THERMO- DIFFUSION AND DIFFUSION - THERMO EFFECTS ON UNSTEADY MHD FLOW PAST AN ACCELERATED VERTICAL PLATE WITH VISCOUS DISSIPATION-FINITE ELEMENT STUDY

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Abstract
An investigation of the effects of thermo-diffusion and diffusion-thermo on an unsteady MHD natural convection flow with heat and mass transfer of an electricallyconducting, incompressible fluid past an accelerated vertical plate in the presence of viscous dissipation. Considered fluid is gray, absorbing-emitting radiation, but a non-scattering medium. The numerical solutions of the governing nonlinear partial differential equations are obtained by finite element method. The numerical values of fluid velocity, fluid temperature and species concentration are displayed graphically. The values of skin friction coefficient,
Nusselt number and Sherwood number for various values of pertinent flow parameters are presented through tables. We have shown that some results are in good agreement with earlier reported studies.

**Key Words:** Diffusion thermo, Thermo Diffusion, MHD, FEM, viscous dissipation.

1 INTRODUCTION

The study of the hydromagnetic flow of an electrically conducting fluid has many applications in science and engineering problems such as magnetohydrodynamic (MHD) generator, plasma studies, nuclear reactors, aerodynamic heating, etc. Elbasheshy [1] studied MHD heat and mass transfer problem along a vertical plate under the combined buoyancy effects on of thermal and spices diffusion. T. Arun kumar and L Anand Babu[2] has analyzed the study of Radiation effect of MHD flow past an impulsive started vertical plate with variable temperature and uniform mass diffusion. A finite element method. Chamkha and Khaled [3] formulated similarity solutions for hydromagnetic simultaneous heat and mass transfer by natural convection from an inclined plate with internal heat generation or absorption. D.Srinivasacharya and B. Mallikarjuna[4] studied Soret and dufour effects on mixed convection along a vertically away surface in porous medium with variable properties. J. Anand Rao and S. Shivaiah [5] analized chemical reaction effects on an unsteady MHD free convective flow past in infinite vertical plate with constant suction and heat source. Tsai and Huang [6] studied the heat and mass transfer for Soret and Dufour effects on Hiemenz flow through porous medium over a stretching surface. Kandasamy et al., [7] have studied the heat and mass transfer under a chemical reaction with a heat source. S.Siviah et al. [8] have studied finite element analysis of chemical reaction and radiation effects on isothermal vertical oscillating plate with variable mass diffusion. Exact solutions for MHD free convective boundary layer flow past a porous vertical surface in the presence of chemical reaction, thermal radiation and suction were carried out by Raju et al. [19]. P. Chandra Reddy et al.[10] have analyzed MHD convective double diffusive laminar boundary layer flow past an accelerated vertical plate using finite difference method.
The object of the present paper is to analyze the Soret and Dufour effects on MHD flow an electrically conducting, viscous and incompressible fluid past an infinite vertical porous plate in the presence of viscous dissipation. The coupled nonlinear governing equations along with boundary conditions are solved by finite element method and the results are presented through graphs.

2 MATHEMATICAL ANALYSIS

In this study, the problem under consideration is incompressible, viscous, radiating, heat absorbing, electrically conducting Newtonian fluid flow past an infinite vertical porous plate in the active presence of both diffusion thermo and thermo diffusion. Along the perpendicular direction to the plate, there is a magnetic field of uniform strength applied. Along the vertically upward direction of the plate \( X^c \) - axis is taken and perpendicular to the plate \( Y^c \) -axis is assumed. At time \( t^c \leq 0 \), the fluid is at rest and the plate is kept at the temperature higher than ambient temperature \( T_Y^c \). At time \( t^c > 0 \), the plate starts accelerating linearly in its own plane with time and the temperature decreases with temperature \( T = \frac{1}{(1+at)} \). Also species concentration starts decreasing linearly with time. An assumption is made that the effect of viscous dissipation is taken in account. Hence, with usual Boussineq’s and boundary layer approximation.

