



DENOISING OF IMAGES USING A NOVEL TWO STAGE ALGORITHM

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Abstract

This paper presents an optimal algorithm for de-noising of images. The present algorithm is based on improved version of local pixels grouping and principal component analysis. The energy of noise evenly spreads over the whole data set and can be easily distinguish the signal from noise over PCA domain. This method uses two stages: image estimation by removing the noise and further refinement of the first stage. The noise is significantly reduced in the first stage, the LPG accuracy will be much improved in the second stage so that the final denoising result is visually much better. The performance of our proposed algorithm can be evaluated by peak signal to noise ratio (PSNR) and structural similarity (SSIM), which are respectively according to the signal to noise ratio in the image and structural similarity of two images. Experimental results show that the presented algorithm has a significant superiority in denoising. Furthermore, the quantities of SSIM and PSNR values are higher in comparison to other methods.

Key Words: PCA, Denoising, LPG, SSIM, PSNR.

1 INTRODUCTION

Noise removal has been extensively studied as it is a primary low-level image processing procedure and many denoising schemes have been proposed, from the earlier smoothing filters and frequency domain denoising methods [25] to the lately developed wavelet [110], curvelet [26] and ridgelet [12] based methods, sparse representation [13] and K-SVD [14] methods, shape-adaptive transform [15], bilateral filtering [16,17], non-local mean based methods [18,19] and non-local collaborative filtering [15]. The rapid development of modern digital imaging devices and their increasingly wide applications in our daily life increases the requirement of new denoising algorithms for higher image quality.

The effective algorithm for noise removal is Wavelet transform(WT)[24].

The input signal is decomposed into multiple scales, which represent different time-frequency components of the original signal. At each scale, some operations, such as thresholding [1,2] and statistical modeling [35], can be performed to suppress noise. Denoising can be achieved by transforming back the processed wavelet coefficients into spatial domain. Late development of WT denoising includes ridgelet [12] and curvelet [26] methods for line structure preservation. Even though WT has demonstrated its efficiency in denoising images, it uses a fixed wavelet basis (with dilation and translation) to represent the image. Rich amount of different local structural patterns for natural images which cannot be well represented by using only one fixed wavelet basis. Therefore, WT-based methods can introduce many visual artifacts in the denoising output.

Spatially adaptive principal component analysis (PCA) based denoising scheme, was proposed by Muresan and Parks[21], to overcome the problem of WT. This algorithm computes the locally fitted basis to transform the image. Elad and Aharon [13,14] proposed sparse redundant representation and K-SVD based denoising algorithm by training a highly over-complete dictionary. Foi et al. [15] applied a shape-adaptive discrete cosine transform (DCT) to the neighborhood, which can achieve very sparse representation of the image and hence lead to effective denoising. All the above methods show better results than the conventional WT-based denoising algorithms.

Recently developed non-local means (NLM) algorithm uses a very

different approach from the above methods in noise removal. The idea of NLM can be traced back to [23], where the similar image pixels are averaged according to their intensity distance. Bilateral filtering methods [16,17] also implement similar ideas, where both the spatial and intensity similarities are used for pixel averaging. In [18], the NLM denoising framework was well established. Each pixel is estimated as the weighted average of all the pixels in the image, and the weights are determined by the similarity between the pixels. The NLM scheme was improved in [19], where the pair-wise hypothesis testing was used in the NLM estimation. The success of NLM methods motivates Dabov et al. [15] to propose a collaborative image denoising scheme by patch matching and sparse 3D transform. They searched for similar blocks in the image by using block matching and grouped those blocks into a 3D cube. A sparse 3D transform was then applied to the cube and noise was suppressed by applying Wiener filtering in the transformed domain. This algorithm was called BM3D which achieves remarkable denoising results but its implementation is a little complex.

In this paper we present an efficient method of denoising by LPG-PCA. PCA is a classical de-correlation technique in statistical signal processing and it is pervasively used in pattern recognition and dimensionality reduction, etc. [11]. The original dataset is transformed into PCA domain and preserving only the most significant principal components, the noise and trivial information can be removed. In [21], a PCA-based moving window scheme was proposed for image denoising to calculate the local statistics, from which the local PCA transformation matrix was estimated. However, this PCA scheme applies directly to the noisy image without data selection and many noise residual and visual artifacts will appear in the denoised outputs.

