MULTI ITEM EOQ MODEL FOR FRESH FRUITS WITH MIXTURE PATTERN DETERIORATION AND MIXTURE PATTERN HOLDING COST WITH PRESERVATION TECHNOLOGY INVESTMENT AND SHORTAGES

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Abstract

Inventory pervades the business world. In a fruit shop, a fruiterer needs to maintain huge inventory of different ranges of fruit items for buyer because of uncertainty in buyer preferences and behavior. But demand of them may differ and different EOQ models for each item are required for optimal level of EOQ. This paper presents a single inventory model to assimilate these varieties. It is assumed that many fruit items bear constant rate of deterioration, some are with time proportional deterioration and some do...
not deteriorate. Refrigerator or freezer storage is necessary for meat, dairy products, eggs, and cut fruits and vegetables. Refrigerator and freezer temperatures do not destroy pathogenic or spoilage microorganisms, but freezer temperatures do stop their growth. To control deterioration of the units in inventory, the advanced preservation technology is considered. The proposed single inventory model of this paper incorporates composite and heterogeneous features of multiple items and presents analysis for obtaining optimal level for output model parameters.

**Keywords:** Mixture Pattern Deterioration, Mixture Pattern holding cost, Multi-items, preservation technology and partial backlogging.

1 INTRODUCTION

Inventory management is one of the key components of any business that can be controlled by a business manager to efficiently and successfully operate in the fiercely competitive modern global market. Therefore, it is very important to build inventory models that are sensitive and responsive to the dynamic real life market situations. The economic order quantity (EOQ), first proposed by Ford Whitman Harris (1913), is the most fundamental result which has generated whole new directions of research in inventory management since its inception.

Inventory control is the supervision of supply, storage and accessibility of items in order to ensure an adequate supply without excessive oversupply. Some authors even claim that More operations research has been directed towards inventory control than toward any other problem area in business and industry” and among these the deteriorating items inventory have gain large emphasis in last decade. The inventory system for instantaneous deteriorating items has been an object of study for a long time, but little is known about the effect of investing in reducing the rate of product deterioration. So in this paper, an inventory model is developed for instantaneous deteriorating items by considering the fact that using the preservation technology the fruiterer can
reduce the deterioration rate by which he can reduce the economic losses, improve the customer service level and increase business competitiveness. Some EOQ models in the literature of early decades do not consider the deterioration of items but later researchers realized the importance and incorporated this in models as a source (see [4], [10], [13], [25], [15], [19] and [18]).

Inventory control management uses methodology to decide how much stock a fruiterer has to store, as per demand and deterioration of various items, in order to optimize the inventory policy of multiple items. To develop an economic order quantity model with varying demand, shortage as partial backlogging, the constant pattern deterioration factor is vital and cannot be ignored [6]. In similar lines, the aspect of time dependent deterioration is equally important (see [3], [20], [12], [26], [22], [23], [24], [27], [28], [29] and [30]). Moreover, deterioration factor of items may be in multiple form of mathematical relation. It has been taken in literature with exponentially increasing demand [17] and with exponentially decreasing demand [11]. A useful policy is drafted [14] for warehouse enterprises in the form of EOQ model with time-dependent demand, inflation and money value. Weibull distribution type deterioration pattern [16] is also a useful idea implemented with time quadratic demand. The marginal increase in total holding cost with respect to inventory level may not be simply regarded as a constant. Some authors designed inventory model when he deterioration rate is time proportional, demand is function of price and holding cost a variable (see [2] and [31]). The one latest contribution [1] is on modern analytic solution to an inventory problem with service reneging of customer with shortage of items. Single price break and convexity of cost function is a useful approach in this area (see [7], [21] and [9]).

The idea of two and three components demand rate could also be considered for obtaining the optimal cost, optimal time and optimal quantities (see [5] and [8]). Inventory management needs to preserve two types of goods. Firstly, seasonal goods (items) which deteriorate only over time, and secondly those items such as soaps, pharmaceuticals, foods whose deterioration depend upon other parameters like temperature, place, humidity etc. One can take deterioration factor which is depending on two or more
parameters. From contributions [11] and [17], motivation is derived to build up a general demand function to assimilate more variations of demand, deterioration for multi items in inventory model with variations in holding cost.

