p, q, r\}$, $\mu_a = \{\phi, X, \{p\}, \{p, q\}, \{q, r\}\}$

\[ \mu_b = \{\phi, X, \{p, q\}, \{p, r\}\}$

\[ \leq 3 = \{(p, p), (q, q), (r, r), (p, q), (p, r)\}. \]
Example 2.13: Let $Z = \{p, q, r\}$, $\mu_a = \{\phi, X, \{p\}, \{p, q\}, \{q, r\}\}$
$\mu_b = \{\phi, X, \{p, q\}, \{p, r\}\}$
$\leq 2 = \{(p, p), (q, q), (r, r), (p, q), (r, p)\}$.

Theorem 2.14: Every weakly $i^{\mu_1\mu_2} - R_0$ space is a weakly R0 space.
Every weakly R0 space is need not be a weakly $d^{\mu_1\mu_2} - R_0$ space.

Example 2.15: Let $Z = \{p, q, r\}$, $\mu_a = \{\phi, X, \{p\}, \{p, q\}, \{q, r\}\}$
$\mu_b = \{\phi, X, \{p, q\}, \{p, r\}\}$
$\leq 3 = \{(p, p), (q, q), (r, r), (p, q), (p, r)\}$.

Theorem 2.16: Every weakly $i^{\mu_1\mu_2} - \alpha - R_0$ space is a weakly $\alpha - R_0$ space.
Every weakly $\alpha - R_0$ space is need not be a weakly $i^{\mu_1\mu_2} - \alpha - R_0$ space.

Example 2.17: Let $Z = \{p, q, r\}$, $\mu_a = \{\phi, X, \{p\}, \{q\}, \{p, q\}, \{p, r\}\}$
$\mu_b = \{\phi, X, \{p, q\}\}$
$\leq 4 = \{(p, p), (q, q), (r, r), (p, q), (r, p), (q, r)\}$.

Theorem 2.18: Every weakly $b^{\mu_1\mu_2} - R_0$ space is a weakly $i^{\mu_1\mu_2} - R_0$ space.
Every weakly $i^{\mu_1\mu_2} - R_0$ space is need not be a weakly $b^{\mu_1\mu_2} - R_0$ space.

Example 2.19: Let $Z = \{p, q, r\}$, $\mu_a = \{\phi, X, \{p\}, \{q\}, \{p, q\}, \{p, r\}\}$
$\mu_b = \{\phi, X, \{p, q\}\}$
$\leq 3 = \{(p, p), (q, q), (r, r), (p, q), (p, r)\}$.

Theorem 2.20: Every weakly $b^{\mu_1\mu_2} - R_0$ space is a weakly $d^{\mu_1\mu_2} - R_0$ space.
Every weakly $d^{\mu_1\mu_2} - R_0$ space is need not be a weakly $b^{\mu_1\mu_2} - R_0$ space.

Example 2.21: Let $Z = \{p, q, r\}$, $\mu_a = \{\phi, X, \{p\}, \{p, q\}, \{q, r\}\}$
$\mu_b = \{\phi, X, \{p, q\}, \{p, r\}\}$
$\leq 2 = \{(p, p), (q, q), (r, r), (p, q), (r, p)\}$.

Theorem 2.22: Every weakly $b^{\mu_1\mu_2} - \alpha - R_0$ space is a weakly
Every weakly semi−$R_0$ space need not be a weakly $b\mu_1\mu_2−\alpha−R_0$ space.

**Example 2.23:** Let $Z = \{p, q, r\}$, $\mu_a = \{\phi, X, \{p\}, \{q\}, \{p, q\}, \{p, r\}\}$

$\mu_b = \{\phi, X, \{p, q\}\}$

$\leq 1 = \{(p, p), (q, q), (r, r), (p, q), (q, r), (p, r)\}$.

