



Markovian Inventory System with Different Types of Service and Negative Customers

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Abstract

This paper analyze a retrial inventory system with Negative Customers under continuous review (s, Q) Policy. Our model considers the scenario having the customer choose two types of service. The arrival of customers constitutes a Poisson process and served exponentially. The lead time and retrial demand time are exponentially distributed. We derive the steady state behavior of this policy and various system performance measures. The cost analysis studied numerically.

Keywords: Continuous review inventory system, Positive lead time, Retrial customers, Negative Customers, Different types of service.

1 Introduction

Research on queueing systems with inventory control has captured much attention of researchers over the last decade. In this system, customers arrive at the service facility one by one and require service. In order to complete the customer service, an item from the inventory is needed. A served customer departs immediately from the system and the on - hand inventory decreases by one at the moment of service completion. This system is called a queueing inventory system [10]. Berman et al. [3] have considered an inventory management system at a service facility which uses one item of inventory for each service provided. Berman and Kim [4] addressed an infinite capacity queueing - inventory system with Poisson arrivals, exponential distributed lead times and service times. The authors identified a replenishment policy which maximized the system profit.

Artalejo et al. [2] were the first to study inventory policies with positive lead-time and retrial of customers who could not get service during their earlier attempts to access the service station. Falin and Artalejo [5] developed a finite source retrial customers. Krishnamoorthy and Jose [9] investigated a comparison of inventory models with positive service time and retrial of customers. Perishable inventory system with finite Population and retrial demands studied Sivakumar [11]. Ushakumari [12] considered a retrial inventory system with classical retrial policy. Jeganathan et al. [7, 8] discussed a retrial inventory system with priority customers and second optional services. Anbazhagan et al. [1] discussed stochastic inventory system with two types of services.

In this paper, we study the retrial inventory system for a continuous reviewed, single server system facing Markovian demand. The customers arriving to the service station are classified as ordinary and negative customers. The customer may choose two types of service. The joint probability distribution of the inventory level, the number of customer in the waiting hall and the number of customers in the orbit is obtained in the steady state case. Various system performance measures in the steady state are derived and the long-run total expected cost rate is calculated.

The remainder of this paper is organized as follows. In Section 2, we present the mathematical model and the notations. Analysis of the model and the steady state solution are given in Section 3. In Section 4, we derive various measures of system performance in steady state. The total expected cost rate is calculated and the results are illustrated numerically in Section 5. The last section is meant for conclusion.

2 Model description

We assume the following to describe the inventory model of our study:

- Continuous time retrial queueing-inventory system with maximum capacity of S units.
- As the (s, Q) replenishment policy, when the on-hand inventory level drops to a prefixed level, say $s(> 0)$ an order for $Q(= S - s > s + 1)$ units is placed. The lead time is exponentially distributed with the rate β .

- Customers arrive at the system one by one in according to a Poisson stream with arrival rate $\lambda(> 0)$.
- The probability that a customer is an ordinary is p and a negative is $q = (1 - p)$. We have assumed that the negative customer removes any one of the ordinary waiting customers from the system including the one at the service point.
- Waiting hall space is limited to accommodate a maximum number N , which includes the customer one who is receiving service.
- Each arriving customer may choose either type 1 service with probability p_1 or type 2 service with probability p_2 . The service time for both types following exponential distribution with rate $p_i\mu(> 0)$, $i = 1, 2$.
- A completion of type-1 service, one customer receiving service only and type-2 service, one customer receiving service and reduces the inventory by one item.
- The waiting hall is full, arriving primary customer enters into orbit of finite size M . When the waiting hall is empty, these orbiting customers compete for their service after an exponentially distributed time with parameter θ . We assume constant retrial policy for the retrial customer. The probability of a repeated attempt is independent of the number of orbiting customers (constant retrial policy).

