DESIGN OF FOPI CONTROLLER
FOR SPEED CONTROL OF BLDC
MOTOR

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Abstract

BLDC motor is a PMSM motors used for several applications. A novel control methodology known as FOPI for speed control of BLDC motor have been proposed. The design of FOPI Controller is realized from Oustaloup filter. FOPI Controller is one type of order change controller. BLDC motor modelled, simulated and FOPI controller is designed using MATLAB-Simulink. FOPI controller tested on BLDC motor for different operating modes and performance of FOPI controller compared to ordinary PI controller.

Key Words: BLDC, PMSM, FOPI controller and conventional PI controller.
1 INTRODUCTION

BLDC motors are popularly used for various industries due to its fast response and consistent performance. BLDC motors are popularly used for different applications some of them are Electric-Vehicles, computer applications, production, and aerospace applications. In [1] PID controller implemented and gains of the PID controller tuned with GA for better performance. A FSMC is demonstrated [2] on BLDC motor drive, the performance of the BLDC motor with FSMC is compared with normal SMC. The effectiveness of GSMC over SMC tested on BLDC motor for uncertain loads are compared. The performance [3] comparison of BLDC motor with linear PID controller and nonlinear SMC are discussed. In [4] discussed variable sampling VSC drive for BLDC motor for position control applications. In [5] an AFSMC is proposed to improve the performance of BLDC motor and performance compared to SMC and discussed about elimination of chattering effect. In [6] discussed an SMO based controller and parameters of the BLDC motor tuned with optimization algorithm. In [7] a cascade SMC with PID controller is designed to improve the performance of BLDC motor. The performance of this cascade controller compared to normal PID controller. In [8] a commutation ripple reduction of BLDC motor discussed for Air-Conditioners. In [9] a LQR optimal regulator for tuning of PID controller for speed control of BLDC motor and performance compared to ordinary PID controller. In[10] an ANFIS controller demonstrated on BLDC motor to overcome the drawbacks with FLC and ANN. In [11] a novel control strategy assisted with PLL is demonstrated for speed control applications of BLDC motor. In [12] a FPGA based digital control is presented to develop low cost drive system for BLDC motor. In [13] a digital implementation of FLC is demonstrated for variable speed control of BLDC motor. In [14] a brain emotional controller proposed and performance is compared to normal PID controller. In [15] a fuzzy PID controller is implemented and effectiveness of proposed controller compared to normal PID controller. In [16] a robust PID controller is designed and parameters are optimized by GA and gains are tuned by online-ANN. A Model Reference Adaptive Controller (MRAC) is implemented and simulation results are compared to normal PID controller, while implementation it is ob-
served that the tuning requirements for MRAC are less compared to normal PID controller.
In this paper section-I deals with literature survey of BLDC motor and mathematical modeling of BLDC motor discussed in section-II. The design of FOPI controller for BLDC motor discussed in section-III finally results are discussed section-IV.

2 MATHEMATICAL MODELING OF BLDC MOTOR

The equations of BLDC motor is represented in state space representation.

\[ \dot{X} = AX + BX \]  

\[ X = [I_d\ I_q\ \omega\ \theta] \]

\[ \frac{dI_d}{dt} = \frac{V_d}{L} - \frac{I_d}{\tau} + \omega I_q \]  

\[ \frac{dI_q}{dt} = \frac{V_q}{L} - \frac{I_q}{\tau} - \omega I_d - \frac{K_e}{L} \omega \]  

\[ \frac{d\omega}{dt} = \frac{PK_e}{j} I_q - \frac{f}{j} \omega + \frac{P}{j} T_L \]  

\[ \frac{d\theta}{dt} = \omega \]

The output equation is given by

\[ Y = CX, Y = [I_d\ I_q]^T \]

\[ C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \]

\( V_d, V_q, I_d, I_q \) are voltages and currents on a \((d,q)\) frame,
\( T_L \) = Load torque
\( \omega \) = electrical angular velocity,
\( K_e \) = factor torque,
\( L \) = inductance,
\( R \) = resistance,
\[ v = [R]i + [L] \frac{di}{dt} + e \] (7)

Where \( R \) is the stator resistance per phase
\[
R = \begin{bmatrix}
R & 0 & 0 \\
0 & R & 0 \\
0 & 0 & R
\end{bmatrix}
\]

\( L \) is the matrix of inductance interms of self and mutual inductance
\[
L = \begin{bmatrix}
L_s & -M & -M \\
-M & L_s & -M \\
-M & -M & L_s
\end{bmatrix}
\]

\( e = [e_a \ e_b \ e_c]^T \) is the vector of the trapezoidal back EMF

\[ \frac{d}{dt} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix}
\frac{1}{L_T} & 0 & 0 \\
0 & \frac{1}{L_T} & 0 \\
0 & 0 & \frac{1}{L_T}
\end{bmatrix} \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} - \begin{bmatrix}
R & 0 & 0 \\
0 & R & 0 \\
0 & 0 & R
\end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} - \begin{bmatrix} e_an \\ e_bn \\ e_cn \end{bmatrix} \] (8)