In this proposed model of LPG-PCA, we consider a pixel and its nearest neighbours as a vector variable. The training samples of this variable are selected by grouping the pixels with similar local spatial structures to the underlying one in the local window. With such an LPG procedure, the local statistics of the variables can be accurately computed so that the image edge structures can be well preserved after shrinkage in the PCA domain for noise removal. The proposed PCA based LPG algorithm is divided into two stages. The first stage gives an estimation of the image by removing most

of the noise and the second stage will further refine the output of the first stage. The two stages have the same procedures except for the parameter of noise level. Since the noise is mostly removed in the first stage, the accuracy will be much improved in the second stage so that the final denoising results are visually much better. Compared with Wavelet Transform which uses a fixed basis function to decompose the image, the proposed LPG-PCA method is a spatially adaptive image representation so that it can better characterize the image local structures. Compared with NLM and the BM3D methods, the proposed PCA based LPG method can use a relatively small local window to group the similar pixels for PCA training, yet it yields competitive results with BM3D algorithm. Remaining part of the paper is organised as follows. In section 2, the procedure of PCA is briefly reviewed. In Section 3 our proposed denoising algorithm is presented in detail. In Section 4 the experimental results are shown and Section 5 concludes the paper.

2 Principal component analysis

Denote an m-component vector variable and denote by

$$X = \begin{bmatrix} x_1^1 & x_1^2 & \dots & x_1^n \\ x_2^1 & x_2^2 & \dots & x_2^n \\ \vdots & \vdots & \ddots & \vdots \\ x_m^1 & x_m^2 & \dots & x_m^n \end{bmatrix} \dots (2.1)$$

the sample matrix of X, where are $x_i^j, j = 1, 2, \dots, n$ are the discrete samples of variable $x_i, i = 1, 2, \dots, m$ The ith row of sample matrix (X) denoted by

$$X_i = [x_i^1 x_i^2 \dots x_i^n] \dots (2.2)$$

is called the sampled vector of x_i . The mean value of X_i is calculated as

$$\mu_i = \frac{1}{n} \sum_{j=1}^n X_i(j) \dots (2.3)$$

and then the sample vector X_i is centralized as

$$\bar{X}_i = X_i - \mu_i = [\bar{x}_i^1 \bar{x}_i^2 \dots \bar{x}_i^n] \dots (2.4)$$

where $\bar{x}_i^j = x_i^j - \mu_i$. Accordingly the centralized matrix of X is

$$\bar{X} = [\bar{x}_1^T \bar{x}_2^T \dots \bar{x}_m^T] \dots (2.5)$$

Finally, the co-variance matrix of the centralized dataset is calculated as

$$\Omega = \frac{1}{n} \bar{X} \bar{X}^T \dots (2.6)$$

The goal of PCA is to find an orthonormal transformation matrix P to de-correlate X , i.e. $\bar{Y} = P\bar{X}$ so that the co-variance matrix of $\bar{Y}$ is diagonal. Since the co-variance matrix Ω is symmetrical, it can be written as:

$$\Omega = \Phi \Lambda \Phi^T \dots (2.7)$$

Where $\Phi = [\Phi_1 \Phi_2 \dots \Phi_m]$ is the $m \times m$ orthonormal eigenvector matrix and $\Lambda = \text{diag}\{\lambda_1, \lambda_2, \dots, \lambda_m\}$ is the diagonal eigen value matrix with $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_m$. The terms $\phi_1, \phi_2, \dots, \phi_m$ and $\lambda_1, \lambda_2, \dots, \lambda_m$ are the eigen vectors and eigen values of Ω . By setting

$$P = \Phi^T \quad (2.8)$$

$\bar{X}$ can be decorrelated, i.e. $\bar{Y} = P\bar{X}$ and $\Lambda = (1/n)\bar{Y}\bar{Y}^T$. An important property of PCA is that it fully de-correlates the original dataset $\bar{X}$. Generally speaking, the energy of a signal will concentrate on a small subset of the PCA transformed dataset, while the energy of noise will evenly spread over the whole dataset. Therefore, the signal and noise can be better distinguished in the PCA domain.