In last two decades a lot of work has been published for controlling the inventory of deteriorating items. The consideration of PT is important due to rapid social changes, and the fact that PT can reduce the deterioration rate significantly. By the efforts of investing in preservation technology, we can reduce the deterioration rate. So in this paper, we made the model of Mishra and Singh [9] more realistic by considering the fact that use of preservation technology can reduce the deterioration rate significantly, which help the retailers to reduce their economic losses.

By valor of amalgamation of multiple variations in input source, one can get the inventory model nearer to the reality. It is hard to have all in a single setup because of rising complexity. But, beginning may be with the few variations which is the main theme of this paper.

2 ASSUMPTIONS AND NOTATIONS

2.1 Assumptions

The mathematical model in this paper was developed based on the following assumptions:

1. Assume that there are six items named A, B, C, D, E and F hold by a supplier to sell at different selling prices having different costs.

2. These products deteriorate at different rates and have the different holding costs. But the Deterioration rate is controlled by using preservation technology during the deterioration period.
3. The backlogging rate is a variable and is dependent on the length of the waiting time for next replenishment, so that the backlogging rate for negative inventory is \( B(t) = \frac{1}{1 + \frac{\delta(T-t)}{1}} \), where \( \delta \) is the backlogging parameter such that \( 0 \leq \delta \leq 1 \) and the factor \((T-t)\) is waiting time \( t_1 \leq t \leq T \). And the remaining fraction \( 1-B(t) \) is lost sale.

4. The demand rate is \( D(t) = \mu a^t \), where \( a \neq 1 \) is a non-zero real number, \( 'c' \) \( (c \neq 0) \) a constant governing trend of demand, and \( 't' \) is time varying over 0 to \( T \).

5. \( T \) is the prefixed time length of a cycle.

6. Due to refrigeration, there will not be any deterioration of fresh fruits for a certain period (Say \( \alpha \), \( \alpha \) is very close to 1) and after that for a long period there will be deterioration of items which we considered as linearly increasing.

7. There is no replacement or repair of deteriorated items in a given cycle but to control the deterioration rate, preservation technology is used.

8. Different suggested rates of demand of six items are:

   **For item A**: Demand is exponentially increasing i.e. \( D(t) = \mu e^t \) [where \( a = e \) and \( c = 1 \)]

   **For item B**: Demand is exponentially decreasing i.e. \( D(t) = \mu e^{-t} \) [where \( a = e \) and \( c = -1 \)]

   **For item C**: Demand is \( D(t) = \mu t^2 \) [where \( a = 2 \) and \( c = -1 \)]

   **For item D**: Demand is \( D(t) = \mu t^3 \) [where \( a = 3 \) and \( c = 1 \)]

   **For item E**: Demand is \( D(t) = \mu \cosh(ct) = \frac{e^{ct} + e^{-ct}}{2} \) [Taking \( a = e \), \( c = 1 \) and \( a = e \), \( c = -1 \) and averaging them]

   **For item F**: Demand is \( D(t) = \mu \sinh(ct) = \frac{e^{ct} - e^{-ct}}{2} \) [Taking \( a = e \), \( c = 1 \) and \( a = -e \), \( c = -1 \) and averaging them]

2.2 Notations

The mathematical model in this paper was developed based on the following notations:

1. \( I_1(t) \) - The inventory level at time \( 't' \), \( 0 \leq t \leq t_1 \).
2. $I_2(t)$ - The inventory level at time 't', $t_1 \leq t \leq T$.

3. $t_1$ - The time at which the inventory level reaches zero, $t_1 \leq 0$.

4. $t_2$ - The length of the period during which shortages are allowed, $t_2 \leq 0$.

5. Holding cost is $H(t) = \left[\beta D + (1 - \beta)(E + Ft)\right]$, D, E and Fare parameters. Here $0 \leq \beta \leq 1$.

6. Initial demand is $\mu \geq 0$ at $t = 0$.

7. Initial stock is $S$ at time $t = 0$.

8. Lead time is zero.

9. $u$ - Preservation technology cost for reducing deterioration rate in order to preserve the product, $u \geq 0$.