Fig 1 Physical flow of the problem
The flow model is as follows:

\[ \frac{\partial \psi}{\partial t} + u \frac{\partial \psi}{\partial x} + \frac{g\beta}{\rho} (T - T_0) \frac{\partial \psi}{\partial x} + \frac{g\beta}{\rho} (C - C_0) \frac{\partial \psi}{\partial x} \frac{A_B \delta}{\rho} \frac{u}{k} \frac{\partial \psi}{\partial t} \]

\[ \frac{\partial \psi}{\partial x} + u \frac{\partial \psi}{\partial x} + \frac{g\beta}{\rho} (T - T_0) \frac{\partial \psi}{\partial x} + \frac{g\beta}{\rho} (C - C_0) \frac{\partial \psi}{\partial x} \frac{A_B \delta}{\rho} \frac{u}{k} \frac{\partial \psi}{\partial t} \]

\[ \frac{\partial C}{\partial t} + D_4 \frac{\partial C}{\partial x} + D_6 \frac{\partial C}{\partial x} \]

With the initial and boundary conditions are as follows:

\[ r \in 0; u = 0, T = T_0, C = C_0 \text{ for all } y \in [0, 1] \]

\[ r \in 0; u = U_0, T = T_0 + \frac{(T_{\infty} - T_0)}{1 + \frac{At}{\tau}}, C = C_0 + \frac{(C_{\infty} - C_0)}{1 + \frac{At}{\tau}} \text{ at } y = 0 \]

\[ u \in [0, 1]; T = T_0, C = C_0 \text{ as } y \in [1, 1] \]

Where \( A = \frac{U_0^2}{u} \), the following non-dimensional quantities are defined as:

\[ t = \frac{u}{u_0}, \quad y = \frac{y}{u_0}, \quad u = \frac{u}{U_0}, \quad \text{Sc} = \frac{u}{D}, \quad \text{Gr} = \frac{g h u (T_{\infty} - T_0)}{U_0^2}, \quad \text{Grm} = \frac{g h u (C_{\infty} - C_0)}{U_0^2}, \quad \text{M} = \frac{g h u \delta}{r U_0^2}, \quad \text{Pr} = \frac{\nu c_p}{k}, \quad \text{Da} = \frac{D_4 \delta}{u_0 c_p (T_{\infty} - T_0)} \]

\[ \text{Ec} = \frac{u_0^2}{c_p (T_{\infty} - T_0)}, \quad \text{Sp} = \frac{D_4 (T_{\infty} - T_0)}{u (C_{\infty} - C_0)}, \quad K = \frac{K d \delta^3}{u^2} \]

With the non-dimensional quantities equations (1), (2) and (3) changes to the following dimensionless form:

\[ \frac{\partial \psi}{\partial t} + \frac{\partial \psi}{\partial x} + \frac{g\beta}{\rho} (T - T_0) \frac{\partial \psi}{\partial x} + \frac{g\beta}{\rho} (C - C_0) \frac{\partial \psi}{\partial x} \frac{A_B \delta}{\rho} \frac{u}{k} \frac{\partial \psi}{\partial t} \]

\[ \frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial x} + \frac{g\beta}{\rho} (T - T_0) \frac{\partial \psi}{\partial x} + \frac{g\beta}{\rho} (C - C_0) \frac{\partial \psi}{\partial x} \frac{A_B \delta}{\rho} \frac{u}{k} \frac{\partial \psi}{\partial t} \]

\[ \frac{\partial C}{\partial t} + D_4 \frac{\partial C}{\partial x} + D_6 \frac{\partial C}{\partial x} \]

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where Gr, Gm, Pr, M, Sc, So, Du, Ec and K are the thermal Grashof number, mass Grashof number, Prandtl number, Magnetic parameter, Schmidt number, Soret number, Dufour number, Eckert number and chemical reaction parameter respectively.

With the initial and boundary conditions in nondimensional form are:

\[ u = 0, q = \beta, C = 0, \text{ for all } y, r \leq 0 \]

\[ r > 0, u = \alpha, q = \frac{1}{\alpha - 1}, C = \frac{1}{1 + \gamma} \text{ as } y = 0 \]

\[ u = 0, q = 0, C = 0 \text{ as } y = \bar{y} \]

3 SOLUTION OF THE PROBLEM

The linear functional for (6) over the typical element \( Y_j \leq y \leq Y_k \) for the boundary value problem can be written as:

\[ \int_{y_j}^{y_k} \left[ \frac{\partial^m u}{\partial y^m} - \frac{\partial^m u}{\partial t^m} - \frac{\partial^m u}{\partial K} \right] \, dy = 0 \]

\[ \text{where,} \]

\[ P = G_r q + G_m C, \quad N = M + \frac{1}{K} \]

\[ \left\{ N^{(e)} \frac{\partial u^{(e)}}{\partial y} \right\} - \int_{y_j}^{y_k} N^{(e)} \frac{\partial u^{(e)}}{\partial t} \, dy + \int_{y_j}^{y_k} N^{(e)} \left[ \frac{\partial u^{(e)}}{\partial t} + Mu^{(e)} - P \right] \, dy = 0 \]