3 Local pixel grouping (LPG)

Grouping the training samples similar to the central block in the Training window is indeed a classification problem and thus different grouping methods, such as block matching, correlation-based matching, K-means clustering, etc., can be employed based on different criteria. Among them, the block matching method may be the simplest yet very efficient one. In this paper, we employ it for LPG. There are totally possible $(L - K + 1)^2$ training blocks of X_v in the $L \times L$ training window. We denote by $\bar{x}_0^v$, the column sample vector containing the pixels in the central $K \times K$ block and denote by $\bar{x}_i^v, i = 1, 2, \dots, (L - K + 1)^2 - 1$, the sample vectors corresponding to the other blocks. Let $\bar{x}_0$ and $\bar{x}_i$, be the associated noiseless sample vectors of $\bar{x}_0^v$ and $\bar{x}_i^v$, respectively. It can be easily calculated that

$$e_i = \frac{1}{m} \sum_{k=1}^m \bar{x}_0^v(k) - \bar{x}_0^v(k)^2 \approx \frac{1}{m} \sum_{k=1}^m \bar{x}_0^v(k) - \bar{x}_0^v(k)^2 + 2\sigma^2 \dots (3.1)$$

In (3.2) we used the fact that noise v is white and uncorrelated with

signal. With (3.2), if
 $e_i < T + 2\sigma^2 \dots (3.2)$

Suppose we select n sample vectors of X_v , including the central vector $\bar{x}_0^v$. For the convenience of expression, we denote these sample vectors as $\bar{x}_0^v, \bar{x}_1^v, \dots, \bar{x}_{n-1}^v$. The noiseless counterparts of $\bar{x}_0, \bar{x}_1, \dots, \bar{x}_{n-1}$ these vectors are denoted as $\bar{x}_0, \bar{x}_1, \dots, \bar{x}_{n-1}$, accordingly. The training dataset for X_v is then formed by $\bar{X}_v = [\bar{x}_0^v, \bar{x}_1^v, \dots, \bar{x}_{n-1}^v]^T$. The noiseless counterpart of $\bar{X}_v$ is denoted as $X = [\bar{x}_0, \bar{x}_1, \dots, \bar{x}_{n-1}]$. To guarantee there are enough samples in computing the PCA transformation matrix, n could not be too small. In practice, we will use at least $c \cdot m$ training samples of X_v in denoising, where constant $c = 8 \sim 10$. That is to say, if $n < c \cdot m$, we will use the best $c \cdot m$ matched samples in PCA training. Usually, the best $c \cdot m$ matched samples are robust to estimate the image local statistics, and this operation makes the algorithm more stable to calculate the PCA transformation matrix.

Now the problem is how to estimate the noiseless data set X from the noisy measurement X_v . Once X is estimated, the central block and consequently the central underlying pixel can be extracted. Applying such procedures to each pixel and then the whole image I_v can be denoised.

4 LPG-PCA based denoising

In the $m \times n$ dataset matrix X_v , each component $x_k^v, k = 1, 2, \dots, m$, of the vector variable X_v has n samples. Denote by X_k^v the row vector containing the n samples of x_k^v . Then the dataset X_v can be represented as $X_v = [(X_1^v)^T \dots (X_m^v)^T]^T$. Similarly, we have $X = [(X_1^T) \dots (X_m^T)]^T$ where X_k is the row vector containing the n samples of x_k , and $X_v = X + V$, where $V = [(V_1^T) \dots (V_m^T)]^T$ is the dataset of noise variable v and V_k is the row sample vector of v_k .