\( L_T = L_s + M \)

The equation of motion is
\[
j \frac{d\omega_r}{dt} = T_{em} - T_L - f\omega_r \] (9)

\( T_{em} = \frac{1}{\omega_r} (e_an i_a + e_bn i_b + e_cn i_c) \) (10)

\( \omega_r \) - mechanical speed [rad/s],
\( T_l \) - load torque [N m],
\( J \) - motor shaft and load inertias [kg m²],
\( f \) - frictional damping coefficient [N m s/rad m],
\( T_{em} \) - electromagnetic torque [N]

3 DESIGN OF A FOPI CONTROLLER FOR BLDC MOTOR

The FOPI have gained more attention by the researchers for various applications in power system and power electronics. FOPI controller
is designed first time for BLDC motor but in recent past authors implemented for power system for different applications. The design of the controller purely depends on realization of $s^\lambda$ in terms of transfer function model of filter can be realized using Oustaloup Filter [18-21].

![Fig-1: Illustrates the block diagram representation of BLDC motor with FOPI controller](image)

![Fig-2: Structure of Fractional Order PI controller](image)

The fractional order calculus derived from ordinary calculus. According to Riemann-Liouville (R-L) the fractional derivative is given by [18]

$$\frac{d^\lambda}{dt^\lambda} f(t) = \frac{1}{\Gamma(n-\lambda)} \frac{d^n}{dt^n} \int_0^t \frac{f(\tau) \, d\tau}{(t-\tau)^{\lambda-n+1}}$$  \hspace{1cm} (10)

Where $n-1 \leq \lambda < n$, $n$ is an integer and $\Gamma(.)$ is the Euler's gamma function.

According to R-L the other definition [18] is

$$\frac{d^\lambda}{dt^\lambda} f(t) = \frac{1}{\Gamma(n-\lambda)} \frac{d^n}{dt^n} \int_0^t \frac{f(\tau) \, d\tau}{(t-\tau)^{\lambda-n+1}}$$  \hspace{1cm} (11)
Where $\Delta^\lambda$ is fractional operator.
According to Riemann-Liouville definition [10] Laplace transformation of fractional derivative is

$$L \Delta^\lambda f(t) = s^\lambda F(s) - \sum_{k=0}^{\lambda-1} s^{\lambda-k-1} f(t)$$

(12)

For $n - 1 \leq \lambda < n$ where $L f(t)$ indicates the normal Laplace transformation. The representation of fractional order transfer function involves infinite number of poles and zeros. Refined Oustaloup filter [18] used in the design of FOPI controller.

The fractional order derivative ($S^\lambda$) represented in terms of recursive distribution of transfer function in pole- zero notation.

$$s^\lambda = K \prod_{n=1}^{\infty} \frac{1 + s/\omega_n}{1 - s/\omega_n}$$

(13)

'K' is an adjusted gain and frequency of poles and zeros are given by the equation.

$$\omega_{p,1} = \omega_p \sqrt{N}$$

(14)

$$\omega_{p,n} = \omega_p \varepsilon^n, \quad n = 1, .., N$$

(15)

$$\omega_{z,n+1} = \omega_z \varepsilon^{n-1}, \quad n = 1, .., N - 1$$

(16)

$$\varepsilon = \frac{\omega_z}{\omega_p}$$

(17)

$$\omega_n = \omega_p \sqrt{\frac{1}{N}}$$

(18)

The commonly used form of FOPI is the $PI^\lambda$, where $\lambda$ is a real integer. The transfer function model of FOPI is given by (19)

$$\sigma(s) = Kp + \frac{Kd}{s^\lambda}$$

(19)

The differential equation of fractional order PI controller is described in equation (20)

$$u(t) = Kp \, e(t) + Kd \, D^{-\lambda} e(t)$$

(20)

**a. Design of Oustaloup Filter:**

The fractional-order operator $S^\lambda$ can be approximated by the fractional-order transfer function as
In the frequency range $\omega_b < \omega < \omega_h$ by using a Taylor series expansion, we obtain

$$K(z) = \left( \frac{\lambda}{\lambda - 1} \right)^\lambda \left( 1 + \frac{z^2 + z}{\lambda^2 + \lambda} \right)^\lambda$$

(21)

Where $0 < \lambda < 1$, $z = j\omega$, $b > 0$, $d > 0$, and

$$K(z) = \left( \frac{\lambda}{\lambda - 1} \right)^\lambda \left( 1 + \frac{z^2 + z}{\lambda^2 + \lambda} \right)^\lambda$$

(22)

In the frequency range $\omega_b < \omega < \omega_h$ by using a Taylor series expansion, we obtain

$$K(z) = \left( \frac{\lambda}{\lambda - 1} \right)^\lambda \left( 1 + \lambda p(z) \frac{10z^2 + 5}{5z^2 + 5} p^2(z) + \ldots \right)$$

$$p(z) = \frac{z^2 - z}{\lambda^2 + \lambda}$$

(23)