**Theorem 2.24:** Every weakly $b\mu_1\mu_2−\alpha−R_0$ space is a weakly pre−$R_0$ space.

Every weakly pre−$R_0$ space need not be a weakly $b\mu_1\mu_2−\alpha−R_0$ space.

**Example 2.25:** Let $Z = \{p, q, r\}$, $\mu_a = \{\phi, X, \{p\}, \{q\}, \{p, q\}\}$

$\mu_b = \{\phi, X, \{p, q\}\}$

$\leq 1 = \{(p, p), (q, q), (r, r), (p, q), (q, r), (p, r)\}$.

**Theorem 2.26:** Every weakly $b\mu_1\mu_2−\alpha−R_0$ space is a weakly $\beta−R_0$ space.

Every weakly $\beta−R_0$ space need not be a weakly $b\mu_1\mu_2−\alpha−R_0$ space.

**Example 2.27:** Let $Z = \{p, q, r\}$, $\mu_a = \{\phi, X, \{p, q\}\}$

$\mu_b = \{\phi, X, \{q, r\}\}$

$\leq 1 = \{(p, p), (q, q), (r, r), (p, q), (q, r), (p, r)\}$.

**Theorem 2.28:** Every weakly $b\mu_1\mu_2−\beta−R_0$ space is a weakly $\beta−R_0$ space.

The converse of the above theorem is not true as it can be seen by the following example.

**Example 2.29:** Let $Z = \{p, q, r\}$, $\mu_a = \{\phi, X, \{p\}, \{q\}, \{p, q\}\}$

$\mu_b = \{\phi, X, \{p, q\}\}$

$\leq 1 = \{(p, p), (q, q), (r, r), (p, q), (q, r), (p, r)\}$.

**Theorem 2.30:** Every weakly $b\mu_1\mu_2−semi−R_0$ space is a weakly semi−$R_0$ space.

Every weakly semi−$R_0$ space need not be a weakly $b\mu_1\mu_2−semi−R_0$ space.

**Example 2.31:** Let $Z = \{p, q, r\}$, $\mu_a = \{\phi, X, \{p\}, \{q\}\}$

$\mu_b = \{\phi, X, \{p, q\}\}$
\[ \leq 1 = \{(p,p), (q,q), (r,r), (p,q), (q,r), (p,r)\}. \]

**Theorem 2.32:** Every weakly $d^{\mu_{1}^{2}} - semi - R_{0}$ space is a weakly

semi $- R_{0}$ space.

Every weakly semi $- R_{0}$ space is need not be a weakly $d^{\mu_{1}^{2}} - semi - R_{0}$ space.

**Example 2.33:** Let $Z = \{p, q, r\}$, $\mu_{a} = \{\phi, X, \{p\}, \{p, q\}, \{q, r\}\}$

$\mu_{b} = \{\phi, X, \{p, q\}, \{q, r\}\}$

\[ \leq 1 = \{(p,p), (q,q), (r,r), (q,p)\}. \]

**Theorem 2.34:** Every weakly $i^{\mu_{1}^{2}} - pre - R_{0}$ space is a weakly

pre $- R_{0}$ space.

Every weakly pre $- R_{0}$ space is need not be a weakly $i^{\mu_{1}^{2}} - pre - R_{0}$ space.

**Example 2.35:** Let $Z = \{p, q, r\}$, $\mu_{a} = \{\phi, X, \{p, q\}, \{p, r\}\}$

$\mu_{b} = \{\phi, X, \{p, q\}, \{p, r\}\}$

\[ \leq 1 = \{(p,p), (q,q), (r,r), (p,q), (q,r), (p,r)\}. \]

**Theorem 2.36:** Every weakly $b^{\mu_{1}^{2}} - \alpha - R_{0}$ space is a weakly

$\alpha - R_{0}$ space.

Every weakly $b^{\mu_{1}^{2}} - \alpha - R_{0}$ space need not be a weakly $b^{\mu_{1}^{2}} - \alpha - R_{0}$ space.