Next we centralized at a set X_v . The mean value of X_k^v is $\mu_k = (1/n) \sum_{i=1}^n X_k^v(i)$, and then X_k^v is centralized by $\bar{X}_k^v = X_k^v - \mu_k$. Since the noise v_k is zero-mean, X_k can also be centralized by $\bar{X}_k^v = X_k^v - \mu_k$. Then the centralized data sets of X_v and X are obtained as $\bar{X}_v = [(\bar{X}_1^v)^T \dots (\bar{X}_m^v)^T]^T$ and $\bar{X} = [(\bar{X}_1^T) \dots (\bar{X}_m^T)]^T$, and we have

$$\bar{X}_v = \bar{X} + V \dots (4.1)$$

By computing the covariance matrix of $\bar{X}$, denoted by $\Omega_{\bar{X}}$,

the PCA transformation matrix $P_{\bar{X}}$ can be obtained. However, the available dataset $\bar{X}_v$ is noise corrupted so that $\Omega_{\bar{X}}$ cannot be directly computed. With the line a model (3.5), we have $\Omega_{\bar{x}} = \frac{1}{n} \bar{X}_v \bar{X}_v^T = \frac{1}{n} (\bar{X} \bar{X}^T + \bar{X} V^T + V \bar{X}^T + V V^T) \dots (4.2)$ since $\bar{X}$ and V are uncorrelated, items $\bar{X} V^T$ and $V \bar{X}^T$ will be nearly zero matrices and thus: $\Omega_{\bar{X}} \approx \frac{1}{n} (\bar{X} \bar{X}^T + V V^T) = \Omega_{\bar{X}} + \Omega_v$ where $\Omega_{\bar{X}} = (1/n) \bar{X} \bar{X}^T$ and $\Omega_v = (1/n) V V^T$. The component $\Omega_v(i, j)$ is the correlation between v_i and v_j . Since v_i and v_j are uncorrelated for $i \neq j$, we know that Ω_v is a $m \times m$ diagonal matrix with all the diagonal components being σ^2 . In other words, Ω_v can be written as $\sigma^2 I$, where I is the identity matrix. Then it can be readily proved that the PCA transformation matrix $P_{\bar{X}}$ associated with $\Omega_{\bar{X}_v}$ is the same as the PCA transformation matrix associated with $\Omega_{\bar{x}_v}$.

As in (2.7), we can decompose $\Omega_{\bar{X}}$ as

$$\Omega_{\bar{x}} = \Phi_{\bar{x}} \Lambda_{\bar{x}} \Phi_{\bar{x}}^T \dots (4.3)$$

where $\Phi_{\bar{x}}$ is the $m \times m$ orthonormal eigen vector matrix and $\Lambda_{\bar{x}}$ is the diagonal eigen value matrix. Since $\phi_{\bar{x}}$ is an orthonormal matrix, we can write Ω_v as $\Omega_v = (\sigma^2 I) \Phi_{\bar{x}} \Phi_{\bar{x}}^T = \Phi_{\bar{x}} \Omega_v \Phi_{\bar{x}}^T$. Thus we have

$$\begin{aligned} \Omega_{\bar{X}_v} &= \Omega_{\bar{X}} + \Omega_v = \Phi_{\bar{X}} \Lambda_{\bar{X}} \Phi_{\bar{X}}^T + \Phi_{\bar{X}} (\sigma^2 I) \Phi_{\bar{X}}^T \\ &= \Phi_{\bar{X}} (\Lambda_{\bar{X}} + \sigma^2 I) \Phi_{\bar{X}}^T = \Phi_{\bar{X}} \Lambda_{\bar{X}_v} \Phi_{\bar{X}}^T \end{aligned}$$

where

$$\Lambda_{\bar{X}_v} = \Lambda_{\bar{X}} + \sigma^2 I \dots (4.4)$$

implies that $\Omega_{\bar{X}_v}$ and $\Omega_{\bar{x}}$ have the same eigen vector matrix $\Phi_{\bar{X}}$. Thus, impractical implementation we can directly compute $\Omega_{\bar{X}_v}$ by decomposing $\Omega_{\bar{x}}$, instead of $\Omega_{\bar{X}_v}$, and then the orthonormal PCA transformation matrix for $\bar{X}$ is set as

$$P_{\bar{X}} = \Phi_{\bar{x}}^T \dots (4.5)$$

Applying $P_{\bar{X}}$ to dataset $\bar{X}$, we have

$$\bar{Y}_v = P_{\bar{X}} \bar{X} + P_{\bar{X}} V = \bar{Y} + V_y$$

where $\bar{Y}_v = P_{\bar{X}} \bar{X}$ is the decorrelated data set for $\bar{X}$ and $V_y = P_{\bar{X}} V$ is the transformed noise dataset for V . Since $\bar{Y}$ and noise V_y are uncorrelated, we can easily derive that the covariance matrix of $\bar{Y}_v$ is