10. $m(u)$ - Reduced deterioration rate due to the use of preservation technology.

11. $\lambda$ - Resultant deterioration rate, $\lambda = 1 - m(u)$.

12. $p$ - Unit purchase cost

13. $A$ - Ordering cost per order

14. $C_1$ - The deterioration cost per unit per cycle

15. $\Pi C_1, 0 \leq \pi < 1$ - Salvage value associated with deteriorated units during a cycle time

16. $C_2$ - The shortage cost for backlogged items per unit per cycle.

17. $C_3$ - The unit cost of lost sales per unit

18. $f t(\alpha \phi + (1 + \alpha)(\theta + \psi t))$ is the mixture pattern deterioration of items depending upon time $t$, here $0 \leq \alpha \leq 1$ and $\phi, \theta, \psi$ are parameters.

19. TAC - Total average cost per unit time.
3 MODEL FORMULATION

The differential equations governing the inventory level at any
time t during the cycle (0,T) are given as follows:

\[
\frac{dI_1(t)}{dt} + [\alpha \phi + (1 - \alpha)(\theta + \psi t)] \lambda I_1(t) = - \mu a^c t, \quad 0 \leq t \leq t_1 \quad (1)
\]

\[
\frac{dI_1(t)}{dt} = - \mu \alpha c t + \delta (T - t) \quad t_1 \leq t \leq T \quad (2)
\]

with boundary condition \( I_1(t_1) = 0, I_2(t_1) = 0 \)

The solution of equation (1) is

\[
I_1(t) = \mu k [a^c t (1 - \lambda \rho t_1 - \lambda k \rho) - a^c (1 + \lambda \rho t - \lambda k \rho)]
\]

Where, \( \rho = [\alpha \phi + (1 - \alpha)\theta] \) and \( k = \frac{1}{e^{\lambda a^c T}} \)

Using \( I_1(0) = s \), we get \( s = \mu k [a^c t_1 (1 + \alpha \phi + (1 - \alpha)\theta) - 1 + \lambda k \rho] \)

The solution of equation (2) is

\[
I_2(t) = \mu k [a^c t (1 - \delta(T - t_1 + K)) - a^c (1 - \delta(T - t + K))]
\]

The maximum backordered inventory \( BI \) is obtained at \( t = T \)

\[
BI = -I_2(T) = - \mu k [a^c t (1 - \delta(T - t_2 + K)) - a^c (1 - \delta K)]
\]

Thus the order size during entire time interval \([0, T]\) is

\[
Q = S + BI = \mu k [a^c t (1 + \lambda \rho t_1 - \lambda k \rho) - 1 + \lambda k \rho] - a^c (1 - \delta(T - t_1 + K)) - a^c (1 - \delta K)
\]

Total average cost per cycle (TAC) = (Set-up cost + inventory holding cost + Purchase cost + deterioration cost + shortage cost + lost sales cost Salvage value) / T.

(i) The Set-up cost during the period \([0, T]\) is

\[
\text{Set up cost} = \frac{A}{T}
\]

(ii) The holding cost during the period \([0, T]\) is

\[
\text{Inventory holding cost} =
\]
Total shortage cost during the period \([0, T]\) is given by

\[ \text{Shortage cost} = \int_0^T [\beta B + (1 - \beta)(E + F) - \frac{\lambda_1}{r}] \, dt \]

The Purchase cost during the period \([0, T]\) is

\[ \text{Purchase cost} = \int_0^T \left[ F(1 - \lambda p) + \frac{\lambda^2 p^2 k E h + (F p - \lambda p^2 E - F) p + \lambda p^3 k E f}{r} - \lambda^2 p^2 F \right] \, dt \]

The deteriorating cost during the period \([0, T]\) is

\[ \text{Deterioration cost} = \int_0^T \left[ \frac{\lambda_1}{r} - \lambda p F \right] \, dt \]

The Salvage value during the period \([0, T]\) is

\[ \text{Salvage value} = 8 \]
\[
\pi \frac{E^T}{2} \left[ \alpha k \left( \frac{k^2}{E^T} (1 + \lambda \tau_t - \lambda k) - 1 + \lambda k \right) - \beta D \tau_t + E \tau_t - \beta E \tau_t - \frac{E^T F}{2} + \frac{\beta E^2}{2} \right]
\]