**Example 2.37:** Let $Z = \{p, q, r\}$, $\mu_{a} = \{\phi, X, \{p\}, \{q\}, \{p, q\}, \{q, r\}\}$

$\mu_{b} = \{\phi, X, \{p, q\}\}$

\[ \leq 8 = \{(p,p), (q,q), (r,r), (p,r)\}. \]

**Theorem 2.38:** Every weakly $i^{\mu_{1}^{2}} - \alpha - R_{0}$ space is independent of weakly $d^{\mu_{1}^{2}} - \alpha - R_{0}$ space.

**Proof:** It follows from the following examples.

**Example 2.39:** Let $Z = \{p, q, r\}$, $\mu_{a} = \{\phi, X, \{p\}, \{q, r\}\}$

$\mu_{b} = \{\phi, X, \{p, q\}, \{q\}, \{p, q\}\}$

\[ \leq 8 = \{(p,p), (q,q), (r,r), (p,r)\}. \]

**Example 2.40:** Let $Z = \{p, q, r\}$, $\mu_{a} = \{\phi, X, \{p\}, \{q\}, \{p, q\}, \{q, r\}\}$

$\mu_{b} = \{\phi, X, \{p, q\}\}$

\[ \leq 7 = \{(p,p), (q,q), (r,r), (q,p)\}. \]
Theorem 2.41: Every weakly $\mu_1 \mu_2 - \text{pre} - R_0$ space is a weakly $\mu_1 \mu_2 - \text{pre} - R_0$ space. Every weakly $\mu_1 \mu_2 - \text{pre} - R_0$ space need not be a weakly $\mu_1 \mu_2 - \text{pre} - R_0$ space. Example 2.42: Let $Z = \{p, q, r\}, \mu_a = \{\phi, X, \{p, q\}, \{q, r\}\} \mu_b = \{\phi, X, \{p, q\}, \{p, q\}\}$ 
$\leq 7 = \{(p, p), (q, q), (r, r), (q, p)\}$.

Theorem 2.43: Every weakly $\beta \mu_1 \mu_2 - \text{pre} - R_0$ space is a weakly $\mu_1 \mu_2 - \text{pre} - R_0$ space. Every weakly $\mu_1 \mu_2 - \text{pre} - R_0$ space need not be a weakly $\mu_1 \mu_2 - \text{pre} - R_0$ space.

Example 2.44: Let $Z = \{p, q, r\}, \mu_a = \{\phi, X, \{p, q\}, \{q, r\}\} \mu_b = \{\phi, X, \{p, q\}, \{p, r\}\}$ 
$\leq 3 = \{(p, p), (q, q), (r, r), (p, q)\}$.

Theorem 2.47: Every weakly $\mu_1 \mu_2 - \text{semi} - R_0$ space is a weakly $\mu_1 \mu_2 - \text{semi} - R_0$ space. Every weakly $\mu_1 \mu_2 - \text{semi} - R_0$ space need not be a weakly $\mu_1 \mu_2 - \text{semi} - R_0$ space. Example 2.46: Let $Z = \{p, q, r\}, \mu_a = \{\phi, X, \{p, q\}, \{q, r\}\} \mu_b = \{\phi, X, \{p, q\}, \{p, r\}\}$ 
$\leq 7 = \{(p, p), (q, q), (r, r), (q, p)\}$.

Theorem 2.49: Every weakly $\mu_1 \mu_2 - \beta - R_0$ space is a weakly $\mu_1 \mu_2 - \beta - R_0$ space. Every weakly $\mu_1 \mu_2 - \beta - R_0$ space need not be a weakly $\mu_1 \mu_2 - \beta - R_0$ space.