Omitting the first term in the above equation, we get

\[ \int_{y_j}^{y_k} \left\{ N^{(e)} \frac{\partial u^{(e)}}{\partial y} \right\} + N^{(e)} \left[ \frac{\partial u^{(e)}}{\partial t} + Nu^{(e)} - P \right] \, dy = 0 \]

Let \( u^{(e)} = N^{(e)} \phi^{(e)} \) be the finite element approximation solution \( ( Y_j \leq y \leq Y_k ) \) where,
are the basis functions. We obtain

\[ N^{(e)} = \begin{bmatrix} N_j & N_i \end{bmatrix}, \phi^{(e)} = \begin{bmatrix} u_j & u_k \end{bmatrix}^T, \quad N_j = \frac{y_k - y}{y_k - y_j}, N_i = \frac{y_j - y_i}{y_k - y_j} \]

are the basis functions. We obtain

\[
\frac{1}{l^{(e)}} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} + \frac{k^{(e)}}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} u_j \\ u_i \end{bmatrix} + \frac{N^{(e)}}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} = \frac{p_j^{(e)}}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}
\]

Where dot denote the differentiation with respect to and write the element equation for the elements \( Y_{i-1} \leq y \leq Y_i \) and \( Y_i \leq y \leq Y_{i+1} \) assemble the elements equations, we obtain

\[
\frac{1}{l^{(e)}} \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} u_{i+1} \\ u_i \\ u_{i-1} \end{bmatrix} + \frac{k^{(e)}}{6} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} u_{i+1} \\ u_i \\ u_{i-1} \end{bmatrix} - \frac{N^{(e)}}{6} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} u_{i+1} \\ u_i \\ u_{i-1} \end{bmatrix} = \frac{f_i}{2} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}
\]  

(11)

Now put row corresponding to the node to zero, from the above equation the difference scheme with \( l^{(e)} = h \) is

\[
\frac{1}{h} \left[ -u_{i+1} + 2u_i - u_{i-1} \right] + \frac{1}{6} \left[ u_{i+1} + 4u_i + u_{i-1} \right] + \frac{N}{6} \left[ u_{i+1} + 4u_i + u_{i-1} \right] = P
\]

Applying the trapezoidal rule, the following equations in Crank-Nicholson method are obtained:

\[
A_d \phi_1^{(e)} + A_d \phi_2^{(e)} + \phi_2^{(e)} = A_d \phi_1^{(e)} + A_d \phi_2^{(e)} + \phi_2^{(e)} + P
\]

(12)

Where,

\[
A_d = A_d = 2 + Nk - 6r, A_4 = A_4 = 2 - Nk + 6r, A_2 = 8 + 12r + 4Nk, A_2 = 8 - 12r - 4Nk
\]

\[
P = 12Pk = 12k(G_0, G_{10} + G_{10})
\]
Similarly applying the Galerkin finite element method for equations (7) and (8) the following equations are obtained:

\[
\begin{align*}
B_1 \beta_1^{(i+1)} + B_3 \beta_2^{(i+1)} + B_4 \beta_3^{(i+1)} - B_7 \beta_4^{(i+1)} + B_5 \beta_5^{(i+1)} + \beta'' &= 0 \\
C_1 \psi_1^{n+1} + C_2 \psi_2^{n+1} - C_3 \psi_3^{n+1} + C_4 \psi_4^{n+1} + \psi'' &= 0
\end{align*}
\]

Where,

\[
\begin{align*}
B_1 &= B_3 = \Pr - 3r, B_4 = B_6 = \Pr + 3r, B_2 = 4\Pr + 6r, B_5 = 4\Pr + 6r, \\
C_1 = C_2 = Sc - 3r, C_3 = C_4 = Sc + 3r, C_5 = 4Sc + 6r, C_6 = 4Sc - 6r
\end{align*}
\]

\[
P'' = 6Prk = 6kPr \left( \frac{\partial^2 C_1}{\partial y^2} + Ec \left( \frac{\partial u}{\partial y} \right)^2 \right), P''' = 6Prk = 6kSc \frac{\partial^3 T_1}{\partial y^3}
\]

Here \(h,k\) are the mesh sizes along \(y\)-direction and \(t\)-direction respectively. Index \(i\) refers to the space and \(n\) refers to the time. In equation (12), (13) and (14), taking \(i=1\to m\) and using (9), the following systems of equations are obtained:

\[
\begin{align*}
A_iX_i = B_i, \quad i = 1(1)m
\end{align*}
\]

