



THE MEMBERSHIP FUNCTION OF A N-TRIANGULAR SHAPE UNCERTAIN NUMBERS COMPLEMENTARY METHODS

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Abstract

Fuzzy numbers are based on membership function, such as triangle shape of triangular fuzzy number with two intervals. Crisp values of triangular fuzzy number is not sufficient when uncertainty arises in more than two interval such as the pentagonal, heptagonal, etc.,. In this paper using Dubios Prade left and right reference function is a distributive and complementary function respectively to find the generalization of the membership function of a n-triangular shape uncertain numbers in a very simple way. We illustrated with few examples also.

Key Words: Distributive function, Complementary function, triangle fuzzy number (TFN), n-triangular shape uncertain numbers (n-TSUN), triangular shape uncertain numbers (TSUN), pentagonal uncertain number (PUN), heptagonal uncertain number (HUN).

1 INTRODUCTION

Sometimes we cannot find the fuzzy membership function using cuts. For example we cannot find the fuzzy membership function for $\exp(x)$. Chou (2009) find the fuzzy membership function for triangular fuzzy number. Ritu parna (2010) find the another method of membership function of fuzzy number. Dubios Prade defined the membership function $X=(a_1, a_2, a_3)$ as follows

$$\mu_F(x) = \begin{cases} L(x) & \text{for } a_1 \leq x \leq a_2 \\ R(x) & \text{for } a_2 \leq x \leq a_3 \\ 0 & \text{for } x < a_1, x > a_3 \end{cases}$$

Where $L(x)$ is a continuous increasing function in $[a_1, a_2]$ and $R(x)$ is a continuous decreasing function in $[a_2, a_3]$ with $L(a_1)=R(a_3)=0$ and $L(a_2)=R(a_2)=1$. Also they denote $L(x)$ and $R(x)$ as left and right reference function of a fuzzy number. According to Debarra, a continuous increasing function of this type is also called a distribution function with reference to a Lebesgue-Stieltjes measure. Also Ritu parna linked the distribution and complementary function and fuzzy membership by means of super imposition of uniformly fuzzy set.

In this paper, we find the generalization of the membership function of a n-triangular shape uncertain numbers using distribution

and complementary function of continuous increasing and decreasing function such as pentagonal, heptagonal, etc., in a very simple way. Also we illustrated with few examples.

2 Preliminaries and Notations

Definition :

A fuzzy set $\tilde{T} \in \tilde{T}(R)$ is said to be a fuzzy number if it has the following definitions. (i) $\tilde{T}$ convex fuzzy set. (ii) $\tilde{T}$ Normal fuzzy set (iii) $\tilde{T}$ is piecewise continuous in membership function.

A. *n*-TRIANGULAR SHAPE UNCERTAIN NUMBERS[8]

Definition :

A fuzzy number $T_{n-\tilde{T}USN}$ is a n-triangular shape uncertain numbers denoted by $T_{n-\tilde{T}USN} = (t_1, t_2, t_3, t_4, \dots, t_{2n-2}, t_{2n-1})$, $n=2,3,4$, are real numbers and its membership function.