Example 2.50: Let $Z = \{p, q, r\}, \mu_a = \{\phi, X, \{p, q\}, \{p, r\}\} \mu_b = \{\phi, X, \{q, r\}, \{p, r\}\}$ 
$\leq 7 = \{(p, p), (q, q), (r, r), (q, p)\}$.
Theorem 2.51: Every weakly $b^{\mu_1\mu_2} - \beta - R_0$ space is a weakly $d^{\mu_1\mu_2} - \beta - R_0$ space. Every weakly $d^{\mu_1\mu_2} - \beta - R_0$ space need not be a weakly $b^{\mu_1\mu_2} - \beta - R_0$ space.

Example 2.52: Let $Z = \{p, q, r\}$, $\mu_a = \{\phi, X, \{p, q\}, \{q, r\}\}$ $\mu_b = \{\phi, X, \{q, r\}, \{p, r\}\}$ $\leq 7 = \{(p, p), (q, q), (r, r), (q, p)\}$.

Theorem 2.53: Every weakly $i^{\mu_1\mu_2} - \beta - R_0$ space is independent of weakly $d^{\mu_1\mu_2} - \beta - R_0$ space.

Proof: It follows from the following examples

Example 2.54: Let $Z = \{p, q, r\}$, $\mu_a = \{\phi, X, \{p\}, \{p, q\}\}$ $\mu_b = \{\phi, X, \{q, p\}, \{p, r\}\}$ $\leq 7 = \{(p, p), (q, q), (r, r), (q, p)\}$.

Theorem 2.55: Let $Z = \{p, q, r\}$, $\mu_a = \{\phi, X, \{p\}, \{p, q\}\}$ $\mu_b = \{\phi, X, \{q, r\}, \{p, r\}\}$ $\leq 7 = \{(p, p), (q, q), (r, r), (q, p)\}$.

Theorem 2.56: Every weakly $i^{\mu_1\mu_2} - pre - R_0$ space is independent of weakly $d^{\mu_1\mu_2} - pre - R_0$ space.

Proof: It follows from the following examples

Example 2.57: Let $Z = \{p, q, r\}$, $\mu_a = \{\phi, X, \{p\}, \{p, q\}\}$ $\mu_b = \{\phi, X, \{q, p\}, \{p, r\}\}$ $\leq 7 = \{(p, p), (q, q), (r, r), (q, p)\}$.

Example 2.58: Let $Z = \{p, q, r\}$, $\mu_a = \{\phi, X, \{q\}, \{p, q\}\}$ $\mu_b = \{\phi, X, \{p, r\}, \{q, r\}\}$ $\leq 3 = \{(p, p), (q, q), (r, r), (q, p), (p, r)\}$.

Theorem 2.59: Every weakly $i^{\mu_1\mu_2} - semi - R_0$ space is independent of weakly $d^{\mu_1\mu_2} - semi - R_0$ space.

Proof: It follows from the following examples
Example 2.60: Let $Z = \{p, q, r\}$, $\mu_a = \{\phi, X, \{p, q\}, \{q, r\}\} \mu_b = \{\phi, X, \{p, q\}, \{p, r\}, \{q, r\}\}$ 
$\leq 5 = \{(p, p), (q, q), (r, r), (q, p)\}$.

Example 2.61: Let $Z = \{p, q, r\}$, $\mu_a = \{\phi, X, \{p, q\}, \{p, r\}, \{q, r\}\} \mu_b = \{\phi, X, \{p, q\}, \{p, r\}\}$ 
$\leq 5 = \{(p, p), (q, q), (r, r), (q, p)\}$.

Theorem 2.62: Every weakly $d^{\mu_1\mu_2} - \text{pre} - R_0$ space is a weakly $\text{pre} - R_0$ space. Every weakly $\text{pre} - R_0$ space need not be a weakly $d^{\mu_1\mu_2} - \text{pre} - R_0$ space.

Example 2.63: Let $Z = \{p, q, r\}$, $\mu_a = \{\phi, X, \{p, q\}, \{q, r\}\} \mu_b = \{\phi, X, \{p, q\}, \{q, r\}\}$ 
$\leq 5 = \{(p, p), (q, q), (r, r), (q, p)\}$.