(vii) Lost sales during the period \((0, T) = \int_{t_1}^{T} \left[ 1 - \frac{1}{\mu(t)} \right] \mu \sigma t dt
\]

\[
= \frac{det(H_1)}{t} \left[ k x'^t + x'' \right] \left( t_1 - T - \lambda k \right)
\]

\(\cdot\) Total average cost per cycle \(TAC(t_1, T, \lambda)\)

\[
\cdot TAC(t_1, T, \lambda) = \frac{\pi}{2} \left[ \frac{k^2}{E^T} \left( \frac{k^2}{E^T} (1 + \lambda \tau_t - \lambda k) - 1 + \lambda k \right) - \frac{E^T}{2} \left( \frac{E^T}{2} (1 + \lambda \tau_t - \lambda k) - 1 + \lambda k \right) - \frac{E^T}{2} \left( \frac{E^T}{2} (1 + \lambda \tau_t - \lambda k) - 1 + \lambda k \right) \right]
\]

\[
= \frac{det(H_1)}{t} \left[ k x'^t + x'' \right] \left( t_1 - T - \lambda k \right)
\]

\[= \frac{\pi}{2} \left[ - \frac{k^2}{E^T} \left( \frac{k^2}{E^T} (1 + \lambda \tau_t - \lambda k) - 1 + \lambda k \right) - \frac{E^T}{2} \left( \frac{E^T}{2} (1 + \lambda \tau_t - \lambda k) - 1 + \lambda k \right) \right]
\]

4 SOLUTION PROCEDURE

The necessary and sufficient conditions for minimizing the total cost \(TAC(t_1, T, \lambda)\) are

\[
\frac{\partial TAC(t_1, T, \lambda)}{\partial t_1} = 0, \frac{\partial TAC(t_1, T, \lambda)}{\partial \lambda} = 0 \quad \text{and} \quad \frac{\partial TAC(t_1, T, \lambda)}{\partial \lambda} = 0 \quad \text{and}
\]

\[
\text{Hessian matrix} (H) =
\]

\[
\begin{bmatrix}
\frac{\partial^2 TAC}{\partial t_1^2} & \frac{\partial^2 TAC}{\partial t_1 \partial \lambda} & \frac{\partial^2 TAC}{\partial \lambda^2} \\
\frac{\partial^2 TAC}{\partial t_1 \partial \lambda} & \frac{\partial^2 TAC}{\partial \lambda^2} & \frac{\partial^2 TAC}{\partial \lambda^2} \\
\frac{\partial^2 TAC}{\partial \lambda^2} & \frac{\partial^2 TAC}{\partial \lambda^2} & \frac{\partial^2 TAC}{\partial \lambda^2}
\end{bmatrix}
\]

is positive definite.

Provided the determinant of principal minors of Hessian matrix \((H\text{-matrix})\) of \(TAC(t_1, T, \lambda)\) are positive definite, i.e \(\det(H_1)\), \(\det(H_2)\), and \(\det(H_3)\) are positive, where \(H_1, H_2\), and \(H_3\) are the principal minor of the \(H\)-matrix.
5 ALGORITHM

Step-1: Start

Step-2: Evaluate $\frac{\partial TAC}{\partial t_1}$, $\frac{\partial TAC}{\partial T}$ and $\frac{\partial TAC}{\partial \lambda}$.

Step-3: Solve the simultaneous equation $\frac{\partial TAC}{\partial t_1} = 0$, $\frac{\partial TAC}{\partial T} = 0$ and $\frac{\partial TAC}{\partial \lambda} = 0$ by initializing the values of $A$, $\mu$, $k$, $\alpha$, $\beta$, $c$, $\rho$, $D$, $E$, $F$, $c_1$, $c_2$, $c_3$, $\phi$, $\theta$, $\delta$, $u$, $p$ and $\pi$.

Step-4: Choose one set of solution from step-3.

Step-5: Evaluate PM1 = det($H_1$), PM2 = det($H_2$) and PM3 = det($H_3$) where $H_1$, $H_2$ and $H_3$ are the principal minor of the H-matrix.