$$\mu_{\tilde{r}_{\text{ISW}}} = \left\{ \begin{array}{ll} \left(\frac{1}{n-1} \right) \left(\frac{x-t_1}{t_2-t_1} \right) & x \in [t_1, t_2] \\ \left(\frac{1}{n-1} \right) + \left(\frac{1}{n-1} \right) \left(\frac{x-t_2}{t_3-t_2} \right) & x \in [t_2, t_3] \\ \left(\frac{2}{n-1} \right) + \left(\frac{1}{n-1} \right) \left(\frac{x-t_3}{t_4-t_3} \right) & x \in [t_3, t_4] \\ \left(\frac{3}{n-1} \right) + \left(\frac{1}{n-1} \right) \left(\frac{x-t_4}{t_5-t_4} \right) & x \in [t_4, t_5] \\ \dots & \dots \\ \left(\frac{n-4}{n-1} \right) + \left(\frac{1}{n-1} \right) \left(\frac{x-t_{n-3}}{t_{n-2}-t_{n-3}} \right) & x \in [t_{n-3}, t_{n-2}] \\ \left(\frac{n-3}{n-1} \right) + \left(\frac{1}{n-1} \right) \left(\frac{x-t_{n-2}}{t_{n-1}-t_{n-2}} \right) & x \in [t_{n-2}, t_{n-1}] \\ \left(\frac{n-2}{n-1} \right) + \left(\frac{1}{n-1} \right) \left(\frac{x-t_{n-1}}{t_n-t_{n-1}} \right) & x \in [t_{n-1}, t_n] \\ 1 - \left(\frac{1}{n-1} \right) \left(\frac{x-t_n}{t_{n+1}-t_n} \right) & x \in [t_n, t_{n+1}] \\ \left(\frac{n-2}{n-1} \right) - \left(\frac{1}{n-1} \right) \left(\frac{x-t_{n+1}}{t_{n+2}-t_{n+1}} \right) & x \in [t_{n+1}, t_{n+2}] \\ \left(\frac{n-3}{n-1} \right) - \left(\frac{1}{n-1} \right) \left(\frac{t-t_{n+2}}{a_{n+3}-a_{n+2}} \right) & x \in [t_{n+2}, t_{n+3}] \\ \dots & \dots \\ \left(\frac{4}{n-1} \right) - \left(\frac{1}{n-1} \right) \left(\frac{x-t_{2n-5}}{t_{2n-4}-t_{2n-5}} \right) & x \in [t_{2n-5}, t_{2n-4}] \\ \left(\frac{3}{n-1} \right) - \left(\frac{1}{n-1} \right) \left(\frac{x-t_{2n-4}}{t_{2n-3}-t_{2n-4}} \right) & x \in [t_{2n-4}, t_{2n-3}] \\ \left(\frac{2}{n-1} \right) - \left(\frac{1}{n-1} \right) \left(\frac{x-t_{2n-3}}{t_{2n-2}-t_{2n-3}} \right) & x \in [t_{2n-3}, t_{2n-2}] \\ \left(\frac{1}{n-1} \right) \left(\frac{t_{2n-1}-x}{t_{2n-1}-t_{2n-2}} \right) & x \in [t_{2n-2}, t_{2n-1}] \\ 0 & \text{otherwise} \end{array} \right.$$

.....(2.1)

Let $T = (t_1, t_2, t_3, t_4, \dots, t_{2n-2}, t_{2n-1}), t_1, t_2, t_3, t_4, \dots, t_{2n-2}, t_{2n-1} \neq 0$,
 $n = 2, 3, 4, \dots$ be a TSUN. Let $F[T] = \{F(t_1), F(t_2), F(t_3), F(t_4), \dots, F(t_{2n-2}), F(t_{2n-1})\}$
 be the TSUN of any function $F(X)$. Suppose TSUN membership as
 in (2.1). The $L(x)$ is increasing from the left with distribution
 function and $R(x)$ is decreasing continuously from the right with
 complementary function respectively. Here we define the density
 function of $L(x)$ and $1-R(x)$ are $f_i(x)$, $i = 1, 2, 3, 4, \dots, n-1$ and $g_j(x)$,
 $j = 1, 2, 3, \dots, n-1$ respectively, that is $f_i(x) = \frac{d}{dx}(L_i(x))$ and $g_j(x) =$
 $\frac{d}{dx}(1 - R_j(x))$. Then we have

$$\left. \begin{aligned} f_1(x) &= \left(\frac{1}{n-1}\right) \left(\frac{1}{t_2 - t_1}\right) = \phi_{t_1}(y), x \in [t_1, t_2] \\ f_2(x) &= \left(\frac{1}{n-1}\right) \left(\frac{1}{t_3 - t_2}\right) = \phi_{t_2}(y), x \in [t_2, t_3], \dots, \\ f_{n-2}(x) &= \left(\frac{1}{n-1}\right) \left(\frac{1}{t_{n-1} - t_{n-2}}\right) = \phi_{t_{n-2}}(y), x \in [t_{n-2}, t_{n-1}] \\ f_{n-1}(x) &= \left(\frac{1}{n-1}\right) \left(\frac{1}{t_n - t_{n-1}}\right) = \phi_{t_{n-1}}(y), x \in [t_{n-1}, t_n] \end{aligned} \right\} \quad , \quad \left. \begin{aligned} g_1(x) &= \left(\frac{1}{n-1}\right) \left(\frac{1}{t_{2n-1} - t_{2n-2}}\right) = \phi_{t_{2n-1}}(y), x \in [t_{2n-2}, t_{2n-1}] \\ g_2(x) &= \left(\frac{1}{n-1}\right) \left(\frac{1}{t_{2n-3} - t_{2n-2}}\right) = \phi_{t_{2n-3}}(y), x \in [t_{2n-3}, t_{2n-2}], \dots, \\ g_{n-2}(x) &= \left(\frac{1}{n-1}\right) \left(\frac{1}{t_{n+2} - t_{n+1}}\right) = \phi_{t_{n+2}}(y), x \in [t_{n+1}, t_{n+2}] \\ g_{n-1}(x) &= \left(\frac{1}{n-1}\right) \left(\frac{1}{t_{n+1} - t_n}\right) = \phi_{t_{n+1}}(y), x \in [t_n, t_{n+1}] \end{aligned} \right\} \quad ,$$