Notations

$\mathbf{0}$	:	Zero matrix.
$\mathbf{I}$	:	Identity matrix.
$\mathbf{e}$	:	A column vector of 1's of appropriate dimension.
$[A]_{ij}$	:	Entry at (i, j) th position of a matrix A .
δ_{ij}	:	$\begin{cases} 1, & \text{if } i = j, \\ 0, & \text{otherwise.} \end{cases}$
$\bar{\delta}_{ij}$	:	$1 - \delta_{ij}$

3 Analysis

Let $X(t)$, $Y(t)$, and $L(t)$ denote the number of customers in the orbit, the number of customers in the waiting hall and the inventory level of the commodity at time t . From the assumptions made on the input and output processes, it can be shown that the quadruplet $\{(X(t), Y(t), L(t)), t \geq 0\}$ is

a continuous time Markov chain with state space given by E , where

$$E = \{(i, k, m) : i = 0, 1, 2, \dots, M, k = 0, 1, 2, \dots, N, m = 0, 1, 2, \dots, S\}$$

To determine the infinitesimal generator

$$A = (a((i, k, m), (j, l, n))), (i, k, m), (j, l, n) \in E$$

of this process we use the following arguments :

Transition due to the arrival of a primary customers:

* $(i, k, m) \rightarrow (i, k + 1, m)$: the rate is $p\lambda$, for $0 \leq i \leq M, 0 \leq k \leq N - 1, 0 \leq m \leq S$

* $(i, N, m) \rightarrow (i + 1, N, m)$: the rate is $p\lambda$, for $0 \leq i \leq M - 1, 0 \leq m \leq S$

Transition due to the arrival of a negative customers:

* $(i, k, m) \rightarrow (i, k - 1, m)$: the rate is $q\lambda$, for $0 \leq i \leq M, 1 \leq k \leq N, 0 \leq m \leq S$

Transitions due to type - 1 service completion in the system:

* $(i, k, m) \rightarrow (i, k - 1, m)$: the rate is $p_1\mu$, for $0 \leq i \leq M, 1 \leq k \leq N, 0 \leq m \leq S$

Transitions due to type - 2 service completion in the system:

* $(i, k, m) \rightarrow (i, k - 1, m - 1)$: the rate is $p_2\mu$, for $0 \leq i \leq M, 1 \leq k \leq N, 1 \leq m \leq S$

Transitions due to retrail request of an orbiting customer:

* $(i, k, m) \rightarrow (i - 1, k + 1, m)$: the rate is θ , for $1 \leq i \leq M, k = 0, 0 \leq m \leq S$

Transitions due to replenishment occurs:

* $(i, k, m) \rightarrow (i, k, m + Q)$: the rate is β , for $0 \leq i \leq M, 0 \leq k \leq N, 0 \leq m \leq s$.

* We observe that no transition other than the above is possible.

Denoting $q = ((q, 0, 0), (q, 0, 1), (q, 0, 2) \dots (q, 0, S), (q, 1, 0), (q, 1, 1), (q, 1, 2) \dots \dots (q, M, S))$ for $q = 0, 1, 2, \dots, M$. By ordering states lexicographically, the infinitesimal generator A can be conveniently expressed in a block partitioned matrix with entries

$$[A]_{ij} = \begin{cases} A_2, & j = i, & i = M, \\ A_1, & j = i, & i = 1, 2, \dots, M-1 \\ A_0, & j = i, & i = 0 \\ B, & j = i+1, & i = 0, 1, 2, \dots, M-1 \\ C, & j = i-1, & i = 1, 2, \dots, M \\ 0, & \text{otherwise} \end{cases}$$

Where

$$[A_0]_{ij} = \begin{cases} D_0, & n = m, & m = 0, \\ D_1, & n = m, & m = 1, 2, \dots, N \\ D_2, & n = m+1, & m = 0, 1, 2, \dots, N-1 \\ D_3, & n = m-1, & m = 1, 2, \dots, N \\ 0, & \text{otherwise} \end{cases}$$

$$[D_0]_{ij} = \begin{cases} -p\lambda, & j = i, & i = S, S-1, \dots, s+1 \\ -(p\lambda + \beta) & j = i, & i = s, s-1, \dots, 1, 0 \\ \beta, & j = i + Q, & i = s, s-1, \dots, 1, 0 \\ 0, & \text{otherwise} \end{cases}$$