$$\Omega_{\bar{Y}_v} = \frac{1}{n} \bar{Y}_v \bar{Y}_v^T = \Omega_{\bar{Y}} + \Omega_{V_y} \dots (4.6)$$

where $\Omega_{\bar{Y}} = \frac{1}{n} \bar{Y} \bar{Y}^T$ is the covariance matrix of decorrelated dataset $\bar{Y}$ and $\Omega_{V_y} = P_{\bar{X}} \Omega_V P_{\bar{X}}^T$ is the covariance matrix of noise data set V_y . In the PCA transformed domain $\bar{Y}_v$, most energy of noiseless dataset $\bar{Y}$ concentrates on the several most important components, while

the energy of noise V_Y distributes much more evenly. The noise in $\bar{Y}_V$ can be suppressed by using the line a minimum mean square-error estimation (LMMSE) technique. Since Y_V is centralized, the LMMSE of $\bar{\bar{Y}}_k$, i.e. the K_{th} row of $\bar{Y}$, Is obtained as $\bar{\bar{Y}}_k = W_k \cdot \bar{\bar{Y}}_v^k$ (4.7)

where the shrinkage coefficient

$$w_k = \Omega_{\bar{y}}(k, k) / (\Omega_{\bar{y}}(k, k) + \Omega_{V_y}(k, k))$$

and $\bar{\bar{Y}}_v^k$, is the k^{th} row of $\bar{y}_v$. In flat zones, $\Omega_{\bar{y}}(k, k)$ is much smaller than $\Omega_{V_y}(k, k)$ so that w_k is close to 0.

Hence most of the noise will be suppressed in $\bar{\bar{Y}}$ by LMMSE operator $\bar{\bar{Y}} = W_k \cdot \bar{\bar{Y}}_v^k$. In implementation we first calculate $\Omega_{\bar{y}}$ from the available noisy dataset $\bar{Y}_v$ and then estimate $\Omega_{\bar{y}}(k, k)$ by $\Omega_{\bar{y}}(k, k) = \Omega_{\bar{y}_v}(k, k) - \Omega_{V_y}(k, k)$. In flat zones, it is of ten that $\Omega_{\bar{y}_v}(k, k) - \Omega_{V_y}(k, k) \leq 0$, and then we set $\Omega_{\bar{y}}(k, k) = 0$. In this case W_k will be exactly 0 and all the noise in $\bar{\bar{y}}_v^k$ will be removed. Denote by $\bar{\bar{Y}}$ the matrix of all $\bar{\bar{Y}}_k$. By transforming $\bar{\bar{Y}}$ back to the time domain, we obtain the denoised result of $\bar{X}_v$ as $\bar{\bar{X}} = P_{\bar{X}}^T \cdot \bar{\bar{Y}}$ (4.8)

In (3.10), we used the fact that $P_{\bar{X}}^{-1} = P_{\bar{X}}^T$. Adding the mean values μ_k back to gives the denoised data set $\bar{\bar{X}}$. The estimation of the central block $\bar{x}_0$, denoted as $\bar{\bar{x}}_0$, can then be extracted $\bar{\bar{X}}$ from and finally the denoised result of the underlying central pixel can be extracted from $\bar{\bar{x}}_0$. Applying the above procedure to each pixel leads to the full denoised image of I_v .

4.1 Removal of noise in stage-2

The noise will be removed by using the denoising procedures described in above Sections However, there is still much visually unpleasant noise residual in the denoised image. We can still see that much noise is residual in the denoising output. The main reasons for the noise residual are, because of the strong noise in the original data set X_v , the covariance matrix $\Omega_{\bar{x}_v}$ is much noise corrupted, which leads to estimation bias of the PCA transformation matrix and hence deteriorates the denoising performance second, the strong noise in the original data set will also lead to LPG errors, which consequently results in estimation bias of the covariance

matrix $\Omega_{\bar{x}}(or \Omega_{\bar{x}_v})$. Therefore, it is essential to further process the denoising output for a better noise reduction. Since much of the noise is removed in the first round of LPG-PCA denoising, the LPG accuracy and the estimation of $\Omega_{\bar{x}}(or \Omega_{\bar{x}_v})$ can be much improved with the denoised image. Thus we can implement the PCA based LPG denoising procedure for the second round to improve the denoising results.