be the density function of $f_1(x), f_2(x), \dots, f_{n-1}(x)$ and $g_{n-1}(x), g_{n-2}(x), \dots, g_1(x)$
 Let $Y = F(x)$ or $x = \phi Y$ or $\frac{d}{dx} = \frac{d}{dy}(\phi(Y))$. Replacing x by $\phi(Y)$ in
 $f_i(x)$ and $g_j(x)$. Then the membership function

$$\mu_F(x) = \begin{cases} \int_{F(t_1)}^x \varphi_{t_1}(y)m(y)dy & \text{for } F(t_1) \leq x \leq F(t_2) \\ \left(\frac{1}{n-1}\right) + \int_{F(t_2)}^x \varphi_{t_2}(y)m(y)dy & \text{for } F(t_2) \leq x \leq F(t_3) \\ \left(\frac{2}{n-1}\right) + \int_{F(t_3)}^x \varphi_{t_3}(y)m(y)dy & \text{for } F(t_3) \leq x \leq F(t_4) \\ \vdots & \vdots \\ \left(\frac{n-3}{n-1}\right) + \int_{F(t_{n-2})}^x \varphi_{t_{n-2}}(y)m(y)dy & \text{for } F(t_{n-2}) \leq x \leq F(t_{n-1}) \\ \left(\frac{n-2}{n-1}\right) + \int_{F(t_{n-1})}^x \varphi_{t_{n-1}}(y)m(y)dy & \text{for } F(t_{n-1}) \leq x \leq F(t_n) \\ \left(\frac{n-2}{n-1}\right) + \left(\frac{1}{n-1}\right) - \int_{F(t_n)}^x \varphi_{t_{n+1}}(y)m(y)dy & \text{for } F(t_n) \leq x \leq F(t_{n+1}) \\ \left(\frac{n-3}{n-1}\right) + \left(\frac{1}{n-1}\right) - \int_{F(t_{n+1})}^x \varphi_{t_{n+2}}(y)m(y)dy & \text{for } F(t_{n+1}) \leq x \leq F(t_{n+2}) \\ \vdots & \vdots \\ \left(\frac{2}{n-1}\right) + \left(\frac{1}{n-1}\right) - \int_{F(t_{2n-4})}^x \varphi_{t_{2n-3}}(y)m(y)dy & \text{for } F(t_{2n-4}) \leq x \leq F(t_{2n-3}) \\ \left(\frac{1}{n-1}\right) + \left(\frac{1}{n-1}\right) - \int_{F(t_{2n-3})}^x \varphi_{t_{2n-2}}(y)m(y)dy & \text{for } F(t_{2n-3}) \leq x \leq F(t_{2n-2}) \\ \left(\frac{1}{n-1}\right) - \int_{F(t_{2n-2})}^x \varphi_{t_{2n-1}}(y)m(y)dy & \text{for } F(t_{2n-2}) \leq x \leq F(t_{2n-1}) \\ 0 & \text{for } \text{otherwise} \end{cases}$$

.....(2.2)

We now progress to express the conclusion with a few examples.

3 The membership function of r^{th} root triangle shape uncertain numbers.

Let TSUN be $T = (t_1, t_2, t_3, \dots, t_{2n-2}, t_{2n-1})$, where $t_1, t_2, t_3, \dots, t_{2n-2}, t_{2n-1} > 0$ & $n=2,3,4,\dots$ its membership function is given in (2.1). Let the fuzzy m^{th} root of the triangle shape uncertain number be $\sqrt[m]{T} = (\sqrt[m]{t_1}, \sqrt[m]{t_2}, \sqrt[m]{t_3}, \dots, \sqrt[m]{t_{2n-2}}, \sqrt[m]{t_{2n-1}})$. Suppose $Y = \sqrt[r]{x}$ then $x = Y^r$. Also we have $\frac{dx}{dy} = ry^{r-1} \rightarrow dx = ry^{r-1}dy$.