$$[D_1]_{ij} = \begin{cases} -(\lambda + p_1\mu + p_2\mu), & j = i, & i = S, S-1, \dots, s+1 \\ -(\lambda + p_1\mu + p_2\mu + \beta), & j = i, & i = s, s-1, \dots, 1 \\ -(\lambda + \beta), & j = i, & i = 0 \\ \beta, & j = i + Q, & i = s, s-1, \dots, 1, 0 \\ 0, & \text{otherwise} \end{cases}$$

$$D_2 = p\lambda I_{S+1}$$

$$[D_3]_{ij} = \begin{cases} p_1\mu, & j = i, & i = S, S-1, \dots, 1 \\ p_2\mu, & j = i-1, & i = S, S-1, \dots, 1 \\ q\lambda, & j = i, & i = 0 \\ 0, & \text{otherwise} \end{cases}$$

$$[A_1]_{ij} = \begin{cases} E_0, & n = m, & m = 0, \\ D_1, & n = m, & m = 1, 2, \dots, N \\ D_2, & n = m+1, & m = 0, 1, 2, \dots, N-1 \\ D_3, & n = m-1, & m = 1, 2, \dots, N \\ 0, & \text{otherwise} \end{cases}$$

$$\begin{aligned}
[E_0]_{ij} &= \begin{cases} -(p\lambda + \theta), & j = i, & i = S, S-1, \dots, s+1 \\ -(p\lambda + \theta + \beta), & j = i, & i = s, s-1, \dots, 1, 0 \\ \beta, & j = i+Q, & i = s, s-1, \dots, 1, 0 \\ 0, & \text{otherwise} \end{cases} \\
[A_2]_{ij} &= \begin{cases} E_0, & n = m, & m = 0, \\ D_1, & n = m, & m = 1, 2, \dots, N-1 \\ D_2, & n = m+1, & m = 0, 1, 2, \dots, N-1 \\ D_3, & n = m-1, & m = 1, 2, \dots, N \\ F_1, & n = m, & m = N \\ 0, & \text{otherwise} \end{cases} \\
[F_1]_{ij} &= \begin{cases} -(q\lambda + p_1\mu + p_2\mu), & j = i, & i = S, S-1, \dots, s+1 \\ -(q\lambda + p_1\mu + p_2\mu + \beta), & j = i, & i = s, s-1, \dots, 1 \\ -(q\lambda + \beta), & j = i, & i = 0 \\ \beta, & j = i+Q, & i = s, s-1, \dots, 1, 0 \\ 0, & \text{otherwise} \end{cases} \\
[B]_{kl} &= \begin{cases} B_1, & n = m, & m = N \\ 0, & \text{otherwise} \end{cases} \\
[B_1] &= p\lambda I_{S+1}
\end{aligned}$$

$$[C]_{kl} = \begin{cases} C_1, & n = 1, \quad m = 0 \\ 0, & \text{otherwise} \end{cases}$$

$$[C_1] = \theta I_{S+1}$$

3.1 Steady State Analysis

It can be seen from the structure of A that the homogeneous Markov process $\{(X(t), Y(t), L(t)), t \geq 0\}$ on the finite state space E is irreducible. Hence the limiting distribution

$$\pi^{(i,j,k)} = \lim_{t \rightarrow \infty} Pr\{X(t) = i, Y(t) = j, L(t) = k | X(0), Y(0), L(0)\},$$

exists. Let

$$\mathbf{\Pi} = (\mathbf{\Pi}^{(0)}, \mathbf{\Pi}^{(1)}, \mathbf{\Pi}^{(2)}, \dots, \mathbf{\Pi}^{(M)})$$

we partition the vector $\mathbf{\Pi}^{(i)}$ as into follows:

$$\mathbf{\Pi}^{(i)} = (\pi^{(i,0)}, \pi^{(i,1)}, \dots, \pi^{(i,N)}), \quad i = 0, 1, 2, \dots, M$$

which is partitioned as follows:

$$\mathbf{\Pi}^{(i,j)} = (\pi^{(i,j,0)}, \pi^{(i,j,1)}, \dots, \pi^{(i,j,S)}), \quad i = 0, 1, 2, \dots, M, j = 0, 1, 2, \dots, N$$