The noise level should be updated in the second stage of LPG-PCA denoising algorithm.

Denote by $\bar{I}$ the denoised version of noisy image I_v in the first stage. We can write $\bar{I}$ as $\bar{I} = I + v_s$, where v_s is the residual in the denoised image. We need to estimate the level of v_s , denoted by $\rho_s = \sqrt{E[v_s^2]}$, and input it to the second stage of LPG-PCA denoising. Here we estimate based on the difference between $\bar{I}$ and $\tilde{I}_v$.

Let $\tilde{I} = I_v - \bar{I} = v - v_s$ (4.1.1)

We have: $E[\tilde{I}^2] = E[v^2] + E[v_s^2] - 2E[v.v_s]$
 $= \rho^2 + \rho_s^2 - 2E[v.v_s]$ (4.1.2)

We approximately view v_s as the smoothed version of noise v , and it contains mainly the low frequency component of v .

Let $\tilde{v} = v - v_s$ be their difference and $\tilde{v}$ contains mainly the high frequency component of v .

There is $E[v.v_s] = E[\tilde{v} - v_s] + E[v_s^2]$ (4.1.3)

In general, $E[\tilde{v}.v_s]$ is much smaller compared with $E[v_s^2]$. For example, after the first stage denoising of noisy image Cameraman (s=20), we have. For the convenience of development, we remove $E[\tilde{v}.v_s]$ from $E[v.v_s]$, and let

$E[v.v_s] = E[\tilde{v}.v_s] + E[v_s^2] \approx E[v_s^2] = \rho_s^2$ (4.1.4)

Thus from(4.1.4) we have

$\rho_s^2 \approx \rho^2 - E[\tilde{I}^2]$ (4.1.5) In practice, v_s will include not only the noise residual but also the estimation error of noiseless image I . Therefore ,in implementation

We let $\rho_s = c_s \sqrt{\rho^2 - E[\tilde{I}^2]}$ (4.1.6)

where $c_s < 1$ is a constant. We experimentally found that c_s getting around 0.35 can lead to satisfying denoising results for most of the testing images. Fig. 2d shows the denoising results(PSNR = 30.1 dB) of Cameraman after the second stage. Although the PSNR is not improved too much on this image, we can clearly see that the visual quality is much improved by effectively removing the noise residual in the first stage.

5 Experimental results

In this proposed PCA based LPG denoising algorithm, most of the computational cost spends on LPG grouping and PCA transformation, and thus the complexity mainly depends on two parameters: the size K of the variable block and the size L of training block. In LPG grouping, it requires additions, $(2K^2 - 1) \cdot (L - K + 1)^2$ multiplications and $K^2 \cdot (L - K + 1)^2$ "less than" logic operations. Suppose in average S training samples are selected, i.e. the dataset X_v is of dimension $K^2 \times S$.

Table 1 The PSNR(db) and SSIM results of the denoised images of Proposed algorithm.

Images	First Stage	Second stage
Image: House.tif		
$\sigma = 12$	psnr1 = 34.9645 ssim1 = 0.8864	psnr2 = 35.3016 ssim2 = 0.9005
$\sigma = 24$	psnr1 = 31.1629 ssim1 = 0.7695	psnr2 = 32.2674 ssim2 = 0.8558
Image: Cameraman.tif		
$\sigma = 12$	psnr1 = 32.3720 ssim1 = 0.8886	psnr2 = 32.4887 ssim2 = 0.9089
$\sigma = 24$	psnr2 = 28.5571 ssim2 = 0.7550	psnr2 = 28.8488 ssim2 = 0.8386

Then in the PCA transformation, it requires $K^2 \cdot S + (S^2 - 1) \cdot K^4 + (K^2 - 1) \cdot K^2 \cdot S$ additions, $K^4 \cdot (S + S^2)$ multiplications, and an SVD decomposition of a $K^2 \times K^2$ definite covariance matrix.