Using (2.2), its membership function

$$\mu_{\exp}(x) = \begin{cases} \int_{\sqrt{x}}^t \varphi_{t_1}(y) y^{n-1} dy & x \in [\sqrt{t_1}, \sqrt{t_2}] \\ \left(\frac{1}{n-1}\right) + \int_{\sqrt{x}}^t \varphi_{t_2}(y) y^{n-1} dy & x \in [\sqrt{t_2}, \sqrt{t_3}] \\ \left(\frac{2}{n-1}\right) + \int_{\sqrt{x}}^t \varphi_{t_3}(y) y^{n-1} dy & x \in [\sqrt{t_3}, \sqrt{t_4}] \\ \dots & \dots \\ \left(\frac{n-3}{n-1}\right) + \int_{\sqrt{x}}^t \varphi_{t_{2n-2}}(y) y^{n-1} dy & x \in [\sqrt{t_{2n-2}}, \sqrt{t_{2n-1}}] \\ \left(\frac{n-2}{n-1}\right) + \int_{\sqrt{x}}^t \varphi_{t_{2n-1}}(y) y^{n-1} dy & x \in [\sqrt{t_{2n-1}}, \sqrt{t_{2n+1}}] \\ \left(\frac{n-2}{n-1}\right) + \left(\frac{1}{n-1}\right) - \int_{\sqrt{x}}^t \varphi_{t_{2n+1}}(y) y^{n-1} dy & x \in [\sqrt{t_{2n+1}}, \sqrt{t_{2n+2}}] \\ \left(\frac{n-3}{n-1}\right) + \left(\frac{1}{n-1}\right) - \int_{\sqrt{x}}^t \varphi_{t_{2n+2}}(y) y^{n-1} dy & x \in [\sqrt{t_{2n+2}}, \sqrt{t_{2n+3}}] \\ \left(\frac{2}{n-1}\right) + \left(\frac{1}{n-1}\right) - \int_{\sqrt{x}}^t \varphi_{t_{2n+3}}(y) y^{n-1} dy & x \in [\sqrt{t_{2n+3}}, \sqrt{t_{2n+4}}] \\ \left(\frac{1}{n-1}\right) + \left(\frac{1}{n-1}\right) - \int_{\sqrt{x}}^t \varphi_{t_{2n+4}}(y) y^{n-1} dy & x \in [\sqrt{t_{2n+4}}, \sqrt{t_{2n+5}}] \\ \left(\frac{1}{n-1}\right) - \int_{\sqrt{x}}^t \varphi_{t_{2n+5}}(y) y^{n-1} dy & x \in [\sqrt{t_{2n+5}}, \sqrt{t_{2n+6}}] \\ 0 & \text{otherwise} \end{cases} = \begin{cases} \left(\frac{1}{n-1}\right) \left(\frac{x' - t_1}{t_2 - t_1}\right) & x \in [\sqrt{t_1}, \sqrt{t_2}] \\ \left(\frac{1}{n-1}\right) + \left(\frac{1}{n-1}\right) \left(\frac{x' - t_2}{t_3 - t_2}\right) & x \in [\sqrt{t_2}, \sqrt{t_3}] \\ \left(\frac{2}{n-1}\right) + \left(\frac{1}{n-1}\right) \left(\frac{x' - t_3}{t_4 - t_3}\right) & x \in [\sqrt{t_3}, \sqrt{t_4}] \\ \dots & \dots \\ \left(\frac{n-3}{n-1}\right) + \left(\frac{1}{n-1}\right) \left(\frac{x' - t_{2n-2}}{t_{2n-1} - t_{2n-2}}\right) & x \in [\sqrt{t_{2n-2}}, \sqrt{t_{2n-1}}] \\ \left(\frac{n-2}{n-1}\right) + \left(\frac{1}{n-1}\right) \left(\frac{x' - t_{2n-1}}{t_{2n+1} - t_{2n-1}}\right) & x \in [\sqrt{t_{2n-1}}, \sqrt{t_{2n+1}}] \\ 1 - \left(\frac{1}{n-1}\right) \left(\frac{x' - t_{2n+1}}{t_{2n+2} - t_{2n+1}}\right) & x \in [\sqrt{t_{2n+1}}, \sqrt{t_{2n+2}}] \\ \left(\frac{n-2}{n-1}\right) - \left(\frac{1}{n-1}\right) \left(\frac{x' - t_{2n+2}}{t_{2n+3} - t_{2n+2}}\right) & x \in [\sqrt{t_{2n+2}}, \sqrt{t_{2n+3}}] \\ \dots & \dots \\ \left(\frac{3}{n-1}\right) - \left(\frac{1}{n-1}\right) \left(\frac{x' - t_{2n+4}}{t_{2n+5} - t_{2n+4}}\right) & x \in [\sqrt{t_{2n+4}}, \sqrt{t_{2n+5}}] \\ \left(\frac{2}{n-1}\right) - \left(\frac{1}{n-1}\right) \left(\frac{x' - t_{2n+5}}{t_{2n+6} - t_{2n+5}}\right) & x \in [\sqrt{t_{2n+5}}, \sqrt{t_{2n+6}}] \\ \left(\frac{1}{n-1}\right) \left(\frac{t_{2n+6} - x'}{t_{2n+6} - t_{2n+5}}\right) & x \in [\sqrt{t_{2n+6}}, \sqrt{t_{2n+7}}] \\ 0 & \text{otherwise} \end{cases}$$

.....(3.1)