Then the limiting probability $\mathbf{\Pi}$ satisfies

$$\mathbf{\Pi}A = 0, \quad \mathbf{\Pi}e = 1 \quad (1)$$

From the structure of A , it is a finite QBD matrix, therefore its steady state vector $\mathbf{\Pi}$ can be computed by using the following algorithm discribed by Gaver et al.[6]

Algorithm :

1. Determine recursively the matrices

$$F_0 = A_0,$$

$$F_i = A_1 + B(-F_{i-1}^{-1})C, \quad i = 1, 2, \dots, M-1,$$

$$F_M = A_2 + B(-F_{M-1}^{-1})C.$$

2. Compute recursively the vectors $\Pi^{(i)}$ using

$$\Pi^{(i)} = \Pi^{(i+1)}B(-F_i^{-1}), \quad i = 0, 1, 2, \dots, M-1$$

3. Solve the system of equations

$$\Pi^{(M)}F_M = 0 \text{ and } \sum_{i=0}^M \Pi^{(i)}e = 1.$$

From the system of equations $\Pi^{(M)}F_M = 0$, vector $\Pi^{(M)}$ could be determined uniquely, upto a multiplicative constant. This constant is decided by

$$\Pi^{(i)} = \Pi^{(i+1)}B(-F_i^{-1}), \quad i = 0, 1, 2, \dots, M-1 \text{ and } \sum_{i=0}^M \Pi^{(i)}e = 1.$$

4 System Performance Measures

In this section we will list a number of system performance measures.

4.1 Expected inventory level

Let ρ_I denote the mean inventory level in the steady state. Then

$$\rho_I = \sum_{i=0}^M \sum_{j=0}^N \sum_{k=1}^S k \left[\pi^{(i,1,k)} \right]$$

4.2 Expected reorder rate

Let ρ_R denote the expected reorder rate from the first supplier in the steady state. Then

$$\rho_R = \sum_{i=0}^M \sum_{j=1}^N p_1 \mu \left[\pi^{(i,j,s+1)} \right]$$

4.3 Expected number of demands in the waiting hall

Let ρ_W denote the expected number of demands in the waiting hall in the steady state. Then

$$\rho_W = \sum_{i=0}^M \sum_{j=1}^N \sum_{k=1}^S j \left[\pi^{(i,j,k)} \right]$$

4.4 Expected number of demands in the orbit

Let ρ_o denote the expected number of demands in the orbit in the steady state. Then

$$\rho_o = \sum_{i=1}^M \sum_{j=1}^N \sum_{k=0}^S i \left[\pi^{(i,j,k)} \right]$$

4.5 Expected blocking rate

Let ρ_B denote the expected number blocking rate in the steady state. Then

$$\rho_B = \sum_{i=0}^{M-1} \sum_{k=0}^S p\lambda \left[\pi^{(i,N,k)} \right]$$

4.6 Mean rate of arrivals of negative customers

Let ρ_{N_g} denote the expected rate of arrivals of negative customers in the steady state. Then

$$\rho_{N_g} = \sum_{i=0}^M \sum_{j=1}^N \sum_{k=0}^S q\lambda \left[\pi^{(i,j,k)} \right]$$

4.7 Overall rate of retrials

Let ρ_R denote the overall rate of retrials in the steady state. Then

$$\rho_R = \sum_{i=1}^M \sum_{j=0}^N \sum_{k=0}^S \theta \left[\pi^{(i,j,k)} \right]$$

4.8 The successful retrial rate

Let ρ_{SR} denote the successful retrial rate in the steady state. Then

$$\rho_{SR} = \sum_{i=1}^M \sum_{k=0}^S \theta \pi^{(i,0,k)}$$

4.9 The fraction of successful rate of retrial

Let ρ_{FSR} denote the fraction of successful retrial rate in the steady state. Then

$$\rho_{FSR} = \frac{\rho_{SR}}{\rho_R}$$

4.10 Cost Analysis

The long-run total expected cost per unit time for this system in the steady state is given by
where

c_h : the inventory holding cost per unit item per unit time

c_s : the inventory setup cost for per unit item per unit time

c_b : cost per blocking customer

c_w : Waiting cost of a customer in the waiting hall per unit time.

c_n : cost of lost per unit time to arrival of a negative customer.

c_o : Waiting cost of a customer in the orbit per unit time.