Where \(A_i\)'s are the matrices of order \(m\) and \(X_iB_i\)'s column matrices having \(m\)-components. The solutions of the above system of equations are obtained by using Thomas algorithm for the velocity(\(u\)), temperature(\(\theta\)), concentration(\(c\)). Also numerical solutions are obtained by MATLAB-program. Computations are carried out until the steady state is reached. In order to prove the convergence of the Galerkin finite element method, the computations are carried out for slight changed values of \(h,k\) by running same MATLAB-program, no significant changes was observed in the values of velocity(\(u\)), temperature(\(\theta\)), concentration(\(C\)). Hence, the finite element method is stable and convergent.

The skin friction, Nusselt number and Sherwood number are important physical parameters for this type of boundary layer flow and are given by

\[
\tau = \left( \frac{\partial u}{\partial y} \right)_{y=0}, \quad Nu = -\left( \frac{\partial \theta}{\partial y} \right)_{y=0} \quad \text{and} \quad Sh = -\left( \frac{\partial C}{\partial y} \right)_{y=0}
\]
TABLE I Comparison of present skin friction ($\tau$) results with previous skin friction ($\tau^*$) results were obtained by P. Chandra Reddy et al.[10] and found to be in good agreement.

\[\begin{array}{|c|c|c|c|c|c|}
\hline
M & K & So & Du & \tau & \tau^* \\
\hline
5 & 0.5 & 0.1 & 0.1 & 0.1079 & 0.1084 \\
10 & 0. & 0.1 & 0.1 & 0.1097 & 0.1098 \\
15 & 0.5 & 0.1 & 0.1 & 0.1112 & 0.1112 \\
20 & 0.5 & 0.1 & 0.1 & 0.1124 & 0.1125 \\
5 & 0.2 & 0.1 & 0.1 & 0.1093 & 0.1093 \\
5 & 0.4 & 0.1 & 0.1 & 0.1086 & 0.1086 \\
5 & 0.6 & 0.1 & 0.1 & 0.1082 & 0.1083 \\
5 & 0.8 & 0.1 & 0.1 & 0.1081 & 0.1082 \\
5 & 0.5 & 1 & 0.1 & 0.0867 & 0.0866 \\
5 & 0. & 2 & 0.1 & 0.0828 & 0.0826 \\
5 & 0.5 & 3 & 0.1 & 0.0787 & 0.0786 \\
5 & 0.5 & 4 & 0.1 & 0.0746 & 0.0746 \\
5 & 0.5 & 0.1 & 1 & 0.0866 & 0.0866 \\
5 & 0. & 0.1 & 1.5 & 0.0846 & 0.0846 \\
5 & 0.5 & 0.1 & 2 & 0.0826 & 0.0826 \\
5 & 0.5 & 0.1 & 2.5 & 0.0806 & 0.0806 \\
\hline
\end{array}\]

TABLE II Variations in Nusselt Number

\[\begin{array}{|c|c|}
\hline
Du & Nu \\
\hline
0.2 & 13.5280 \\
0.4 & 13.1683 \\
0.6 & 12.8124 \\
0.8 & 12.4560 \\
\hline
\end{array}\]
TABLE III Variations in Sherwood number

<table>
<thead>
<tr>
<th>Sc</th>
<th>So</th>
<th>Sh</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.22</td>
<td>0.1</td>
<td>6.9882</td>
</tr>
<tr>
<td>0.60</td>
<td>0.1</td>
<td>10.0194</td>
</tr>
<tr>
<td>0.78</td>
<td>0.1</td>
<td>11.7981</td>
</tr>
<tr>
<td>0.96</td>
<td>0.1</td>
<td>13.0790</td>
</tr>
<tr>
<td>0.22</td>
<td>0.1</td>
<td>6.9880</td>
</tr>
<tr>
<td>0.22</td>
<td>0.2</td>
<td>6.8095</td>
</tr>
<tr>
<td>0.22</td>
<td>0.3</td>
<td>6.6311</td>
</tr>
<tr>
<td>0.22</td>
<td>0.4</td>
<td>6.4527</td>
</tr>
</tbody>
</table>

4 GRAPHS

Figure 2 Velocity profile for different values of $M$

Pr = 0.71; Gr = 6; Sc = 0.22; Ec = 0.001; t = 0.1
Du = 0.2; K = 0.5; Gm = 5; So = 0.2; a = 2
Figure 3 Velocity profile for different values of $Gr$