4 The membership function of $\exp(x)$ of triangle shape uncertain numbers

Let TSUN be $T = (t_1, t_2, t_3, \dots, t_{2n-2}, t_{2n-1})$, where $t_1, t_2, t_3, \dots, t_{2n-2}, t_{2n-1} > 0$ & $n=2, 3, 4, \dots$ its membership function is given in (2.1). Let the fuzzy $\exp(x)$ of the triangle shape uncertain number be $e^{(T)} = (e^{(t_1)}, e^{(t_2)}, e^{(t_3)}, e^{(t_4)}, \dots, e^{(t_{2n-2})}, e^{(t_{2n-1})})$. Suppose $Y = e^{(x)}$ then $x = \ln(y)$. Also we have $\frac{dx}{dy} = \frac{1}{y} \rightarrow dx = \frac{1}{y} dy$. Using (2.2), its membership function

$$\mu_{x'}(x) = \begin{cases} \int_{\theta^{(1)}}^x \varphi_{z_1}(y) \left(\frac{1}{y}\right) dy & x \in [\theta^{(1)}, \theta^{(2)}] \\ \left(\frac{1}{n-1}\right) + \int_{\theta^{(2)}}^x \varphi_{z_2}(y) \left(\frac{1}{y}\right) dy & x \in [\theta^{(2)}, \theta^{(3)}] \\ \dots & \dots \\ \left(\frac{n-3}{n-1}\right) + \int_{\theta^{(n-2)}}^x \varphi_{z_{n-2}}(y) \left(\frac{1}{y}\right) dy & x \in [\theta^{(n-2)}, \theta^{(n-1)}] \\ \left(\frac{n-2}{n-1}\right) + \int_{\theta^{(n-1)}}^x \varphi_{z_{n-1}}(y) \left(\frac{1}{y}\right) dy & x \in [\theta^{(n-1)}, \theta^{(n)}] \\ \left(\frac{n-2}{n-1}\right) + \left(\frac{1}{n-1}\right) - \int_{\theta^{(n)}}^x \varphi_{z_{n+1}}(y) \left(\frac{1}{y}\right) dy & x \in [\theta^{(n)}, \theta^{(n+1)}] \\ \left(\frac{n-3}{n-1}\right) + \left(\frac{1}{n-1}\right) - \int_{\theta^{(n+1)}}^x \varphi_{z_{n+2}}(y) \left(\frac{1}{y}\right) dy & x \in [\theta^{(n+1)}, \theta^{(n+2)}] \\ \dots & \dots \\ \left(\frac{1}{n-1}\right) + \left(\frac{1}{n-1}\right) - \int_{\theta^{(2n-1)}}^x \varphi_{z_{2n-1}}(y) \left(\frac{1}{y}\right) dy & x \in [\theta^{(2n-1)}, \theta^{(2n)}] \\ \left(\frac{1}{n-1}\right) - \int_{\theta^{(2n)}}^x \varphi_{z_{2n}}(y) \left(\frac{1}{y}\right) dy & x \in [\theta^{(2n)}, \theta^{(2n+1)}] \\ 0 & \text{otherwise} \end{cases} = \begin{cases} \left(\frac{1}{n-1}\right) \left(\frac{In(x) - t_1}{t_2 - t_1}\right) & x \in [\theta^{(1)}, \theta^{(2)}] \\ \left(\frac{1}{n-1}\right) + \left(\frac{1}{n-1}\right) \left(\frac{In(x) - t_2}{t_3 - t_2}\right) & x \in [\theta^{(2)}, \theta^{(3)}] \\ \dots & \dots \\ \left(\frac{n-3}{n-1}\right) + \left(\frac{1}{n-1}\right) \left(\frac{In(x) - t_{n-2}}{t_{n-1} - t_{n-2}}\right) & x \in [\theta^{(n-2)}, \theta^{(n-1)}] \\ \left(\frac{n-2}{n-1}\right) + \left(\frac{1}{n-1}\right) \left(\frac{In(x) - t_{n-1}}{t_n - t_{n-1}}\right) & x \in [\theta^{(n-1)}, \theta^{(n)}] \\ 1 - \left(\frac{1}{n-1}\right) \left(\frac{In(x) - t_n}{t_{n+1} - t_n}\right) & x \in [\theta^{(n)}, \theta^{(n+1)}] \\ \left(\frac{n-2}{n-1}\right) - \left(\frac{1}{n-1}\right) \left(\frac{In(x) - t_{n+1}}{t_{n+2} - t_{n+1}}\right) & x \in [\theta^{(n+1)}, \theta^{(n+2)}] \\ \dots & \dots \\ \left(\frac{2}{n-1}\right) - \left(\frac{1}{n-1}\right) \left(\frac{In(x) - t_{2n-2}}{t_{2n-1} - t_{2n-2}}\right) & x \in [\theta^{(2n-1)}, \theta^{(2n)}] \\ \left(\frac{1}{n-1}\right) - \left(\frac{t_{2n+1} - In(x)}{t_{2n+1} - t_{2n+2}}\right) & x \in [\theta^{(2n)}, \theta^{(2n+1)}] \\ 0 & \text{otherwise} \end{cases}$$