Step-6: If the values of PM1, PM2 and PM3 are greater than zero, then this set of solution is optimal & go to step 7. Otherwise go to step 5.

Step-7: Evaluate TAC($t_1$, $T$, $\lambda$)

Step-8: End

6 NUMERICAL EXAMPLE

The above theory can be illustrated by considering the following numerical examples.

Example 1

Let us take $A = $2500, $\mu = 6.1$, $k = 0.3$, $\alpha = 0.9$, $\beta = 0.9$, $a = 2$, $c = 1$, $\rho = 0.59$, $D = 2.1$, $E = 1.8$, $F = 2.5$, $c_1 = $20, $c_2 = $17, $c_3 = $10, $\phi = 0.6$ units, $\theta = 0.5$, $\varphi = 9.1$, $\lambda = 0.5$, $u = $0.5, $p = $25 and $\pi = $0.6. The outputs of the software program are $t_1 = 0.91$ units, $T = 2.88$ units, $\lambda = $ $3.62$ and TAC($t_1$, $T$, $\lambda$) = $1269.15$, i.e. the optimal value of $t_1$ at which the inventory level become zero is 0.91 unit time, an optimum cycle time is 2.88 unit time and the optimal value of preservation technology cost is $3.62$ per unit the
optimum total cost is $1269.15.

Example 2
Let us take $A= $2800, $\mu=7.4$, $k=0.6$, $\alpha=0.8$, $\beta=0.6$, $a=5$, $c=3$, $\rho=0.62$, $D=2.6$, $E=2.2$, $F=2.8$, $c_1=$ $23$, $c_2=$ $19$, $c_3=$ $11$, $\varphi=0.8$ units, $\theta=0.6$, $\phi=9.9$, $\delta=0.7$, $u=$ $0.6$, $p= $ $28$ and $\pi= $ $0.8$. The outputs of the software program are $t_1=2.16$ units, $T=3.95$ units, $\lambda=4.15$ and $TAC(t_1,T,\lambda)=1641.22$, i.e. the optimal value of $t_1$ at which the inventory level become zero is 2.16 unit time, an optimum cycle time is 3.95 unit time and the optimal value of preservation technology cost is $4.15$ per unit & the optimum total cost is $1641.22$.

Example 3
Let us take $A= $2950, $\mu=8.5$, $k=0.8$, $\alpha=0.4$, $\beta=0.7$, $a=6$, $c=6$, $\rho=0.66$, $D=2.9$, $E=2.6$, $F=3.1$, $c_1=$ $26$, $c_2=$ $20$, $c_3=$ $13$, $\varphi=0.9$ units, $\theta=0.7$, $\phi=10.5$, $\delta=0.8$, $u=$ $0.8$, $p= $ $30$ and $\pi= $ $0.9$. The outputs of the software program are $t_1=2.45$ units, $T=4.23$ units, $\lambda=4.96$ and $TAC(t_1,T,\lambda)=1876.91$, i.e. the optimal value of $t_1$ at which the inventory level become zero is 2.45 unit time, an optimum cycle time is 4.23 unit time and the optimal value of preservation technology cost is $4.96$ per unit & the optimum total cost is $1876.91$.

7 SENSITIVITY ANALYSIS

We studied the effects due to change in parameters $\alpha$, $\beta$, $\lambda$ and on the optimal values of $t_1$, $T$ and $TAC(t_1,T,\lambda)$.

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$t_1$</th>
<th>$T$</th>
<th>$TAC(t_1,T,\lambda)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.08</td>
<td>2.20</td>
<td>3.59</td>
<td>2091.84</td>
</tr>
<tr>
<td>3.09</td>
<td>2.16</td>
<td>3.21</td>
<td>1997.85</td>
</tr>
<tr>
<td>3.10</td>
<td>2.10</td>
<td>3.10</td>
<td>1993.80</td>
</tr>
<tr>
<td>3.11</td>
<td>2.03</td>
<td>2.99</td>
<td>1892.13</td>
</tr>
<tr>
<td>3.12</td>
<td>1.94</td>
<td>2.83</td>
<td>1774.17</td>
</tr>
<tr>
<td>3.13</td>
<td>1.80</td>
<td>2.74</td>
<td>1602.90</td>
</tr>
</tbody>
</table>
Table 2: Effect of the parameter (δ):