In this paper, we set $K=5$ and $L=41$ in all the experiments to test the denoising performance. The threshold T in the LPG grouping is set to 25. In the implementation of LPG-PCA denoising, actually the complete $K \times K$ block centered on the given pixel will be denoised. Therefore, the finally restored value at a pixel can be set as the average of all the estimates obtained by all windows containing the pixel. This strategy was also used in [21]. By our

experiments, this can increase about 0.3dB the noise reduction for most of the test images.

We evaluate and compare the different methods by using two measures: PSNR and SSIM [22]. Although PSNR can measure the intensity difference between two images, it is well-known that it may fail to describe the visual perception quality of the image. On the other hand, how to evaluate the visual quality of an image is a very difficult problem and it is currently an active research topic. The SSIM index proposed in [22] is one of the most commonly used measures for image visual quality assessment. Compared with PSNR, SSIM can better reflect the structure similarity between the target image and the reference image.

We will first verify the improvement of the noise removal in the second stage of the LPG-PCA method. Table1 lists the PSNR and SSIM measures of the first stage and second stage denoising outputs on the test image set. We can see that in the second stage we can improved the PSNR values for different images under different noise level(ρ is 12 and 24). But for some images the second stage will not improve the PSNR measures, but the SSIM measures, can better reflect the image visual quality, can be much improved. For instance, for image House with noise level $\rho = 24$ the SSIM measure is much increased from 0.8864 to 0.7695 after the second stage denoising, while the PSNR is increased by only 0.5dB.

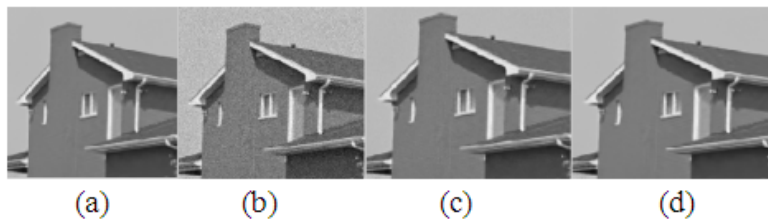


Fig. 1. (a) Original image House; (b) noisy image; (c) denoised image after the first stage of the proposed method (PSNR= 32.372 dB) and (d) denoised image after the second stage of the proposed method (PSNR=32.488 dB). We see that the visual quality is much improved after the second stage refinement.

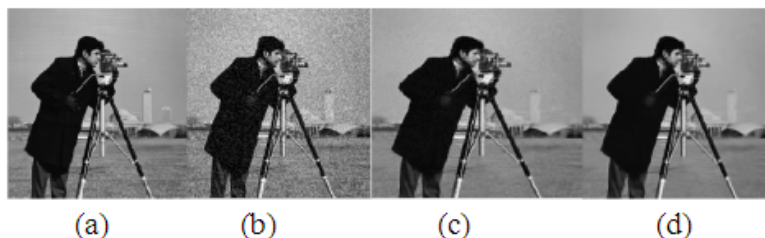


Fig. 2. (a) Original image Cameraman; (b) noisy image; (c) denoised image after the first stage of the proposed method (PSNR=32.372 dB) and (d) denoised image after the second stage of the proposed method (PSNR=35.3016dB). We see that the visual quality is much improved after the second stage refinement.

6 Conclusion

In this paper, a two-stage adaptive algorithm has been proposed for denoising the images. To preserve the local image structures when denoising, we modelled a pixel and its nearest neighbours as a vector variable, and the denoising of the pixel was converted into the estimation of the variable from its noisy observations. The LPG-PCA technique was used such that an estimation and PCA transformation matrix was adaptively trained from the local window of the image. However, in a local window there can have very different structures from the underlying one; therefore, a training sample selection procedure is necessary. The block matching based local pixel grouping (LPG) was used for such a purpose and it guarantees that only the similar sample blocks to the given one are used in the PCA transform matrix estimation. The PCA transformation coefficients were then shrunk to remove noise. In LPG-PCA denoising procedure was iterated more than one time to improve the denoising performance. Our experimental results shows that LPG-PCA can effectively preserve the image fine structures while smoothing noise. It presents a competitive denoising algorithm compared with state-of-the-art denoising algorithms, such as BM3D.

***Thanks for authors of [27], used code and paper as reference.**

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