.....(4.1)

5 The membership function of In(x) triangle shape uncertain numbers

Let TSUN be $T = (t_1, t_2, t_3, \dots, t_{2n-2}, t_{2n-1})$, where $t_1, t_2, t_3, \dots, t_{2n-2}, t_{2n-1} > 0$ & $n=2, 3, 4, \dots$ its membership function is given in (2.1). Let the fuzzy In(x) of the triangle shape uncertain number be $In(T) = (In(t_1), In(t_2), In(t_3), \dots, In(t_{2n-2}), In(t_{2n-1}))$. Suppose $Y = In(x)$ then $x = {}^{(y)}$. Also we have $\frac{dx}{dy} = e^{(y)} \rightarrow dx = e^{(y)} dy$.

Using (2.2), its membership function

$$\mu_{h(T)}(x) = \begin{cases} \int_{h(t_1)}^x \varphi_{t_1}(y) e^{(y)} dy & x \in [h(t_1), h(t_2)] \\ \left(\frac{1}{n-1}\right) + \int_{h(t_2)}^x \varphi_{t_2}(y) e^{(y)} dy & x \in [h(t_2), h(t_3)] \\ \left(\frac{2}{n-1}\right) + \int_{h(t_3)}^x \varphi_{t_3}(y) e^{(y)} dy & x \in [h(t_3), h(t_4)] \\ \dots & \dots \\ \left(\frac{n-2}{n-1}\right) + \int_{h(t_{n-2})}^x \varphi_{t_{n-2}}(y) e^{(y)} dy & x \in [h(t_{n-2}), h(t_{n-1})] \\ \left(\frac{n-2}{n-1}\right) + \left(\frac{1}{n-1}\right) - \int_{h(t_{n-1})}^x \varphi_{t_{n-1}}(y) e^{(y)} dy & x \in [h(t_{n-1}), h(t_{n+1})] \\ \left(\frac{n-3}{n-1}\right) + \left(\frac{1}{n-1}\right) - \int_{h(t_{n+1})}^x \varphi_{t_{n+1}}(y) e^{(y)} dy & x \in [h(t_{n+1}), h(t_{n+2})] \\ \dots & \dots \\ \left(\frac{1}{n-1}\right) + \left(\frac{1}{n-1}\right) - \int_{h(t_{n+2})}^x \varphi_{t_{n+2}}(y) e^{(y)} dy & x \in [h(t_{n+2}), h(t_{n+3})] \\ \left(\frac{1}{n-1}\right) - \int_{h(t_{n+3})}^x \varphi_{t_{n+3}}(y) e^{(y)} dy & x \in [h(t_{n+3}), h(t_{n+4})] \\ \text{otherwise} & \text{otherwise} \end{cases} = \begin{cases} \left(\frac{1}{n-1}\right) \left(\frac{e^{(x)} - t_1}{t_2 - t_1}\right) & x \in [h(t_1), h(t_2)] \\ \left(\frac{1}{n-1}\right) + \left(\frac{1}{n-1}\right) \left(\frac{e^{(x)} - t_2}{t_3 - t_2}\right) & x \in [h(t_2), h(t_3)] \\ \left(\frac{2}{n-1}\right) + \left(\frac{1}{n-1}\right) \left(\frac{e^{(x)} - t_3}{t_4 - t_3}\right) & x \in [h(t_3), h(t_4)] \\ \dots & \dots \\ \left(\frac{n-2}{n-1}\right) + \left(\frac{1}{n-1}\right) \left(\frac{e^{(x)} - t_{n-2}}{t_n - t_{n-2}}\right) & x \in [h(t_{n-2}), h(t_{n-1})] \\ 1 - \left(\frac{1}{n-1}\right) \left(\frac{e^{(x)} - t_n}{t_{n+1} - t_n}\right) & x \in [h(t_{n-1}), h(t_{n+1})] \\ \left(\frac{n-2}{n-1}\right) - \left(\frac{1}{n-1}\right) \left(\frac{e^{(x)} - t_{n+1}}{t_{n+2} - t_{n+1}}\right) & x \in [h(t_{n+1}), h(t_{n+2})] \\ \dots & \dots \\ \left(\frac{2}{n-1}\right) - \left(\frac{1}{n-1}\right) \left(\frac{e^{(x)} - t_{n+2}}{t_{n+3} - t_{n+2}}\right) & x \in [h(t_{n+2}), h(t_{n+3})] \\ \left(\frac{1}{n-1}\right) - \left(\frac{1}{n-1}\right) \left(\frac{t_{n+3} - e^{(x)}}{t_{n+4} - t_{n+3}}\right) & x \in [h(t_{n+3}), h(t_{n+4})] \\ \text{otherwise} & \text{otherwise} \end{cases}$$