Figure 4 Velocity profile for different values of $Gm$
Figure 5 Velocity profile for different values of $K$

Figure 6 Velocity profile for different values of $Pr$
Figure 7 Velocity profile for different values of $S_0$

Figure 8 Velocity profile for different values of $D_u$
Figure 9 Velocity profile for different values Pr

Figure 10 Temperature profile for different values of So
Figure 11 Temperature profile for different values of $Du$

Figure 12 Concentration profile for different values of $Sc$
Figure 13 Temperature profile for different values of $S_o$

Figure 14 Concentration profile for different values of $D_u$
5 RESULTS AND DISCUSSION

The velocity profile, temperature and Concentration profile are illustrated in figures 1 to 16 for various parameters like, thermal-Grashof number (Gr), mass Grashof number (Gm), Prandtl number (Pr), Magnetic parameter (M), Schmidt number (Sc), Soret number (So), Dufour number (Du), Eckert number (Ec) and chemical reaction parameter (K) respectively.

Figures 2-8 exhibit the differences of the fluid velocity under the results of different parameters. Figure 2 depicts the effect of magnetic parameter on velocity profile. It is found that the velocity decreases with increasing values of magnetic parameter. It is known fact that the application of transverse magnetic field which is applied normal to the flow, results in a flow resistive force called the Lorentz force which acts in the opposite direction of the flow. This force has the effect of slowing the motion of the fluid. The influence of Grashof number and modified Grashof number on velocity is described in Figures 3 and 4. An observation is made that velocity increases on increase in either value which occurs due to the fact that fluid
velocity is enhanced with the buoyancy acting on the fluid particles due to gravitational forces.

From figure 5 illustrating the effect of porous permeability parameter on velocity, it can be made clear that increase in porosity parameter values induces velocity. To explain physically, gradual increase in the permeability of porous medium creates a rise in the flow of fluid through it. It can also be stated that when the holes of the porous medium become large, the acting resistance on the medium may be neglected.

From figure 6, velocity distribution and its effects by Prandtl number state that the velocity tends to decrease with an increase in Prandtl number. This can be clearly explained stating that the fluid of low Prandtl number has high thermal diffusivity thereby it acquires a higher temperature in steady state. This in turn creates more buoyancy force i.e. higher velocity of fluid as compared to that of high Prandtl numbered fluid.

Figure 7 shows the effect of Soret number on velocity boundary-layer stating that the velocity boundary layer thickness goes higher with an increase in the value of Soret number. Figure 8 shows the effect of Dufour number on velocity boundary layer that its thickness increases with an increase in the Dufour number. With the assumptions made in the problem, maximum temperature of the fluid is found at the plate surface and it tends to decrease exponentially away from the plate to reach the free stream zero value.

Temperature is affected by varying Prandtl number as shown in figure 9. Surface temperature is observed to reduce with an increase in Prandtl number. This occurs because reduced fluid velocity would mean heat is not convicted readily and hence surface temperature decreases.

Figures 10 and 11 describes the temperature affected by Soret number, it temperature increases with increasing values of Soret number and Dufour number.

Concentration is affected by Schmidt number; this is shown in Figure 12, describes the Schmidt number increase leads to a decrease trend in the concentration field. Alternatively, far away from the plate, such significance is not found to exist.

Figure 13 illustrate the variation of the concentration distribution with the effect of Soret number. It is noticed that the concentration boundary layer thickness increases with an increase in Soret
number. Figure 14 demonstrate the variation of the concentration profile with the Dufour number. It is seen that the concentration boundary layer thickness decreases with an increase in the Dufour number. Figure 15 shows that the temperature of the fluid increases with an increase in Eckert number.

6 CONCLUSION

The non-dimensional governing equations of the problem are solved by using finite element method. The variations in the velocity, temperature and concentration with the effects of various parameters encountered in the problem are studied through graphs. Also the effects some of the above parameters on skin friction, Nusselt number and Sherwood number are observed. The following are some of the notable conclusions:

- The fluid velocity increases with an increase of and.
- The fluid velocity reduces for increasing values of and.
- Temperature profiles decreases with an increase of , whereas it increases in the case of and.
- With an increase of values results in rising of the concentration, but it decreases under the influence of the and.
- Skin friction decreases for increasing values of both Soret and Dufour numbers.
- Nusselt number increases with increasing values of Dufour numbers.
- The rate of mass transfer is enhanced with increasing values of Schmidt number and decreasing values of Soret number.

References