The long run total expected cost rate is given by

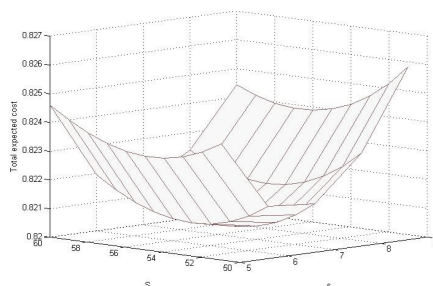
$$TC(s, S) = c_h \rho I + c_s \rho R + c_b \rho B + c_w \rho W + c_o \rho o + c_n \rho N_g$$

5 Numerical Examples

The simple numerical search procedures are used to find the local optimal values for any two of the decision variable(s, S) by consider in small set of integer values for this variable.

Table 1 presents the optimal value of the total expected cost rate for various combinations of S and s . We have assumed the constant values for other parameters and costs. Namely $N = 6, M = 4, \lambda = 4, \theta = 0.61, \beta = 0.5, p_1 = 0.8, p_2 = 0.2, \mu = 2.6, p = 0.6, q = 1 - p, c_h = 0.03, c_s = 23, c_b = 0.02, c_w = 0.4, c_o = 0.2, c_n = 0.02$.

The optimal value of the total expected cost rate is $TC^*(S^*, s^*, 6, 4) = 0.820365$ for the values of $S^* = 55$ and $s^* = 7$. The optimal cost for each S is shown in underlined and the optimal cost for each s is bold. Figure 1 refers the changes of S and s are how to affect the total expected cost rate.



$N = 6, M = 4, \lambda = 4, \theta = 0.61, \beta = 0.5, p_1 = 0.8, p_2 = 0.2, \mu = 2.6,$
 $p = 0.6, q = 1 - p, c_h = 0.03, c_s = 23, c_b = 0.02, c_w = 0.4, c_0 = 0.2, c_n = 0.02$

Figure 1: Convexity of the total cost for various combinations of s and S

Table 1: Total expected cost rate as a function of S and s
 $N = 6, M = 4, \lambda = 4, \theta = 0.61, \beta = 0.5, p_1 = 0.8, p_2 = 0.2, \mu = 2.6,$
 $p = 0.6, q = 1 - p, c_h = 0.03, c_s = 23, c_b = 0.02, c_w = 0.4, c_0 = 0.2, c_n = 0.02$

s	5	6	7	8	9
S					
51	0.824036	0.821630	<u>0.821508</u>	0.823054	0.825831
52	0.823653	0.821209	<u>0.821027</u>	0.822490	0.825165
53	0.823399	0.820921	<u>0.820682</u>	0.822068	0.824648
54	0.823267	0.820757	<u>0.820464</u>	0.821778	0.824270
55	0.823248	0.820709	<u>0.820365</u>	0.821612	0.824020
56	0.823335	0.820768	<u>0.820377</u>	0.821561	0.823892
57	0.823521	0.820928	<u>0.820494</u>	0.821617	0.823875
58	0.823801	0.821184	<u>0.820708</u>	0.821775	0.823965
59	0.824167	0.821528	<u>0.821014</u>	0.822028	0.824153
60	0.824616	0.821957	<u>0.821405</u>	0.822371	0.824434

6 Conclusion

In this paper, We discussed continuous review inventory system with Different Types of service. The customers are of two type:ordinary and negative. Various system performance measures are derived in the steady state. The results are illustrated with numerically. The model discussed here is useful in studying a different types of service for the (s, Q) ordering policy.

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