.....(5.1)

6 .The membership function of inverse triangle shape uncertain numbers

Let TSUN be $T = (t_1, t_2, t_3, \dots, t_{2n-2}, t_{2n-1})$, where $t_1, t_2, t_3, \dots, t_{2n-2}, t_{2n-1} > 0$ & $n=2,3,4,\dots$ its membership function is given in (2.1). Let the fuzzy x^{-1} of the triangle shape uncertain number be $(T)^{-1} = (t_{2n-1}^{-1}, t_{2n-2}^{-1}, \dots, t_3^{-1}, t_2^{-1}, t_1^{-1})$. Suppose $Y=x^{-1}$ then $x = y^{-1}$. Also we have $\frac{dx}{dy} = -Y^{-2} \rightarrow dx = Y^{-2}dy$.

Using (2.2), its membership function

$$\mu_{T_n}(x) = \begin{cases} \left(\frac{n-1}{n-1} \right) \int_{(t_{2n-1})^+} \varphi_{t_{2n-1}}(y) \left(\frac{-dy}{y^2} \right) & x \in \left[\frac{1}{t_{2n-1}}, \frac{1}{t_{2n-2}} \right] \\ \left(\frac{1}{n-1} \right) - \int_{(t_{2n-1})^+} \varphi_{t_{2n-1}}(y) \left(\frac{-dy}{y^2} \right) & x \in \left[\frac{1}{t_{2n-2}}, \frac{1}{t_{2n-3}} \right] \\ \left(\frac{2}{n-1} \right) - \int_{(t_{2n-1})^+} \varphi_{t_{2n-1}}(y) \left(\frac{-dy}{y^2} \right) & x \in \left[\frac{1}{t_{2n-3}}, \frac{1}{t_{2n-4}} \right] \\ \vdots & \vdots \\ \left(\frac{n-3}{n-1} \right) - \int_{(t_{2n-1})^+} \varphi_{t_{2n-1}}(y) \left(\frac{-dy}{y^2} \right) & x \in \left[\frac{1}{t_{2n-4}}, \frac{1}{t_{2n-5}} \right] \\ \left(\frac{n-2}{n-1} \right) - \int_{(t_{2n-1})^+} \varphi_{t_{2n-1}}(y) \left(\frac{-dy}{y^2} \right) & x \in \left[\frac{1}{t_{2n-5}}, \frac{1}{t_{2n-6}} \right] \\ \left(\frac{n-2}{n-1} \right) + \left(\frac{1}{n-1} \right) \int_{(t_{2n-1})^+} \varphi_{t_{2n-1}}(y) \left(\frac{-dy}{y^2} \right) & x \in \left[\frac{1}{t_{2n-6}}, \frac{1}{t_{2n-7}} \right] \\ \left(\frac{n-3}{n-1} \right) + \left(\frac{1}{n-1} \right) \int_{(t_{2n-1})^+} \varphi_{t_{2n-1}}(y) \left(\frac{-dy}{y^2} \right) & x \in \left[\frac{1}{t_{2n-7}}, \frac{1}{t_{2n-8}} \right] \\ \left(\frac{n-4}{n-1} \right) + \left(\frac{1}{n-1} \right) \int_{(t_{2n-1})^+} \varphi_{t_{2n-1}}(y) \left(\frac{-dy}{y^2} \right) & x \in \left[\frac{1}{t_{2n-8}}, \frac{1}{t_{2n-9}} \right] \\ \vdots & \vdots \\ \left(\frac{2}{n-1} \right) + \left(\frac{1}{n-1} \right) \int_{(t_{2n-1})^+} \varphi_{t_{2n-1}}(y) \left(\frac{-dy}{y^2} \right) & x \in \left[\frac{1}{t_{2n-9}}, \frac{1}{t_{2n-10}} \right] \\ \left(\frac{1}{n-1} \right) + \left(\frac{1}{n-1} \right) \int_{(t_{2n-1})^+} \varphi_{t_{2n-1}}(y) \left(\frac{-dy}{y^2} \right) & x \in \left[\frac{1}{t_{2n-10}}, \frac{1}{t_{2n-11}} \right] \\ \left(\frac{1}{n-1} \right) + \int_{(t_{2n-1})^+} \varphi_{t_{2n-1}}(y) \left(\frac{-dy}{y^2} \right) & x \in \left[\frac{1}{t_{2n-11}}, \frac{1}{t_{2n-12}} \right] \\ 0 & \text{otherwise} \end{cases} = \begin{cases} \left(\frac{1}{n-1} \right) \left(\frac{1 - x t_{2n-1}}{(t_{2n-2} - t_{2n-1})x} \right) & x \in \left[\frac{1}{t_{2n-1}}, \frac{1}{t_{2n-2}} \right] \\ \left(\frac{1}{n-1} \right) + \left(\frac{1}{n-1} \right) \left(\frac{1 - x t_{2n-2}}{(t_{2n-3} - t_{2n-2})x} \right) & x \in \left[\frac{1}{t_{2n-2}}, \frac{1}{t_{2n-3}} \right] \\ \left(\frac{2}{n-1} \right) + \left(\frac{1}{n-1} \right) \left(\frac{1 - x t_{2n-3}}{(t_{2n-4} - t_{2n-3})x} \right) & x \in \left[\frac{1}{t_{2n-3}}, \frac{1}{t_{2n-4}} \right] \\ \vdots & \vdots \\ \left(\frac{n-3}{n-1} \right) + \left(\frac{1}{n-1} \right) \left(\frac{1 - x t_{2n-4}}{(t_{2n-5} - t_{2n-4})x} \right) & x \in \left[\frac{1}{t_{2n-4}}, \frac{1}{t_{2n-5}} \right] \\ \left(\frac{n-2}{n-1} \right) + \left(\frac{1}{n-1} \right) \left(\frac{1 - x t_{2n-5}}{(t_{2n-6} - t_{2n-5})x} \right) & x \in \left[\frac{1}{t_{2n-5}}, \frac{1}{t_{2n-6}} \right] \\ \left(\frac{n-2}{n-1} \right) + \left(\frac{1}{n-1} \right) \left(\frac{1 - x t_{2n-6}}{(t_{2n-7} - t_{2n-6})x} \right) & x \in \left[\frac{1}{t_{2n-6}}, \frac{1}{t_{2n-7}} \right] \\ \left(\frac{n-3}{n-1} \right) + \left(\frac{1}{n-1} \right) \left(\frac{1 - x t_{2n-7}}{(t_{2n-8} - t_{2n-7})x} \right) & x \in \left[\frac{1}{t_{2n-7}}, \frac{1}{t_{2n-8}} \right] \\ \vdots & \vdots \\ \left(\frac{2}{n-1} \right) + \left(\frac{1}{n-1} \right) \left(\frac{1 - x t_{2n-9}}{(t_{2n-10} - t_{2n-9})x} \right) & x \in \left[\frac{1}{t_{2n-9}}, \frac{1}{t_{2n-10}} \right] \\ \left(\frac{1}{n-1} \right) + \left(\frac{1}{n-1} \right) \left(\frac{1 - x t_{2n-10}}{(t_{2n-11} - t_{2n-10})x} \right) & x \in \left[\frac{1}{t_{2n-10}}, \frac{1}{t_{2n-11}} \right] \\ \left(\frac{1}{n-1} \right) \left(\frac{1 - x t_{2n-11}}{(t_{2n-12} - t_{2n-11})x} \right) & x \in \left[\frac{1}{t_{2n-11}}, \frac{1}{t_{2n-12}} \right] \\ 0 & \text{otherwise} \end{cases} \quad \dots (6.2)$$