It is then found that

$$z^\lambda = \frac{(\omega_b)^{\lambda - 1}}{1 + \lambda p(z) + \frac{10z^2 + 5}{5z^2 + 5} p^2(z) + \ldots} \left( \frac{1 + \frac{\lambda}{\lambda - 1} z}{1 + \frac{\lambda}{\lambda - 1} z} \right)^\lambda$$

(24)

$S^\lambda$ approximately written as

$$z^\lambda \approx \frac{(\omega_b)^{\lambda - 1}}{\lambda^2 + \lambda(\lambda - 1)} \left( \frac{1 + \frac{\lambda}{\lambda - 1} z}{1 + \frac{\lambda}{\lambda - 1} z} \right)^\lambda$$

(25)

Thus, the fractional-order differentiator is defined as

$$z^\lambda \approx \frac{(\omega_b)^{\lambda - 1}}{\lambda^2 + \lambda(\lambda - 1)} \left( \frac{1 + \frac{\lambda}{\lambda - 1} z}{1 + \frac{\lambda}{\lambda - 1} z} \right)^\lambda$$

(26)

Expression (26) is stable if and only if all the poles are on the left-hand side of the complex s-plane. It is easy to check that expression (26) has three poles:

- One of the poles is located at $-b\omega_h/d$ which is a negative real pole since $\omega_h > 0$, $b > 0$, $d > 0$;
The two other poles are the roots of the equation

\[ d(1 - \lambda)s^2 + a\omega_b + d\lambda = 0 \]  

Whose real parts are negative since \(0 < \lambda < 1\). Thus, the above transfer function model is stable with in band of frequencies \((\omega_b, \omega_h)\). The fractional-order part of expression approximated as

\[
k(z) = \lim_{n \to 0} \prod_{k=n}^{\infty} \frac{\omega_{\lambda/k}^{+}}{\omega_{\lambda/k}^{-}}
\]

According to the recursive distribution of real zeros and poles, the zero and pole of rank \(K\) can be written as

\[
\omega_{K^{-}} = \left(\frac{d\omega_b}{b}\right)^{\frac{a-2K}{a+b}}, \omega_{K^{+}} = \left(\frac{b\omega_b}{d}\right)^{\frac{a+2K}{a+b}}
\]

Thus, the continuous rational transfer function model can be obtained as

\[
s^\lambda \approx \left(\frac{b\omega_b}{d}\right)^{\lambda} \left(\frac{d\omega_b}{b^{2}}\right)^{\frac{a+b\lambda}{a}} \prod_{k=n}^{\infty} \frac{s^{\lambda}}{s^{\omega_{\lambda/k}^{+}} - s^{\omega_{\lambda/k}^{-}}}
\]

\(S^\lambda\) realized in terms of transfer function model of Oustaloup Filter.

4 SIMULATION RESULTS

In this proposed work a FOPI designed to enhance the performance characteristics of BLDC motor. The performance of this novel controller compared to ordinary PI controller under constant speed mode and constant torque mode.
Fig-2: Illustrates load current, Torque-speed responses of BLDC motor with PI controller during starting under constant speed mode.

Fig-3: Illustrates load current, Torque-speed responses of BLDC motor with FOPI controller during starting load under constant speed mode.
Fig-4: Illustrates the torque-speed responses of BLDC motor with PI controller (Blue thick line) and FOPI controller (orange thick lined) during starting under constant speed mode.

Fig-5: Illustrates load current, Torque-speed responses of BLDC motor with PI controller during Generating mode under constant speed mode.
Fig-6: Illustrates load current, Torque-speed responses of BLDC motor with FOPI controller during Generating Mode under constant speed mode.

Fig-7: Illustrates the torque - speed responses of BLDC motor with PI controller (Blue thick line) and FOPI controller (orange thick lined) during Generating mode under constant speed mode.
Fig-8: Illustrates load current, Torque-speed responses of BLDC motor with PI controller during Full load under constant speed mode.

Fig-9: Illustrates load current, torque-speed responses of BLDC motor with FOPI controller during Full load condition under constant speed mode.
Fig-10: Illustrates the torque - speed responses of BLDC motor with PI controller (Blue thick line) and FOPI controller (orange thick lined) during Full load condition under constant speed mode.

Fig-11: Illustrates the torque - speed responses of BLDC motor with PI controller (Blue thick line) and FOPI controller (orange thick lined) during starting under constant torque mode.
5 CONCLUSION

BLDC motor popularly used for aerospace, industrial applications, servo drive applications. This proposed Fractional Order PI Con-
controller (FOPI) is realised with Oustaloup filter. The desired response obtained by adjusting λ in $\frac{1}{S^\lambda}$. The designed FOPI controller tested on BLDC motor for various operating modes and performance compared to normal PI controller. Simulation results shows that the performance of BLDC motor with FOPI controller is quite good under various operating modes (constant speed and constant torque mode) compared to performance of BLDC motor with normal PI controller.

References


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