<table>
<thead>
<tr>
<th>δ</th>
<th>t₁</th>
<th>T</th>
<th>TAC(t₁, T, λ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>2.16</td>
<td>3.92</td>
<td>3797.79</td>
</tr>
<tr>
<td>0.8</td>
<td>2.10</td>
<td>3.58</td>
<td>3808.80</td>
</tr>
<tr>
<td>0.7</td>
<td>2.05</td>
<td>2.94</td>
<td>3821.70</td>
</tr>
<tr>
<td>0.6</td>
<td>2.00</td>
<td>2.84</td>
<td>3834.87</td>
</tr>
<tr>
<td>0.5</td>
<td>1.95</td>
<td>2.79</td>
<td>3847.38</td>
</tr>
<tr>
<td>0.4</td>
<td>1.90</td>
<td>2.52</td>
<td>3858.47</td>
</tr>
</tbody>
</table>

Table 3: Effect of parameter (α):

<table>
<thead>
<tr>
<th>α</th>
<th>t₁</th>
<th>T</th>
<th>TAC(t₁, T, λ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>2.15</td>
<td>3.95</td>
<td>3431.25</td>
</tr>
<tr>
<td>0.8</td>
<td>2.40</td>
<td>4.01</td>
<td>3520.15</td>
</tr>
<tr>
<td>0.7</td>
<td>2.56</td>
<td>4.15</td>
<td>3608.67</td>
</tr>
<tr>
<td>0.6</td>
<td>2.60</td>
<td>4.22</td>
<td>3695.28</td>
</tr>
<tr>
<td>0.5</td>
<td>2.66</td>
<td>4.32</td>
<td>3780.25</td>
</tr>
<tr>
<td>0.4</td>
<td>2.71</td>
<td>4.41</td>
<td>3864.05</td>
</tr>
</tbody>
</table>

Table 4: Effect of the parameter (β):

<table>
<thead>
<tr>
<th>β</th>
<th>t₁</th>
<th>T</th>
<th>TAC(t₁, T, λ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>1.86</td>
<td>2.99</td>
<td>2503.04</td>
</tr>
<tr>
<td>0.8</td>
<td>2.10</td>
<td>3.36</td>
<td>3208.80</td>
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<tr>
<td>0.7</td>
<td>2.25</td>
<td>3.47</td>
<td>3521.17</td>
</tr>
<tr>
<td>0.6</td>
<td>2.26</td>
<td>3.51</td>
<td>3632.44</td>
</tr>
<tr>
<td>0.5</td>
<td>2.29</td>
<td>3.63</td>
<td>3940.06</td>
</tr>
<tr>
<td>0.4</td>
<td>2.31</td>
<td>3.72</td>
<td>4004.35</td>
</tr>
</tbody>
</table>

8 MANAGERIAL IMPLICATIONS

From the above tables, we survey some interesting facts. We notice that the total average cost TAC(t₁, T, λ) of the system increases with the increment in parameters α, β, and δ. While parameter λ increase the total average cost TAC(t₁, T, λ) of the system decreases. The study found that, if we increase the preservation parameter λ,
there may be a chance to face high risk. It may affect the business and may lead to loss.

9 CONCLUSION

In this paper, a multi-item inventory model is established which helps to find the combined optimal policy of different items, when shortages are taken into consideration along with varying demand and varying mixture pattern deteriorations. This model is very practical for the wholesale fruit shop in which the holding cost is also a mixture pattern depending upon the time. The concept of unified multi-item model is more realistic for competitive environment. Herein shown that it is possible to construct one such incorporated inventory model for multiple items with multiple parametric variations. The contribution of this paper helps the business decision makers for choosing an optimal level of inventory, service and inventory management strategy under many variations.

In upcoming research on this problem, it would be remarkable to extend this model for non-instantaneous deterioration rate. Also probabilistic demand, probabilistic deterioration, Quantity discounts, inflation, etc. may be added in this paper for further study.

References


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