.....(6.1)

Note: The triangular fuzzy numbers and n- triangular shape uncertain numbers are same for the points have the equal intervals. The triangular fuzzy numbers and n- triangular shape uncertain numbers are distinct for the points have the unequal intervals. In this paper we discussed only for the distinct points have the unequal intervals. Suppose $m = 2, 3, 4, \dots, n$ then we will get 3, 5, 7, 9,, (2m- 1) number of different points namely triangular, pentagonal heptagonal,,m-gonal triangular shape uncertain numbers. Suppose (2n-1) -number of different points has the (2n-1) different triangular shape uncertain numbers. Then this reduced to 3, 5, 7, 9,, (2m -5),(2m-3) triangular shape uncertain numbers. The reduced triangular shape uncertain numbers have the (2m-1)Cr where $r = 1, 3, \dots, (2m -5)$ numbers of triangular shape uncertain numbers. i.e.) reduced triangular shape uncertain numbers is triangular, pentagonal, . . . ,(2m-3)-gonal. These are all applicable for all the above numbers.

7 CONCLUSION

By using Dubois-Prade left and right reference function , we find the generalization of the membership function of n-Triangular shape

uncertain number by means of super imposition of uniformly fuzzy set and also its satisfies the standard fuzzy operations. The illustrated with few examples also.

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