Theorem 2.64: Every weakly $d^{\mu_1\mu_2} - \alpha - R_0$ space is a weakly $\alpha - R_0$ space. Every weakly $\alpha - R_0$ space need not be a weakly $d^{\mu_1\mu_2} - \alpha - R_0$ space. Example 2.65: Let $Z = \{p, q, r\}$, $\mu_a = \{\phi, X, \{p, q\}, \{p, r\}\} \mu_b = \{\phi, X, \{p, q\}\}$ 
$\leq 5 = \{(p, p), (q, q), (r, r), (q, p)\}$.

Theorem 2.66: Every weakly $d^{\mu_1\mu_2} - \beta - R_0$ space is a weakly $\beta - R_0$ space. Every weakly $\beta - R_0$ space need not be a weakly $d^{\mu_1\mu_2} - \beta - R_0$ space. Example 2.67: Let $Z = \{p, q, r\}$, $\mu_a = \{\phi, X, \{p, q\}, \{p, r\}\} \mu_b = \{\phi, X, \{p, q\}\}$ 
$\leq 5 = \{(p, p), (q, q), (r, r), (q, p)\}$.

Theorem 2.68: Every weakly $b^{\mu_1\mu_2} - \alpha - R_0$ space is a weakly $b^{\mu_1\mu_2} - \alpha - R_0$ space. Every weakly $b^{\mu_1\mu_2} - \alpha - R_0$ space need not be a weakly $b^{\mu_1\mu_2} - \alpha - R_0$ space. Example 2.69: Let $Z = \{p, q, r\}$, $\mu_a = \{\phi, X, \{p, q\}, \{p, r\}, \{q, r\}\} \mu_b = \{\phi, X, \{p, q\}\}$ 
$\leq 5 = \{(p, p), (q, q), (r, r), (q, p)\}$.

Theorem 2.70: Every weakly $b^{\mu_1\mu_2} - \text{pre} - R_0$ space is a weakly $b^{\mu_1\mu_2} - \beta - R_0$ space. Every weakly $b^{\mu_1\mu_2} - \beta - R_0$ space need not be a weakly $b^{\mu_1\mu_2} - \beta - R_0$ space.
$pre-R_0$ space.

**Example 2.71:** Let $Z = \{p, q, r\}$, $\mu_a = \{\phi, X, \{p, q\}\}$ $\mu_b = \{\phi, X, \{p, q\}, \{p, q, r\}\}$

$\leq 7 = \{(p, p), (q, q), (r, r), (q, p)\}$.

**Theorem 2.72:** Every weakly $b^{\mu_1\mu_2} - pre - R_0$ space is a weakly $\beta - R_0$ space.

Every weakly $\beta - R_0$ space need not be a weakly $b^{\mu_1\mu_2} - pre - R_0$ space.

**Example 2.73:** Let $Z = \{p, q, r\}$, $\mu_a = \{\phi, X, \{p, q\}\}$ $\mu_b = \{\phi, X, \{p\}, \{q\}, \{p, q\}, \{p, r\}\}$

$\leq 1 = \{(p, p), (q, q), (r, r), (q, p), (q, r), (p, r)\}$.

**Theorem 2.74:** Every weakly $b^{\mu_1\mu_2} - \alpha - R_0$ space is a weakly $i^{\mu_1\mu_2} - pre - R_0$ space.

Every weakly $i^{\mu_1\mu_2} - pre - R_0$ space need not be a weakly $b^{\mu_1\mu_2} - \alpha - R_0$ space.

**Example 2.75:** Let $Z = \{p, q, r\}$, $\mu_a = \{\phi, X, \{p, q\}\}$ $\mu_b = \{\phi, X, \{p\}, \{q\}, \{p, q\}, \{p, r\}\}$

$\leq 1 = \{(p, p), (q, q), (r, r), (q, p), (q, r), (p, r)\}$.

**References**


